Accessing x-dependent GPDs from lattice QCD

Martha Constantinou

Temple University

DNP Fall Meeting 2023
APS & JPS

Invited Workshop:
3D Hadron Structure from Next-Generation Scattering Experiment & Lattice QCD

November 27, 2023

Outline

- ★ Approaches to access information on GPDs from lattice QCD
- ★ Definition of light-cone GPDs vs Euclidean lattice definition (quasi GPDs)
- ★ New Lorentz covariant approach to access x-dependence of GPDs
- ★ Results on $H, E, \widetilde{H}$ GPDs
- ★ Future extensions - Other developments

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

arXiv:2310.13114v1 [hep-lat]

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

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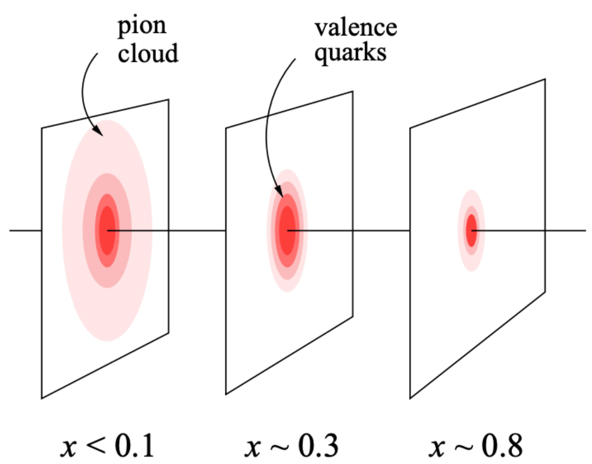
Motivation for GPDs studies



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

1_{mom} + 2_{coord} tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

  (SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs

- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

- and more challenges …

Essential to complement the knowledge on GPD from lattice QCD

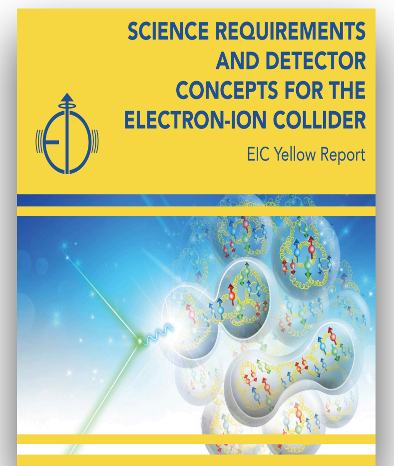
Hadron structure at core of nuclear physics

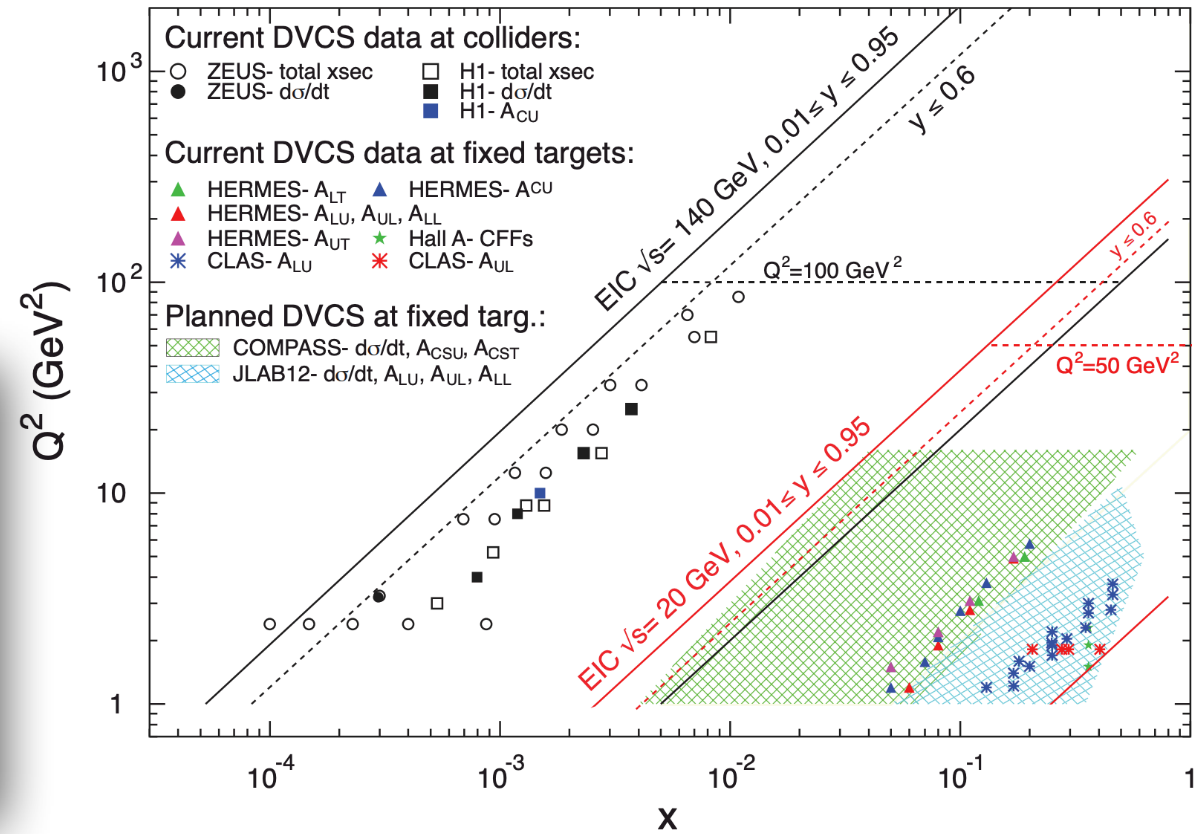
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





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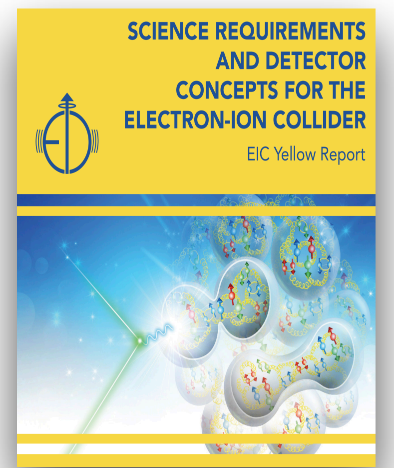

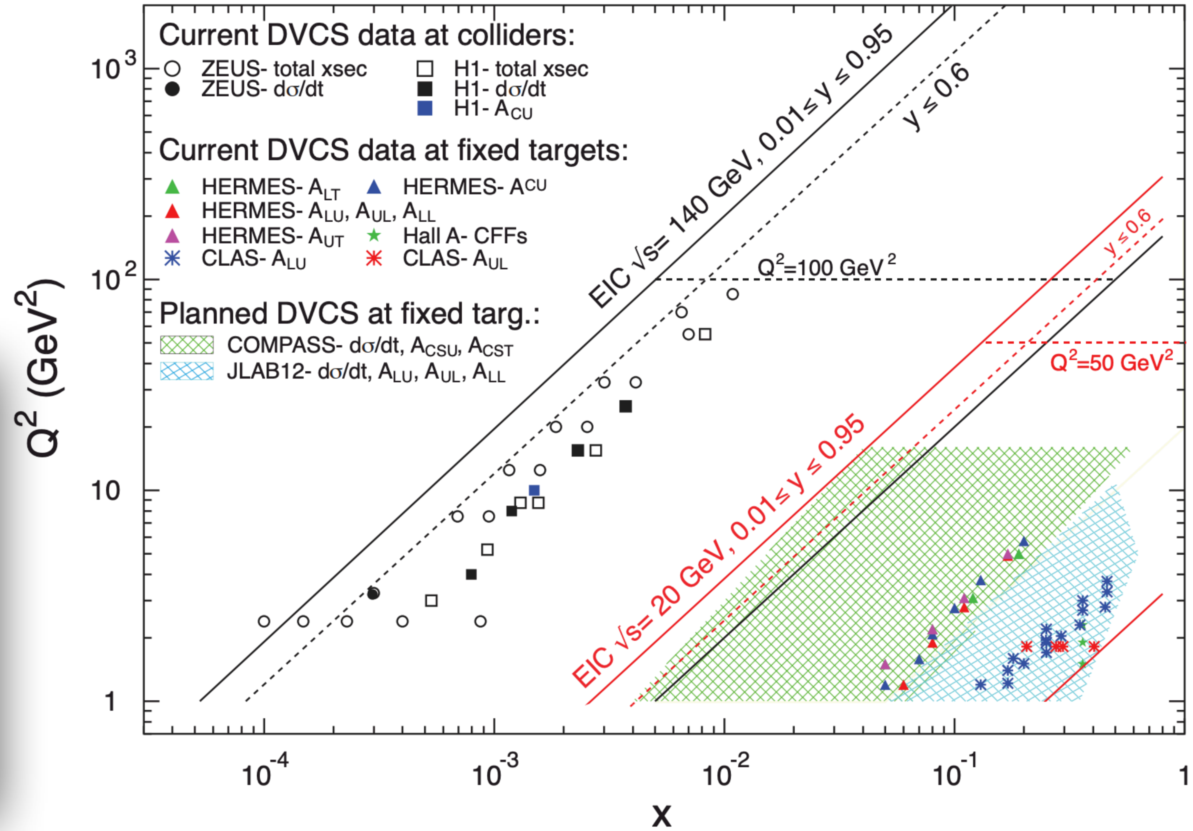

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QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number: DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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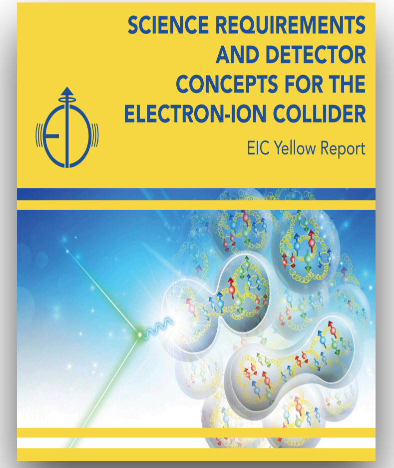

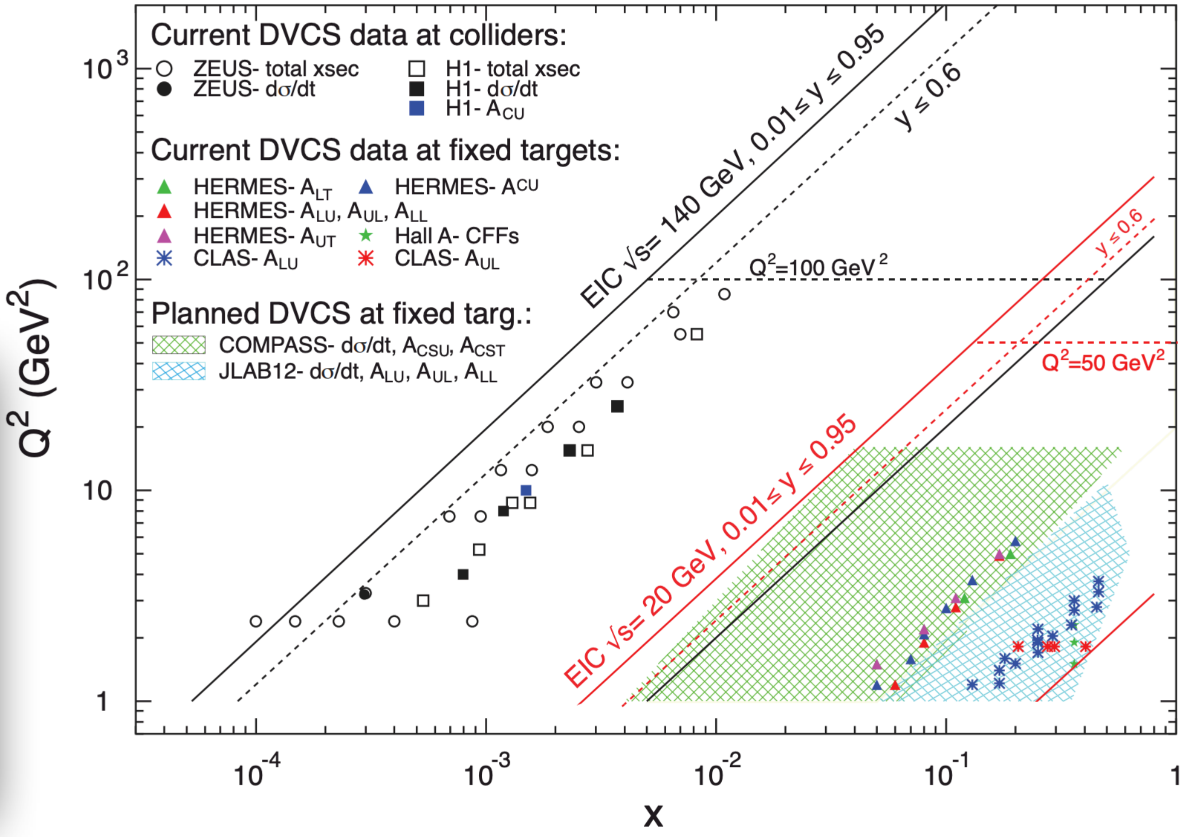

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Advances of lattice QCD are timely

Twist-classification of PDFs, GPDs, TMDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

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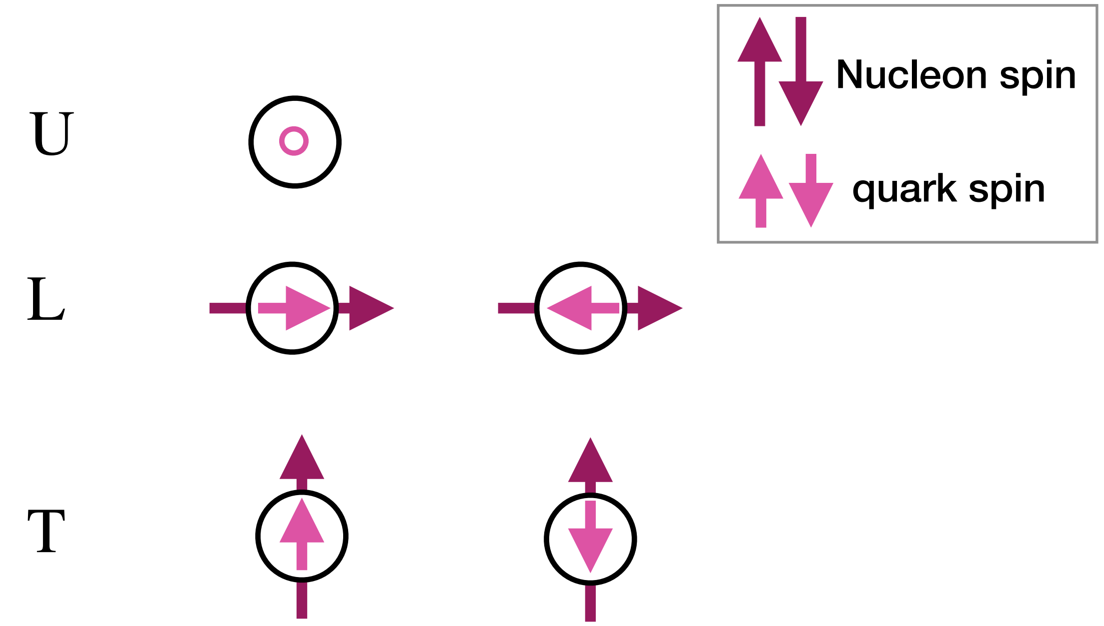
Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



★ Provide a correlation between the transverse position and the longitudinal momentum of the quarks in the hadron and its mechanical properties (OAM, pressure, etc.)

[M. Burkardt, PRD62 071503 (2000), hep-ph/0005108] [M. V. Polyakov, PLB555 (2003) 57, hep-ph/0210165]

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$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

3D Nucleon Structure Minisymposium: Constantinou, Tue @ 7 pm

CEU poster session, Wed @ 2pm: Sarah Lambreich

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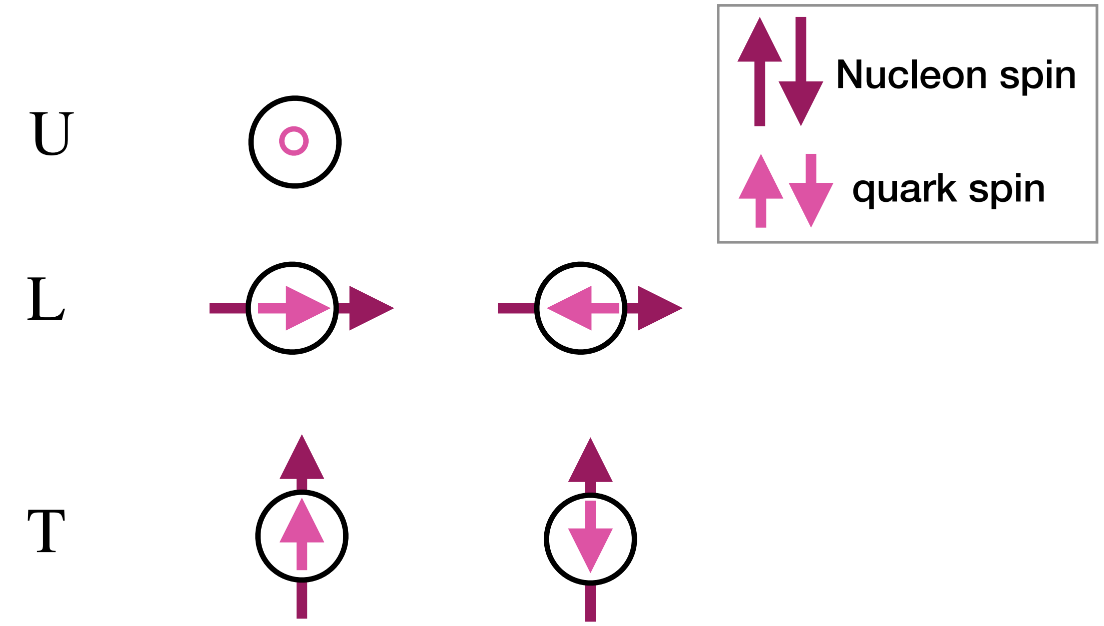

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Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]}_{\text{local operators}}$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Bigg\}$$

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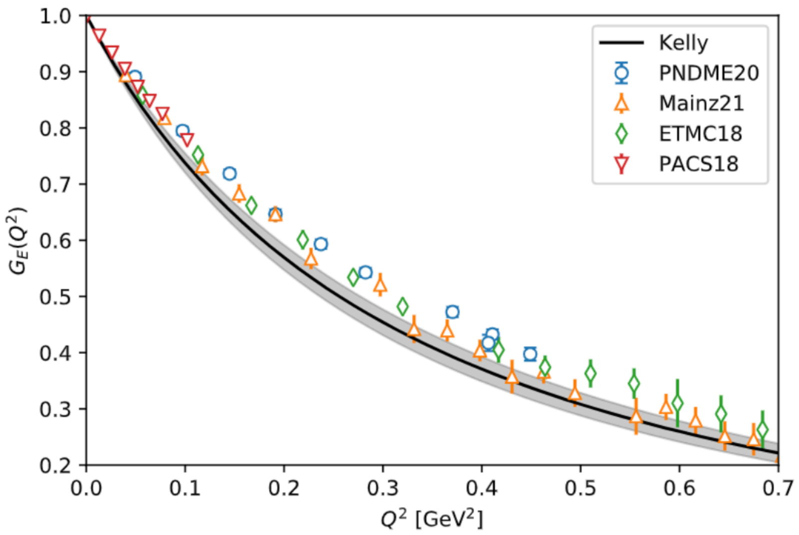

Wide -t range that comes at the cost of 1 (in the majority of cases)

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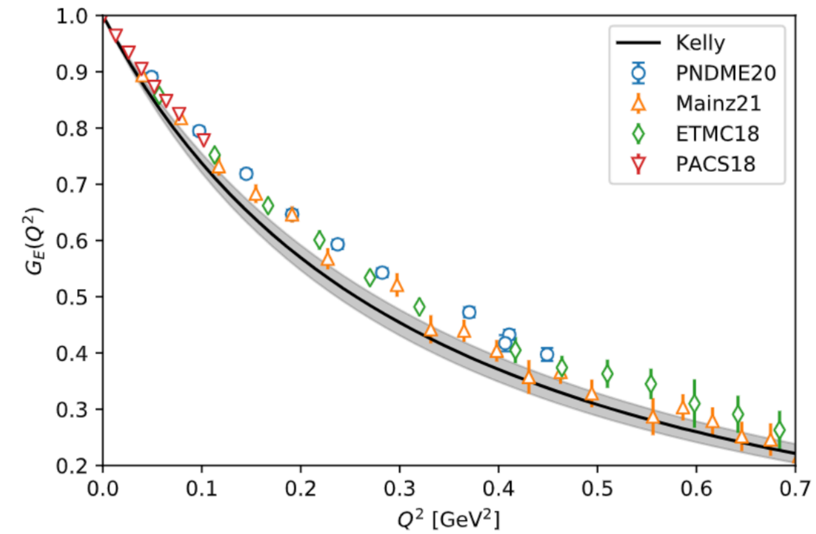

Wide -t range that comes at the cost of 1 (in the majority of cases)

★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

$$\langle N(P')|O_V^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

GPDs

Through non-local matrix elements of momentum-boosted hadrons

GPDs

Through non-local matrix elements of momentum-boosted hadrons

I will usually refer to unpolarized GPDs as an example

Access of PDFs/GPDs on a Euclidean Lattice

- ★ Matrix elements of momentum-boosted hadrons coupled to nonlocal (equal-time) operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

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Accessing -t dependence: Computationally intensive

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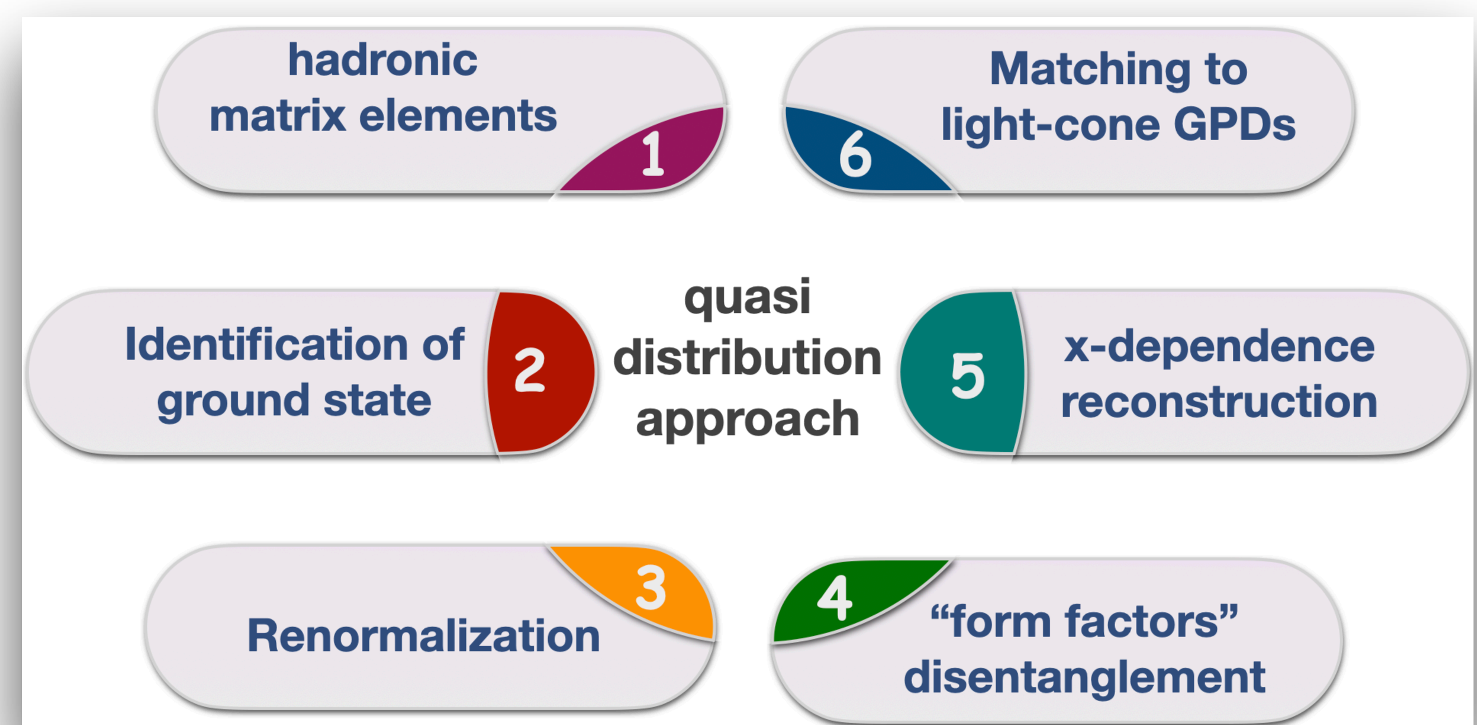

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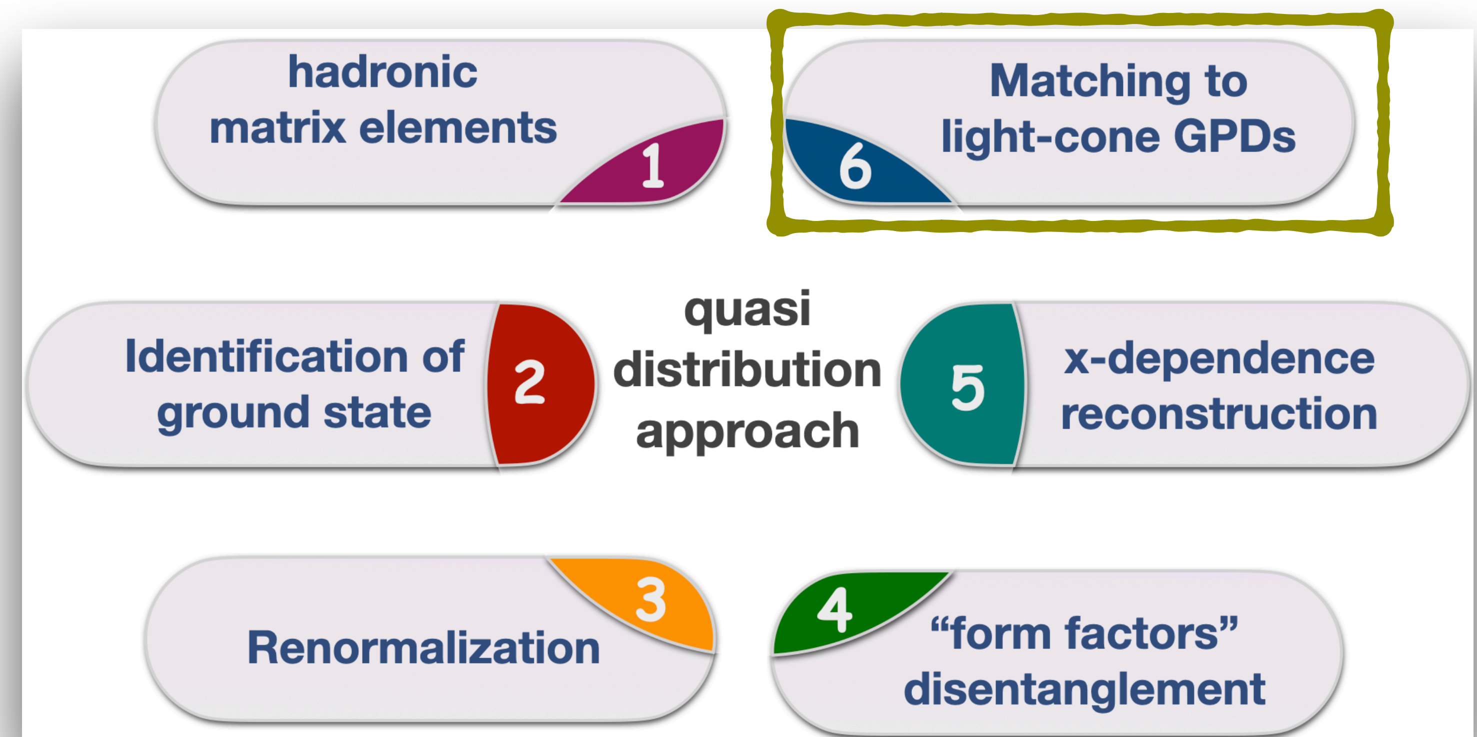

GPDs on the lattice

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar\psi(-\tfrac{z}{2})\,\gamma^+\,\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,|p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar u(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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→ finite mixing with scalar [Constantinou & Panagopoulos (2017)]

- Lorentz non-invariant parametrization
- Typically used in symmetric frame
- A non-symmetric setup may result to different functional form for GPDs compared to the symmetric one

Definition of GPDs in Euclidean lattice

★ Calculation expected to be performed in symmetric frame to extract "standard" GPDs

★ Symmetric frame requires separate calculations at each t

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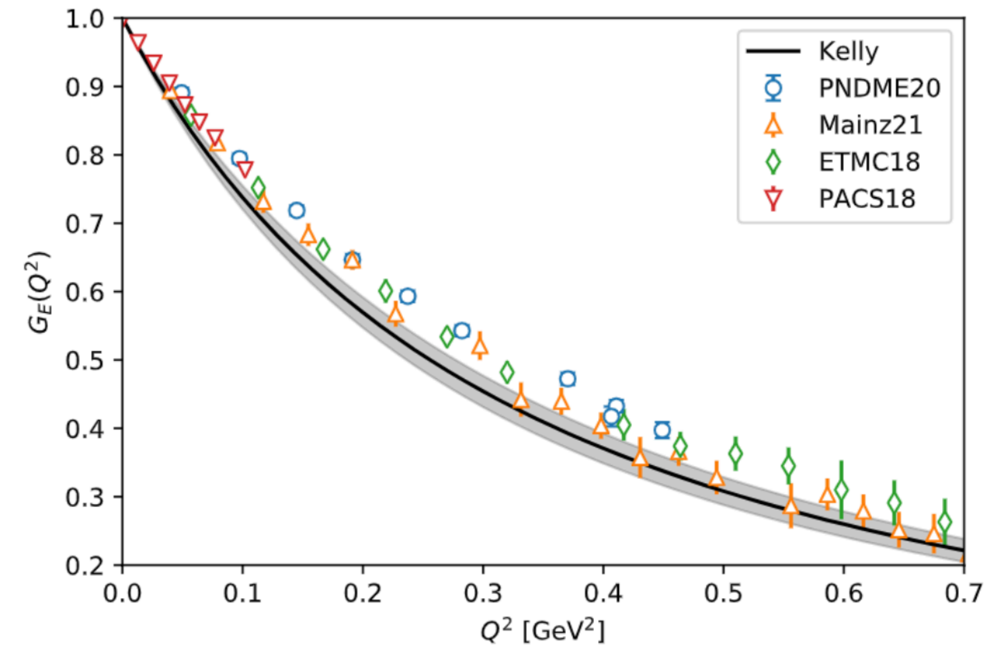

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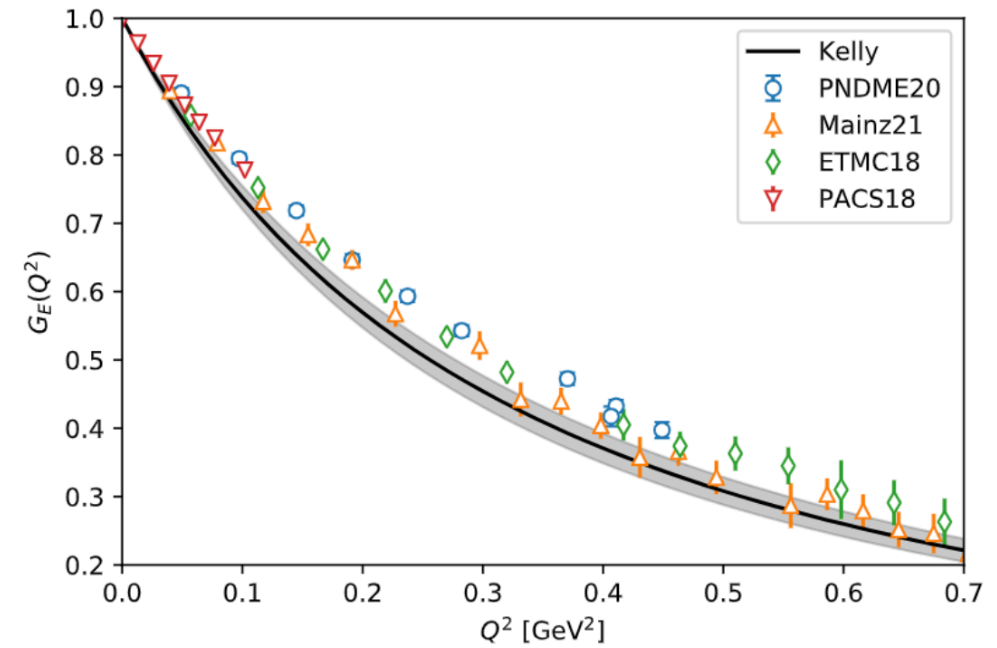

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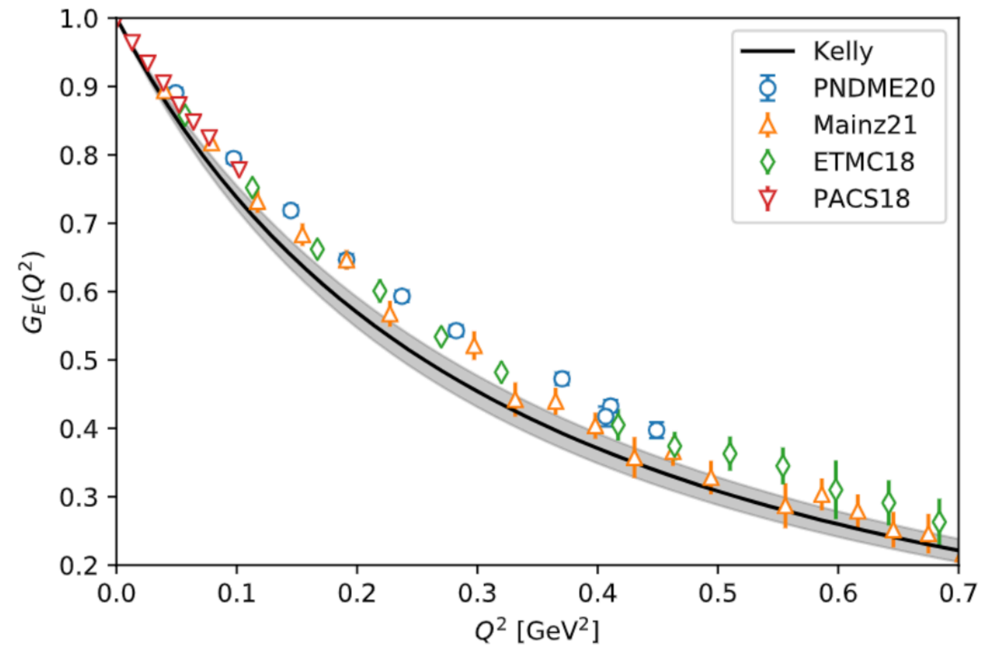

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★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard H, E GPDs
- Quasi H, E may be redefined (Lorentz covariant) to eliminate $1/P_3$ contributions:

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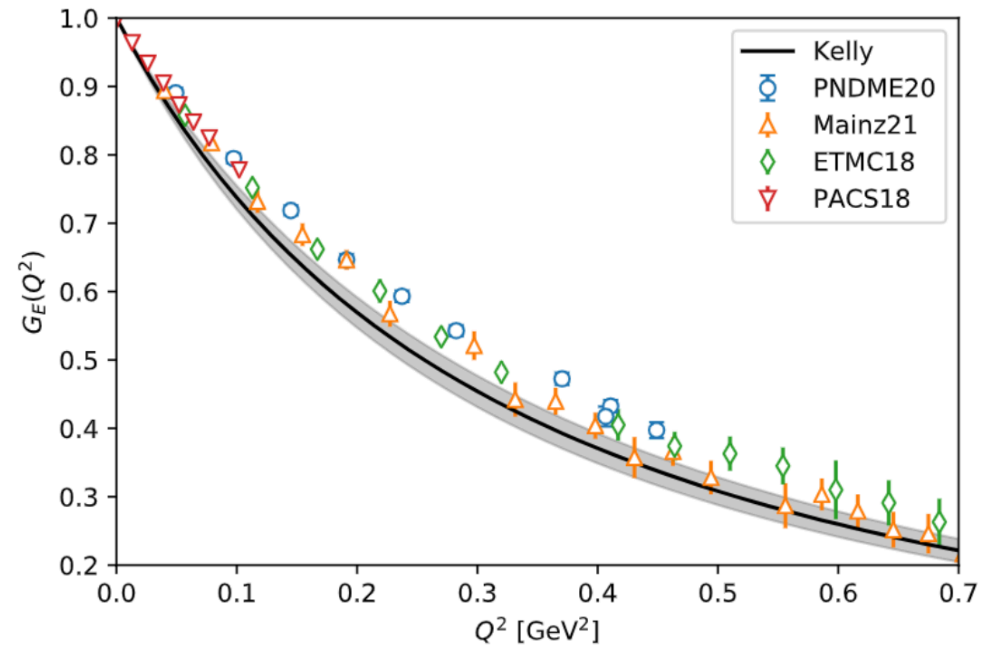

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Theoretical setup

★ Unique relation between H, E GPDs and Amplitudes

$$\mathcal{H}_0^s(A_i^s; z) = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6 \,,$$

$\xi = 0$

$$\mathcal{E}_0^s(A_i^s; z) = -A_1 - \frac{m^2 z}{P_3} A_4 + 2A_5 - \frac{z\left(4E^2 + \Delta_1^2 + \Delta_2^2\right)}{2P_3} A_6 \,.$$

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➡ **Proof-of-concept calculation ($\xi = 0$):**

- **symmetric frame:** $\vec{p}_f^{\,s} = \vec{P} + \frac{\vec{Q}}{2}$, $\vec{p}_i^{\,s} = \vec{P} - \frac{\vec{Q}}{2}$ $\quad -t^s = \vec{Q}^2 = 0.69\, GeV^2$
- **asymmetric frame:** $\vec{p}_f^{\,a} = \vec{P}$, $\vec{p}_i^{\,a} = \vec{P} - \vec{Q}$ $\quad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\, GeV^2$

Matrix element decomposition

Symmetric

$$C_s = \frac{2m^2}{E(E+m)}$$

$$\Gamma_0 = \frac{1}{2}(1+\gamma^0)$$

$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j=1,2,3)$$

$$\Pi^0_s(\Gamma_0) = C_s\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right)$$

$$\Pi^0_s(\Gamma_1) = i\,C_s\left(\frac{EP_3Q_2}{4m^3}A_1 - \frac{(E+m)P_3Q_2}{2m^3}A_5 - \frac{E\left(P_3^2+m(E+m)\right)zQ_2}{2m^3}A_6\right)$$

$$\Pi^0_s(\Gamma_2) = i\,C_s\left(-\frac{EP_3Q_1}{4m^3}A_1 + \frac{(E+m)P_3Q_1}{2m^3}A_5 + \frac{E\left(P_3^2+m(E+m)\right)zQ_1}{2m^3}A_6\right)$$

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Novel feature: z-dependence

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No definite symmetries for Π_μ^a

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{\left(m(E+m)+P_3^2\right)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3 Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

$$A_5 = -\frac{E}{Q_1}\Pi_2^s(\Gamma_3)$$

$$A_6 = \frac{P_3}{2Ez(E+m)}\Pi_0^s(\Gamma_0) + i\,\frac{\left(P_3^2 - E(E+m)\right)}{EQ_1 z(E+m)}\Pi_0^s(\Gamma_2) + \frac{P_3}{EQ_1 z}\Pi_2^s(\Gamma_3)$$

Asymmetric

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3 m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
$$+i\,\frac{2(E_i-E_f)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_1^a(\Gamma_0)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

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- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

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$=$

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
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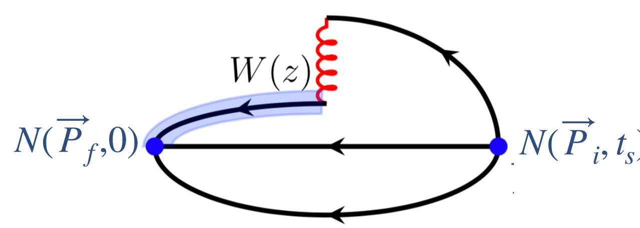
Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover improvement

Pion mass: 260 MeV
Lattice spacing: 0.093 fm
Volume: 32^3 x 64
Spatial extent: 3 fm

★ Calculation:

- isovector combination
- zero skewness
- T_{sink}=1 fm



frame	P_3 [GeV]	$\mathbf{Q}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
symm	1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
non-symm	1.25	(±2,0,0), (0,±2,0)	0.64	0	8	269	8	17216

★ Computational cost:

- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

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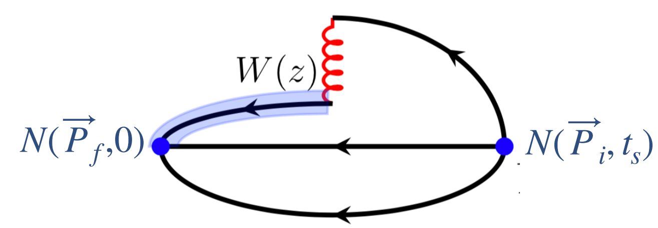

frame	P_3 [GeV]	$\mathbf{Q}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
symm	1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
non-symm	1.25	(±2,0,0), (0,±2,0)	0.64	0	8	269	8	17216

Small difference: $t^s = -\vec{Q}^2$ $\quad$ $t^a = -\vec{Q}^2 + (E_f - E_i)^2$

$$A(-0.65\text{GeV}^2) \sim A(-0.69\text{GeV}^2)$$

★ **Computational cost:**

- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

Results: matrix elements

★ Eight independent matrix elements needed to disentangle the A_i

asymmetric frame

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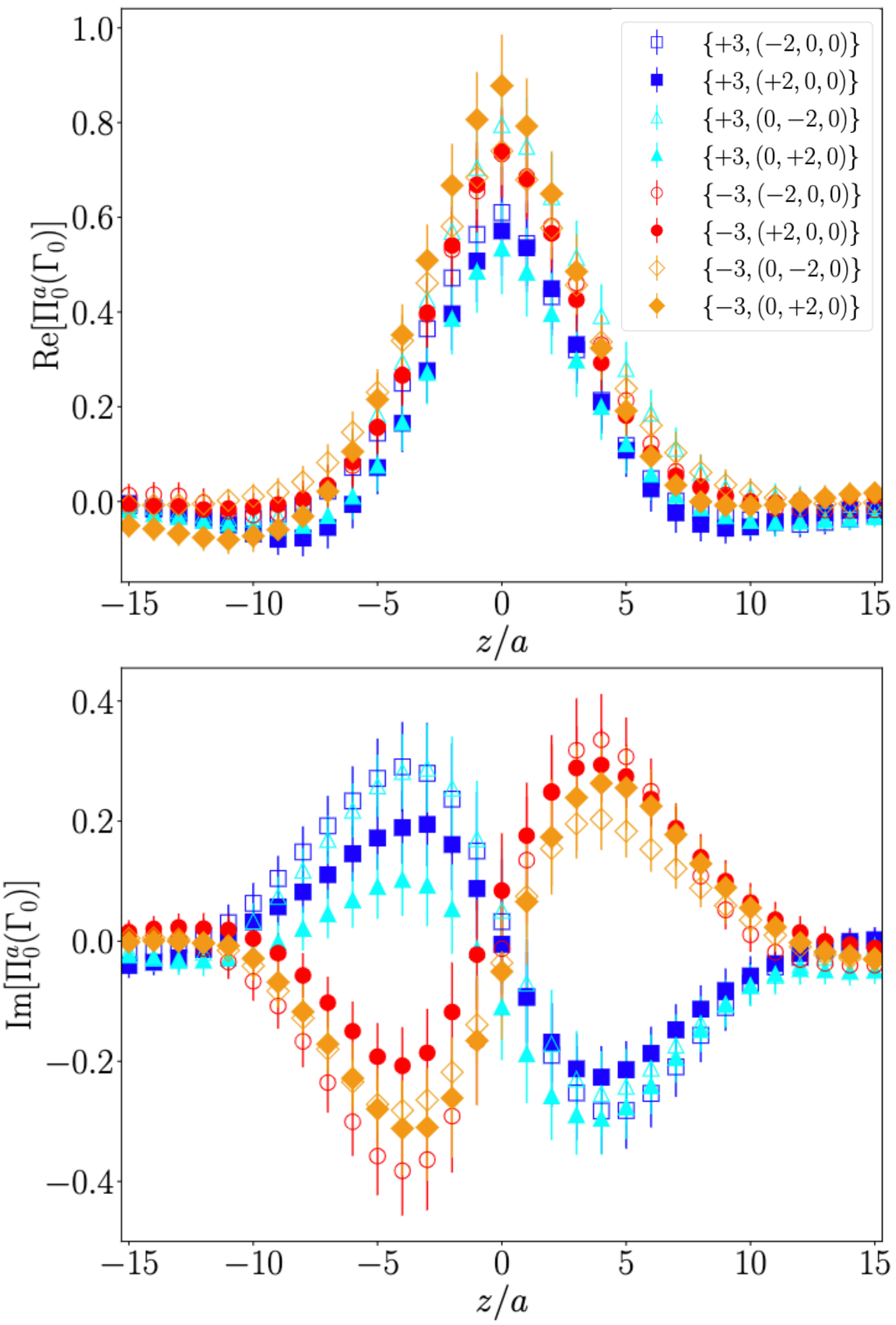

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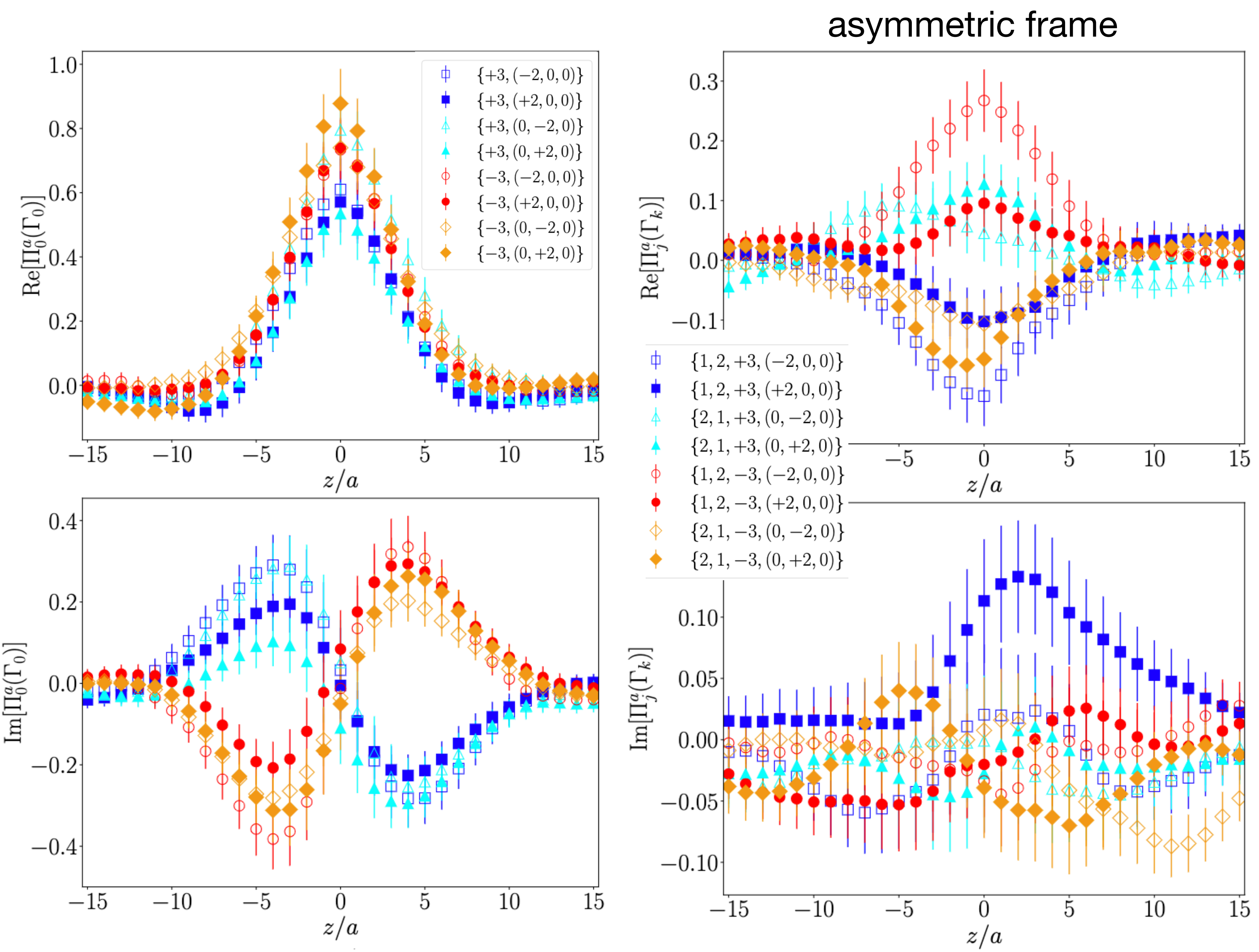

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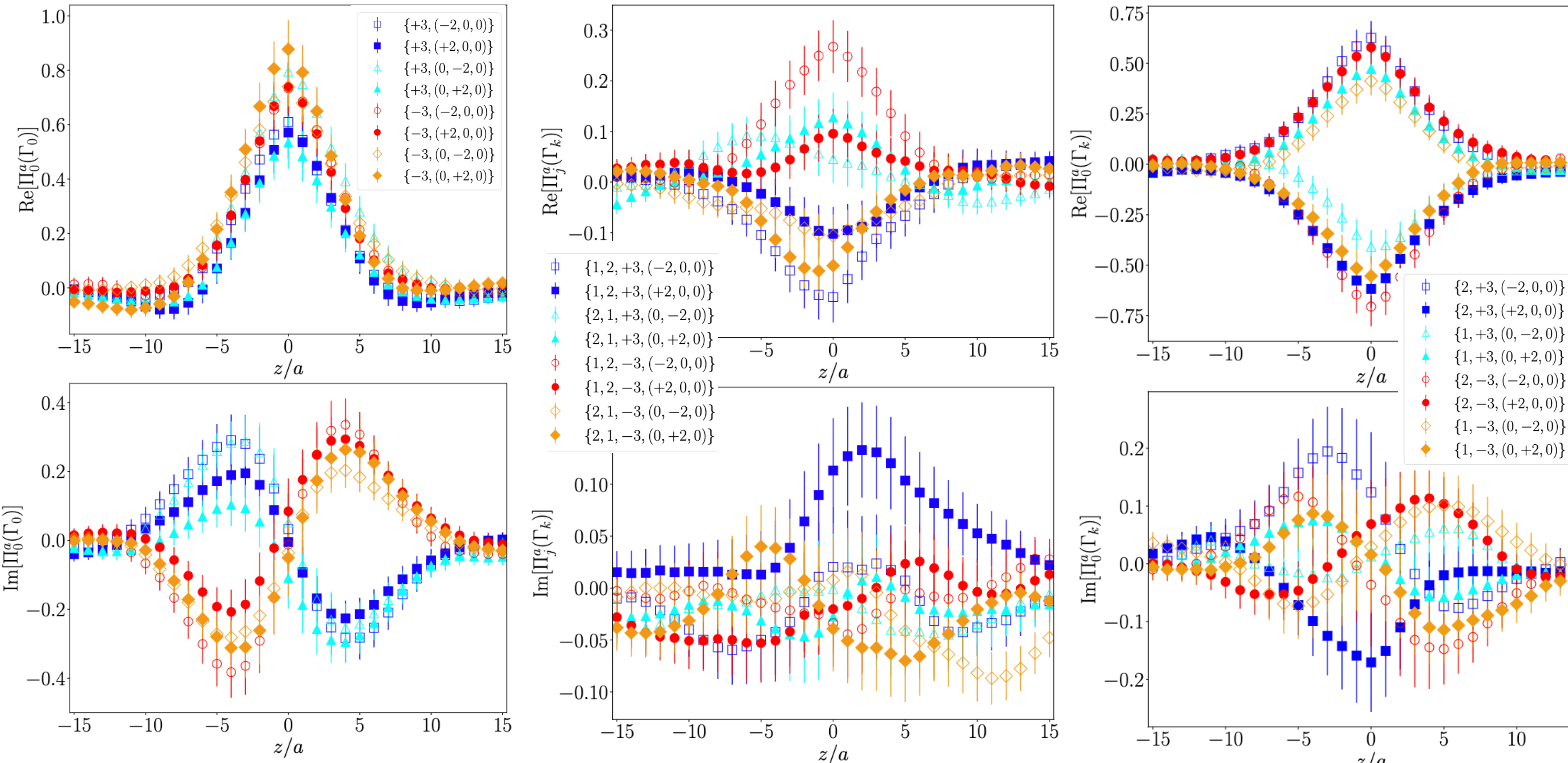

Results: matrix elements

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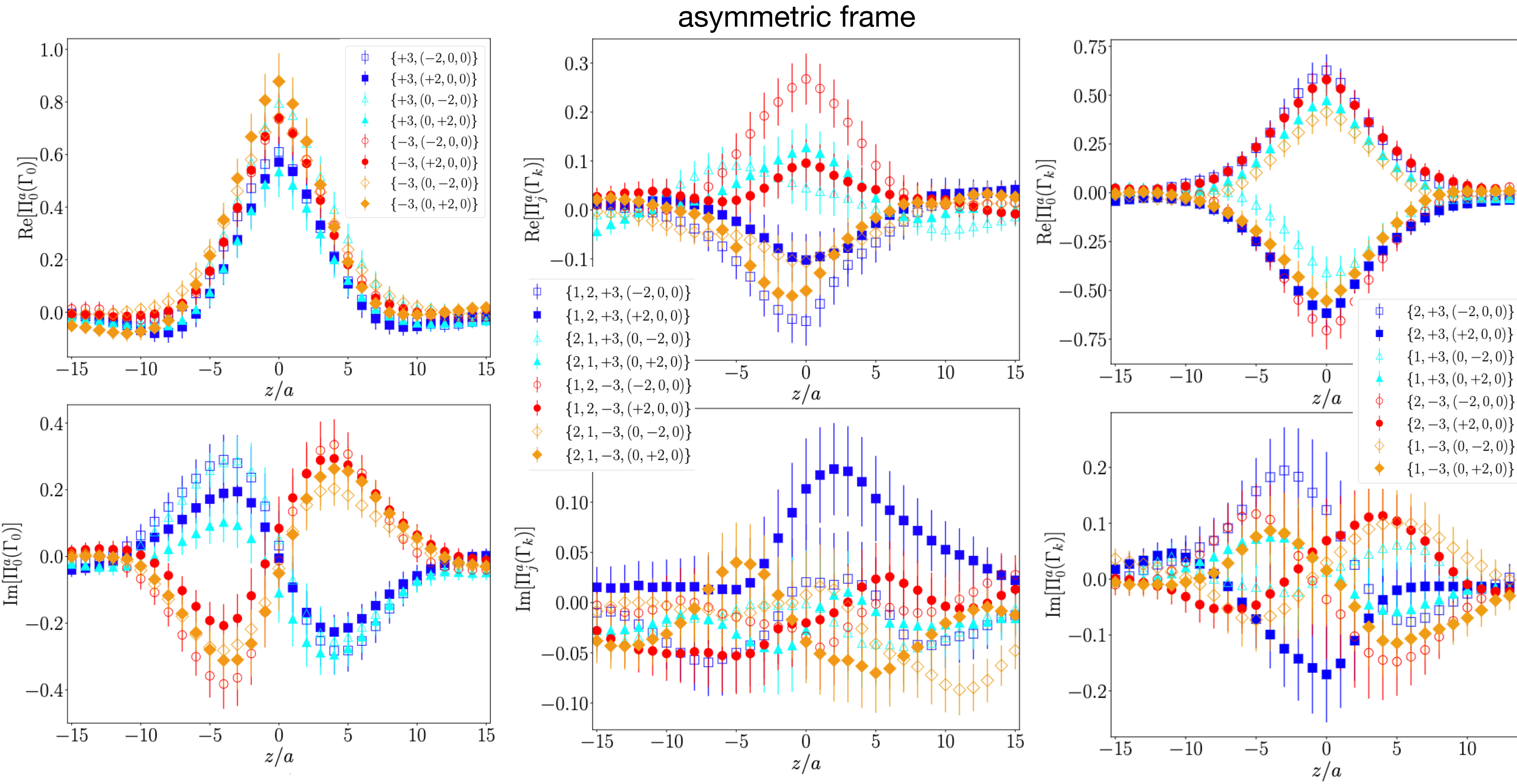
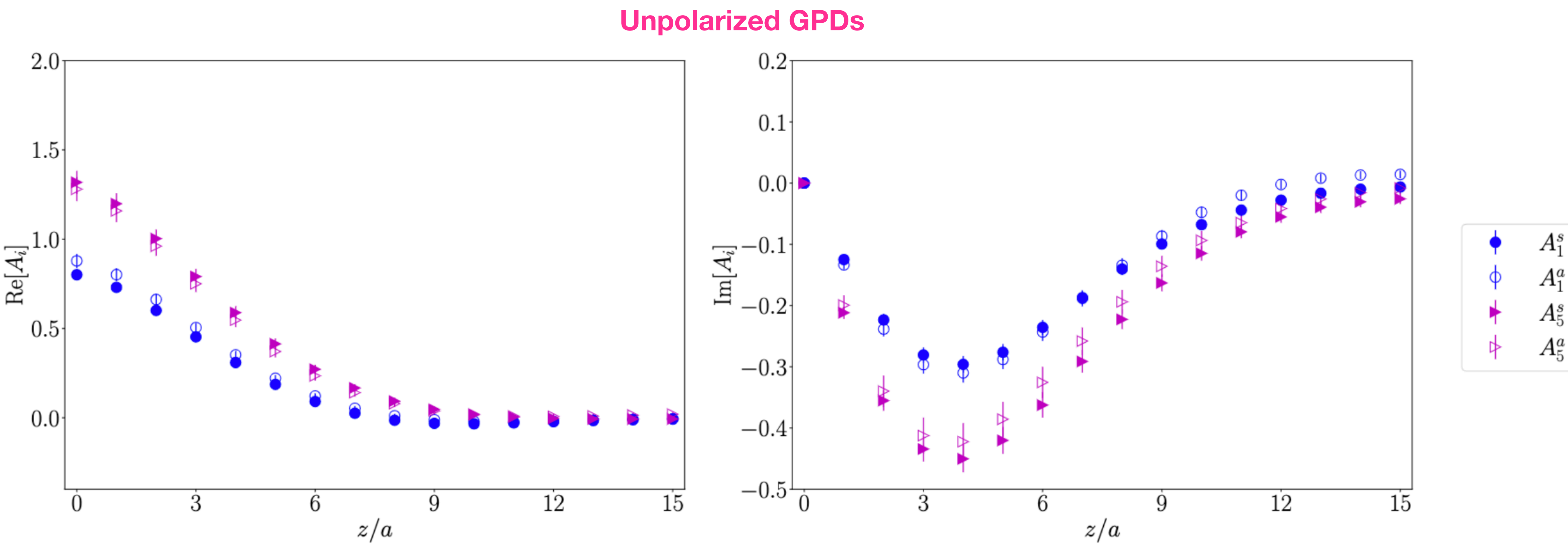


- ★ Asymmetric frame: ME do not have definite symmetries in $\pm P_3, \pm Q, \pm z$
- ★ Noisy ME lead to challenges in extracting A_i of sub-leading magnitude

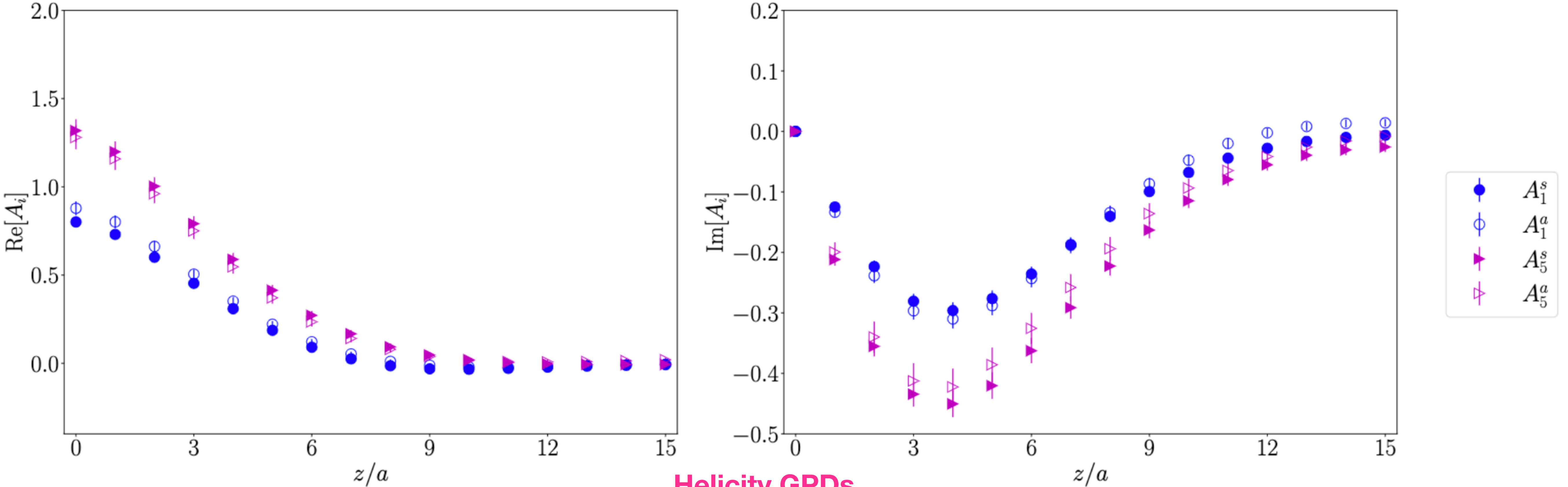
Comparison of A_i in two frames



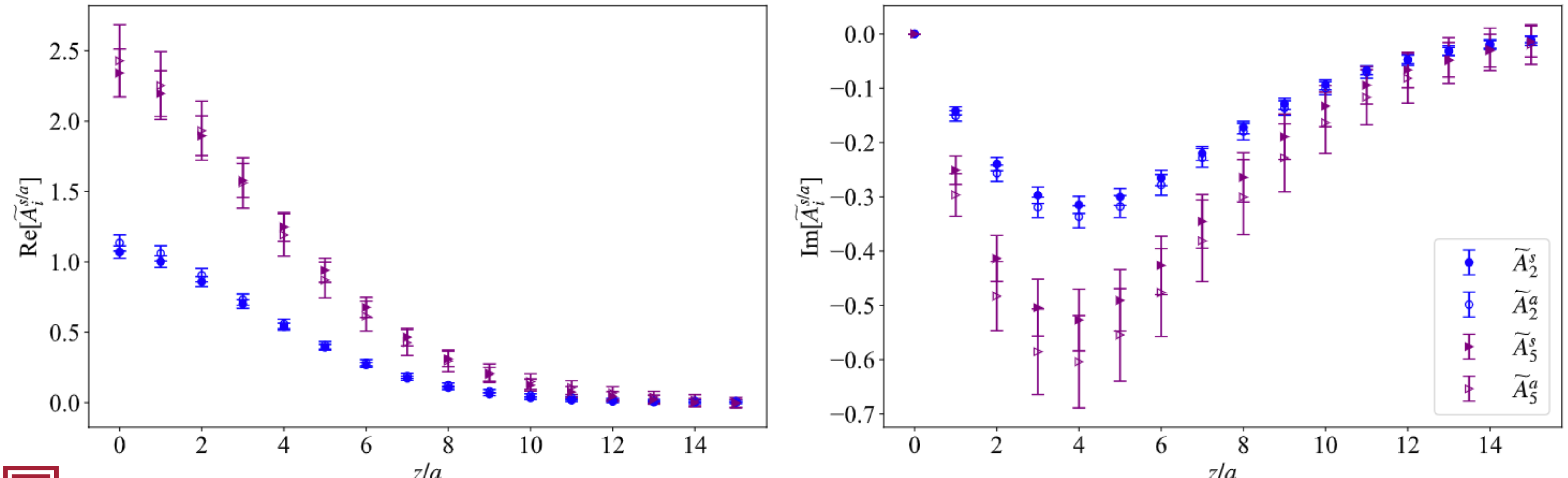
- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ A_3, A_4, A_8 zero at $\xi = 0$
- ★ A_2, A_6, A_7 suppressed (at least for this kinematic setup and $\xi = 0$)

Comparison of A_i in two frames

Unpolarized GPDs



Helicity GPDs



Extension of asymm. frame calculation

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

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Symmetric frame very expensive computationally

H, E light-cone GPDs

- quasi-GPDs transformed to momentum space
- Matching formalism to 1 loop accuracy level
- +/-x correspond to quark and anti-quark region
- Anti-quark region susceptible to systematic uncertainties

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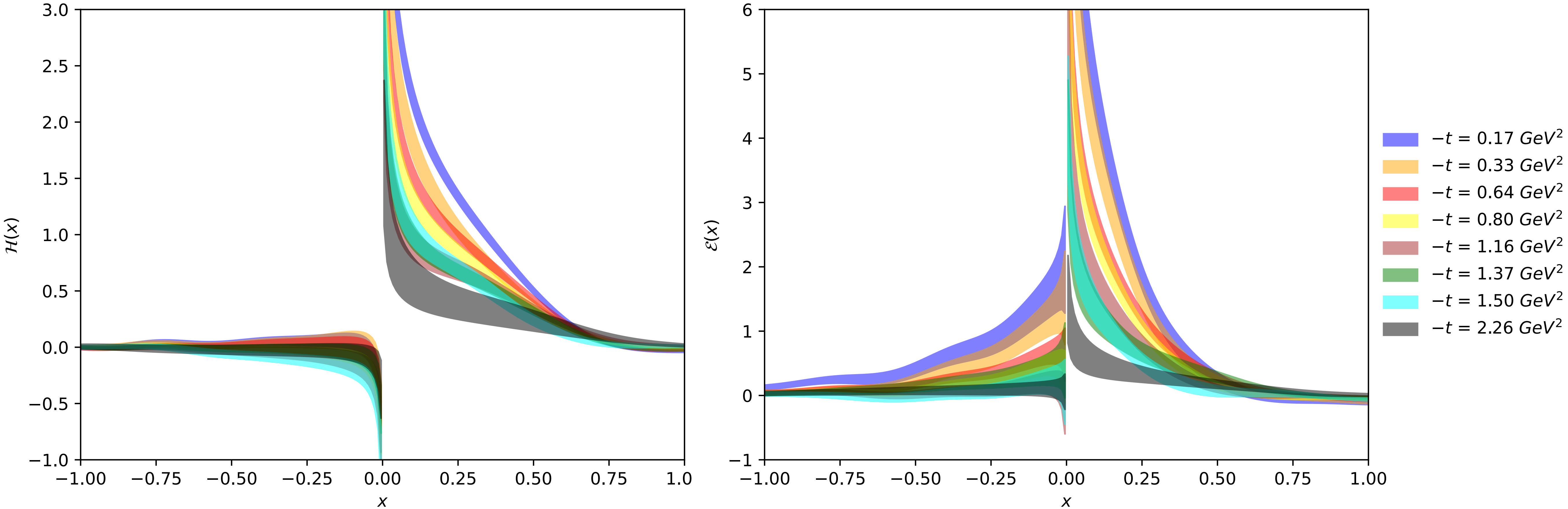

- ★ small-x region not reliably extracted
- ★ perturbative matching breaks down at $\xi = x$

Helicity GPDs

★ $$F^{[\gamma^3\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^0}\bar{u}(p_f,\lambda')\left[\gamma^3\gamma_5\widetilde{\mathcal{H}}_3(x,\xi,t;P^3) + \frac{\Delta^3\gamma_5}{2m}\widetilde{\mathcal{E}}_3(x,\xi,t;P^3)\right]u(p_i,\lambda)$$

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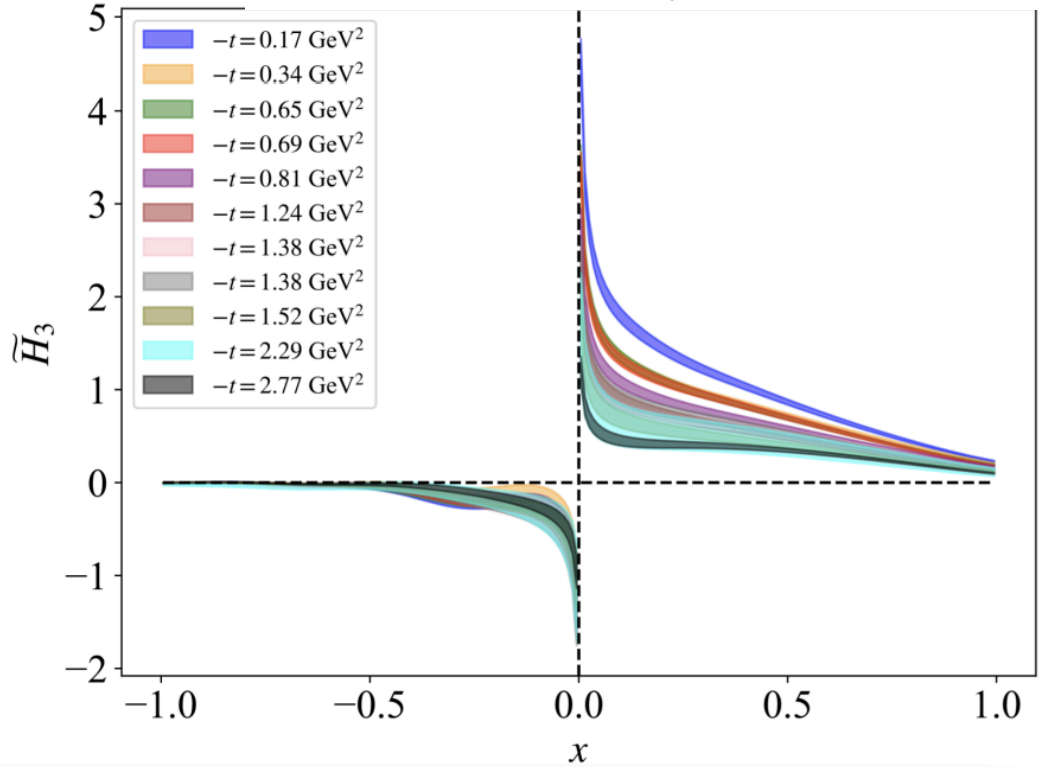

Helicity GPDs

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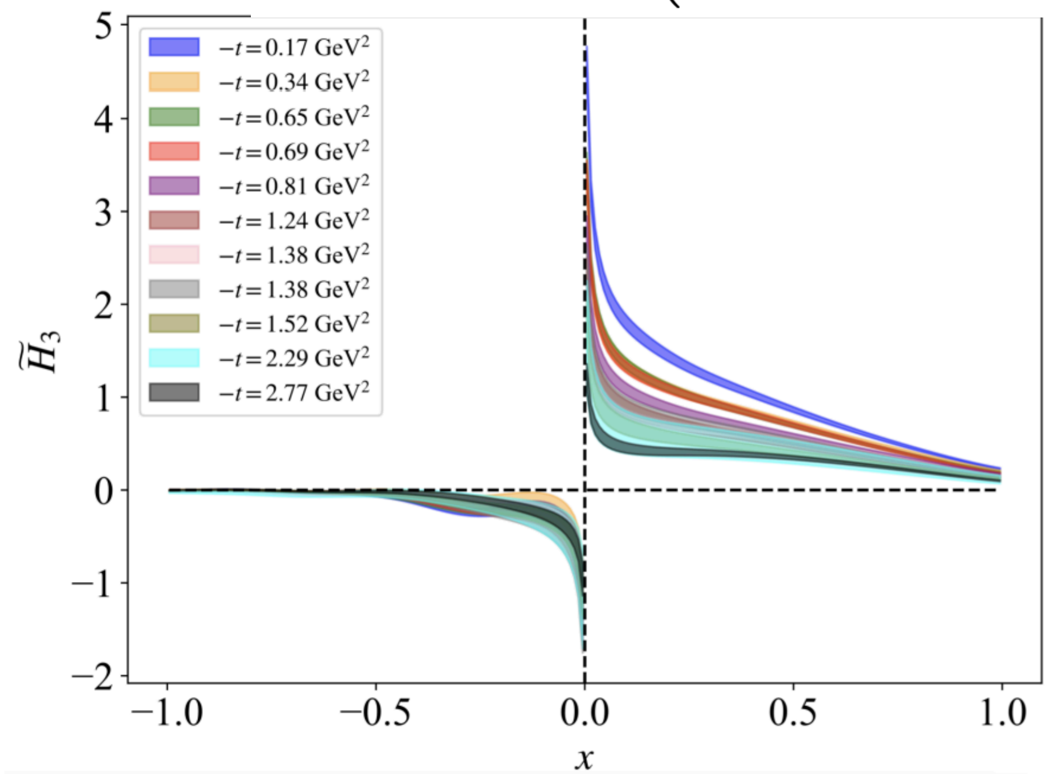

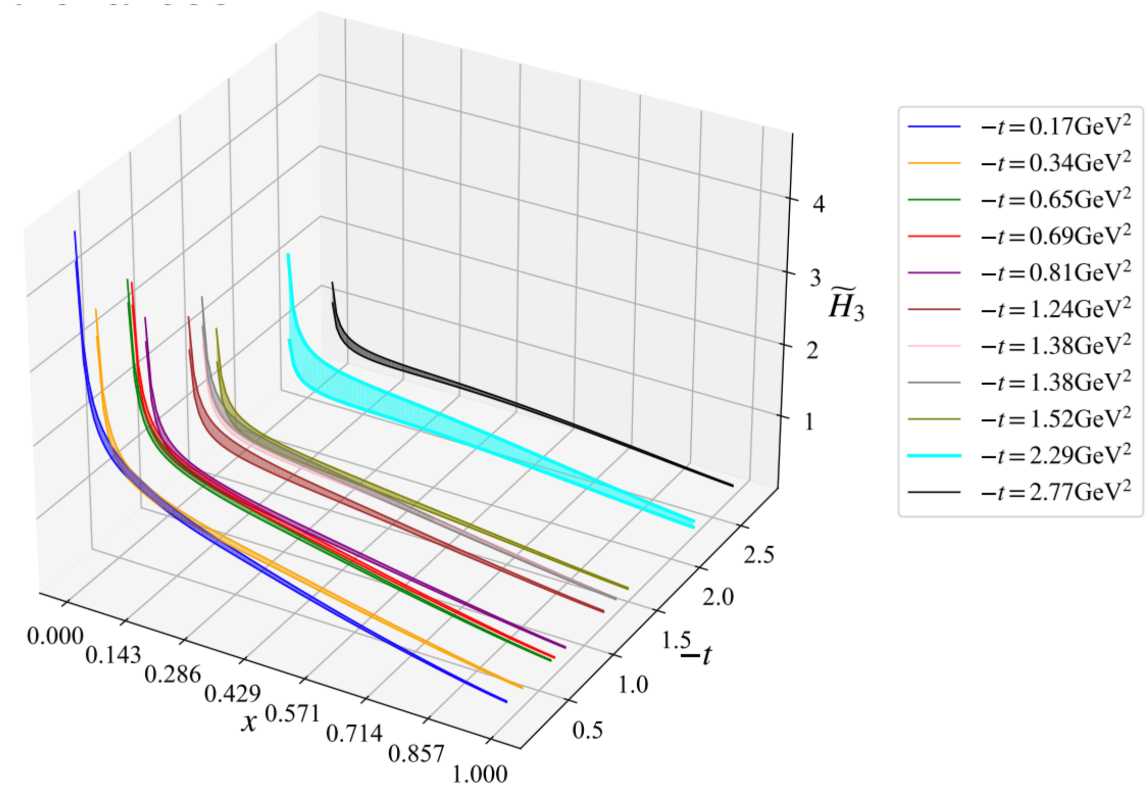

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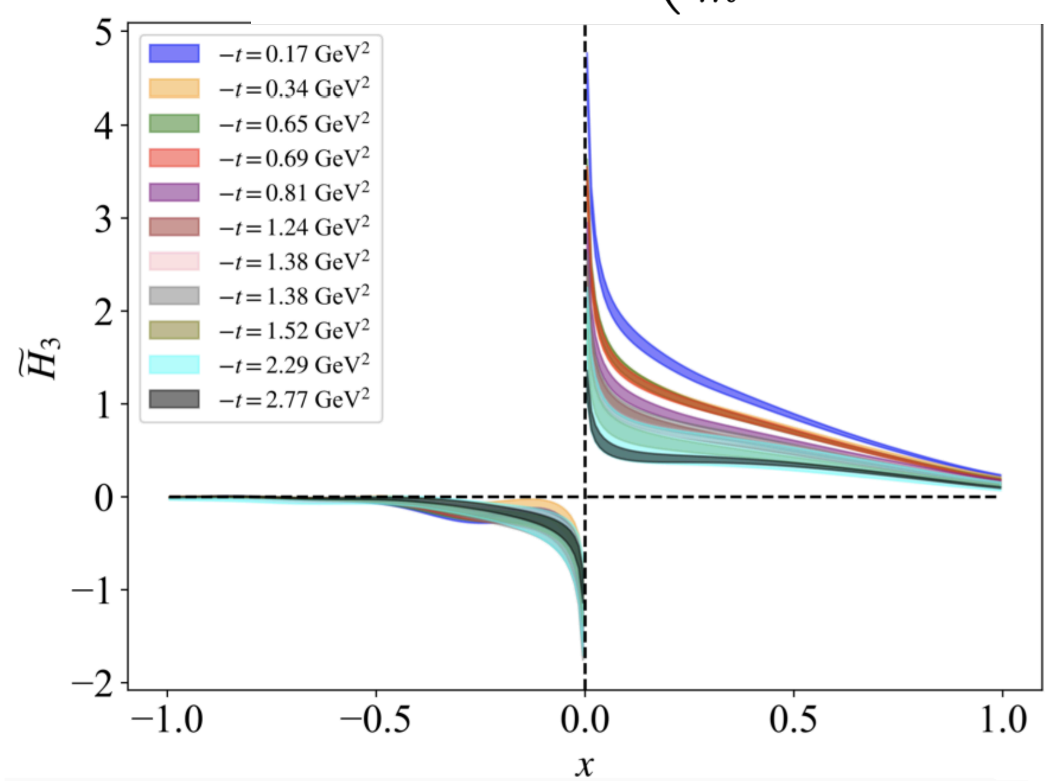

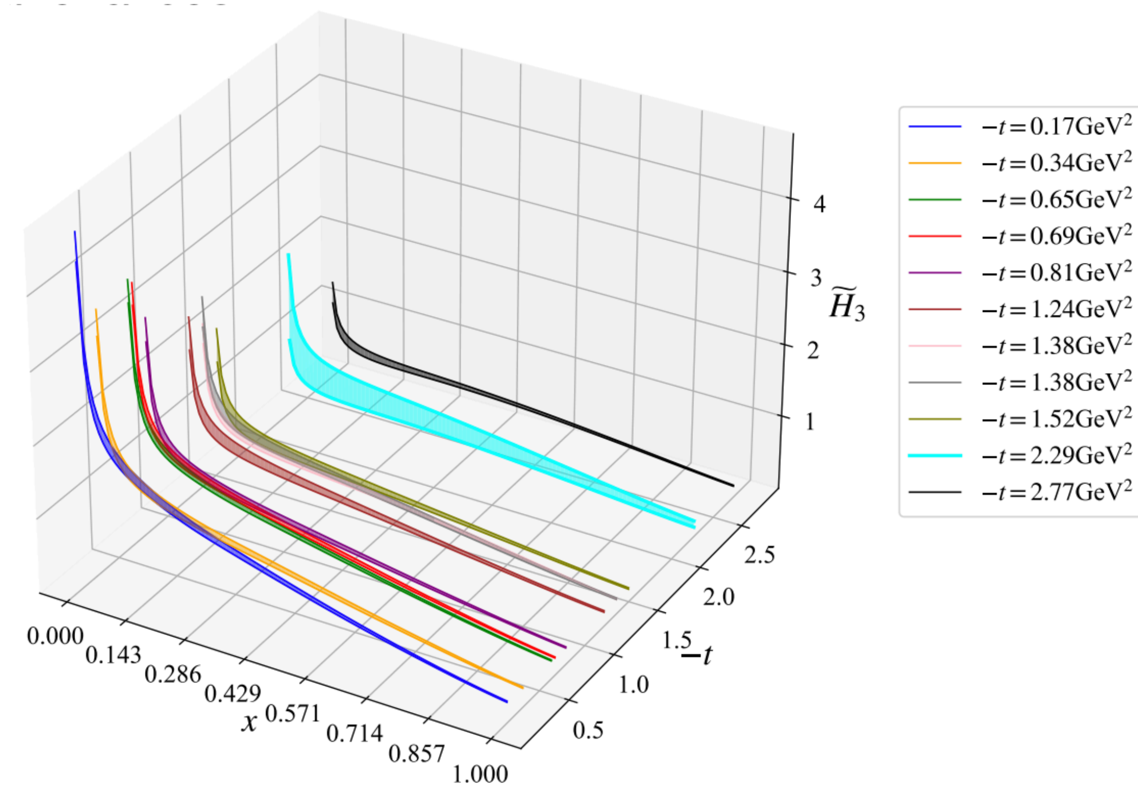

★ Large values of $-t$ not reliably extracted

★ $\widetilde{E}$-GPD cannot be extracted directly at $\xi = 0$

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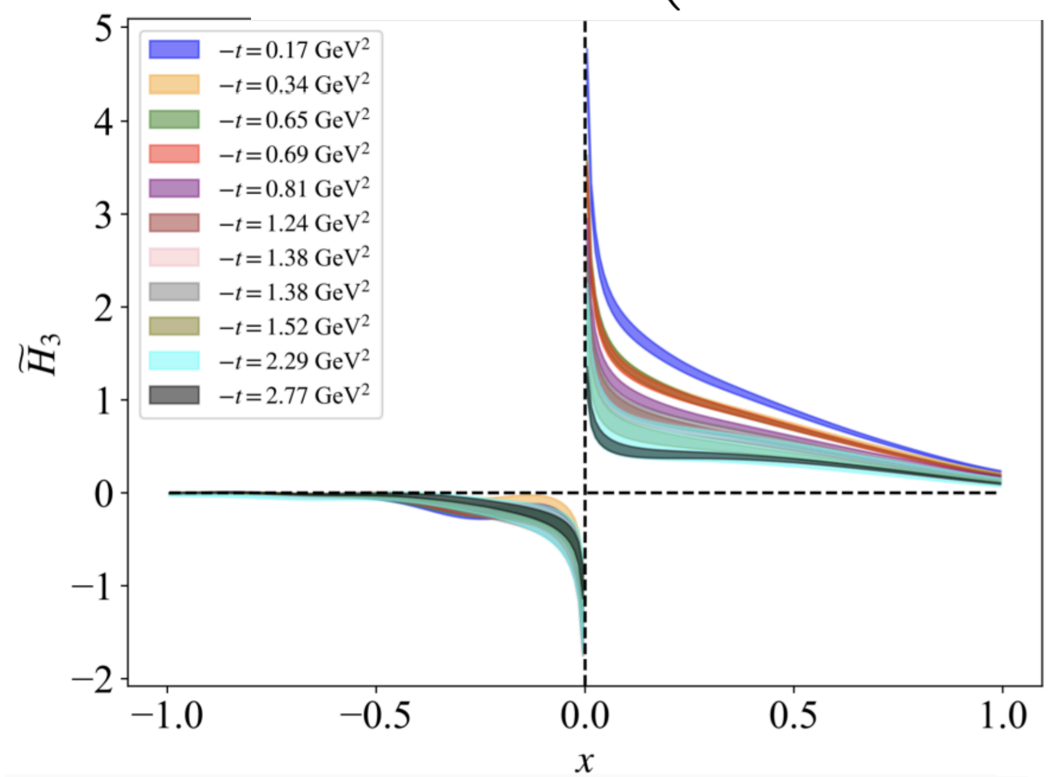

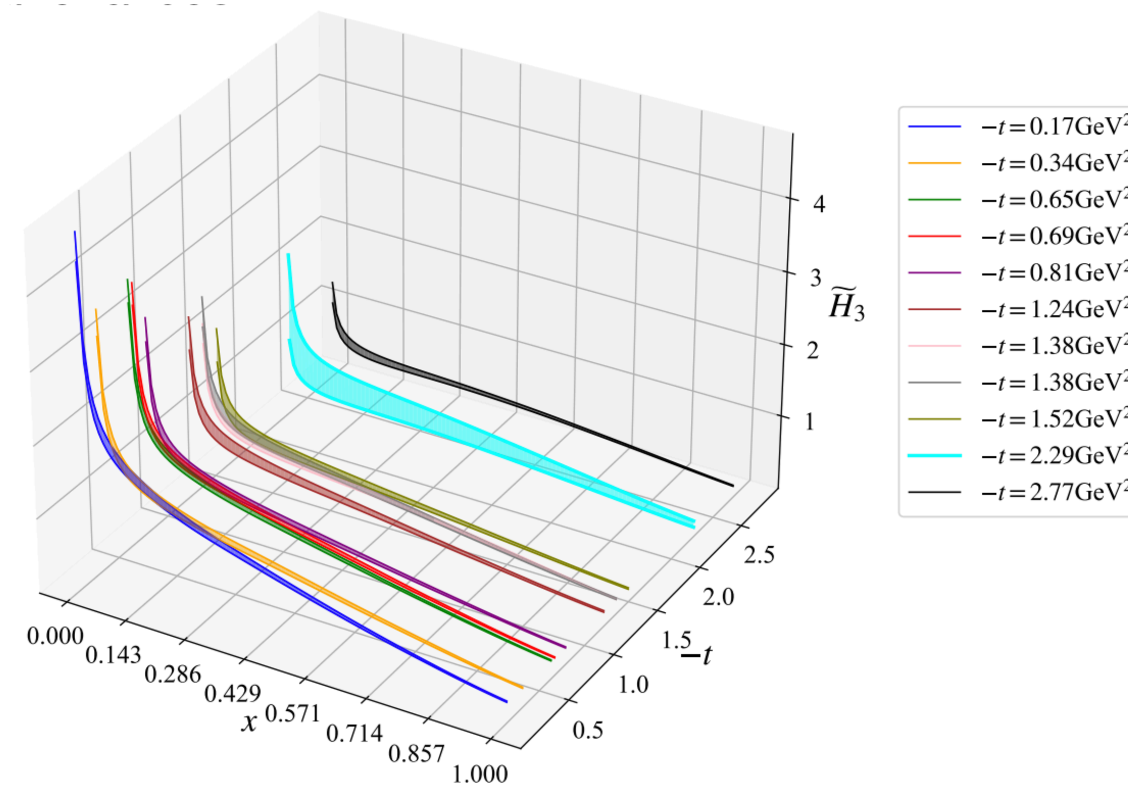

Glimpse of $\widetilde{E}$ from twist-3 GPDs

★ Large values of $-t$ not reliably extracted

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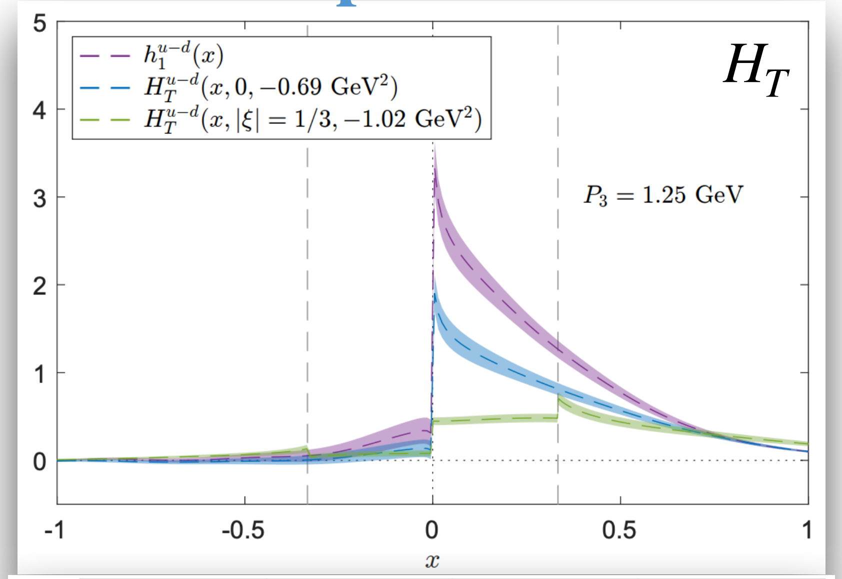
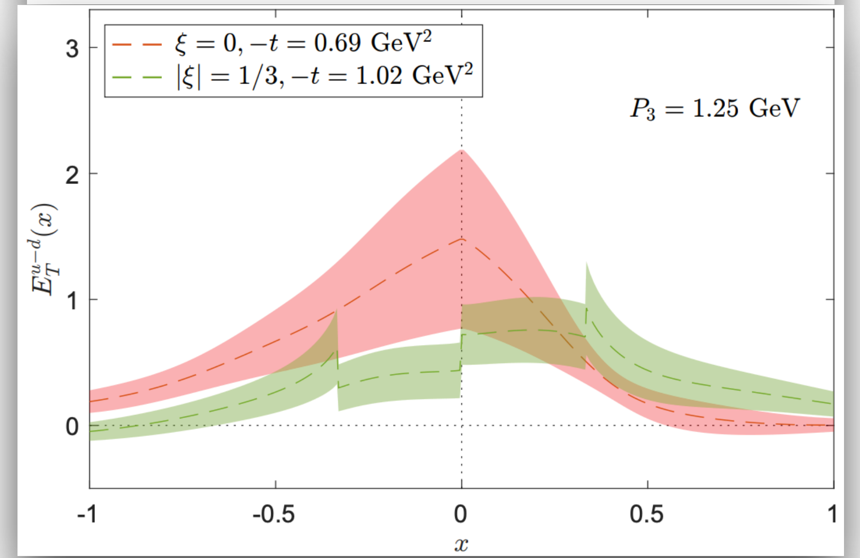
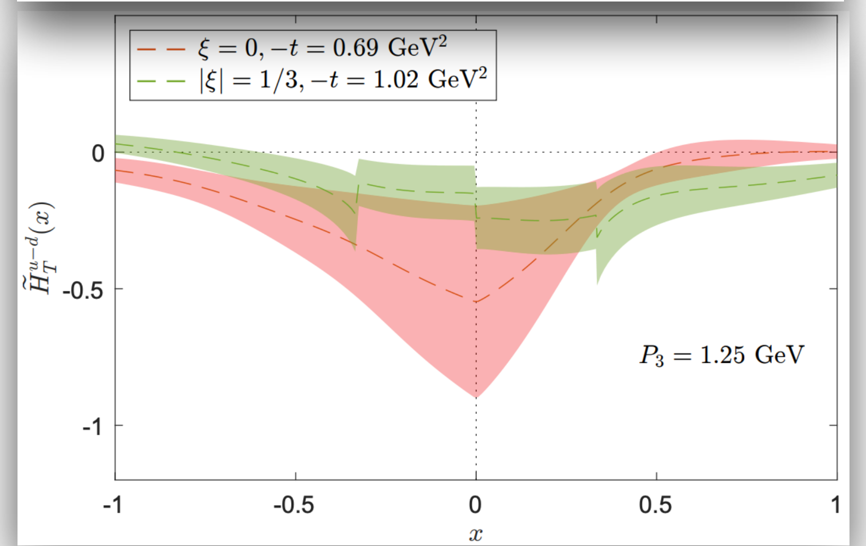
3D Nucleon Structure Minisymposium: Constantinou, Tue @ 7 pm

Transversity GPDs

Standard parametrization







[C. Alexandrou et al., PRD 105, 034501 (2022)]

Transversity GPDs

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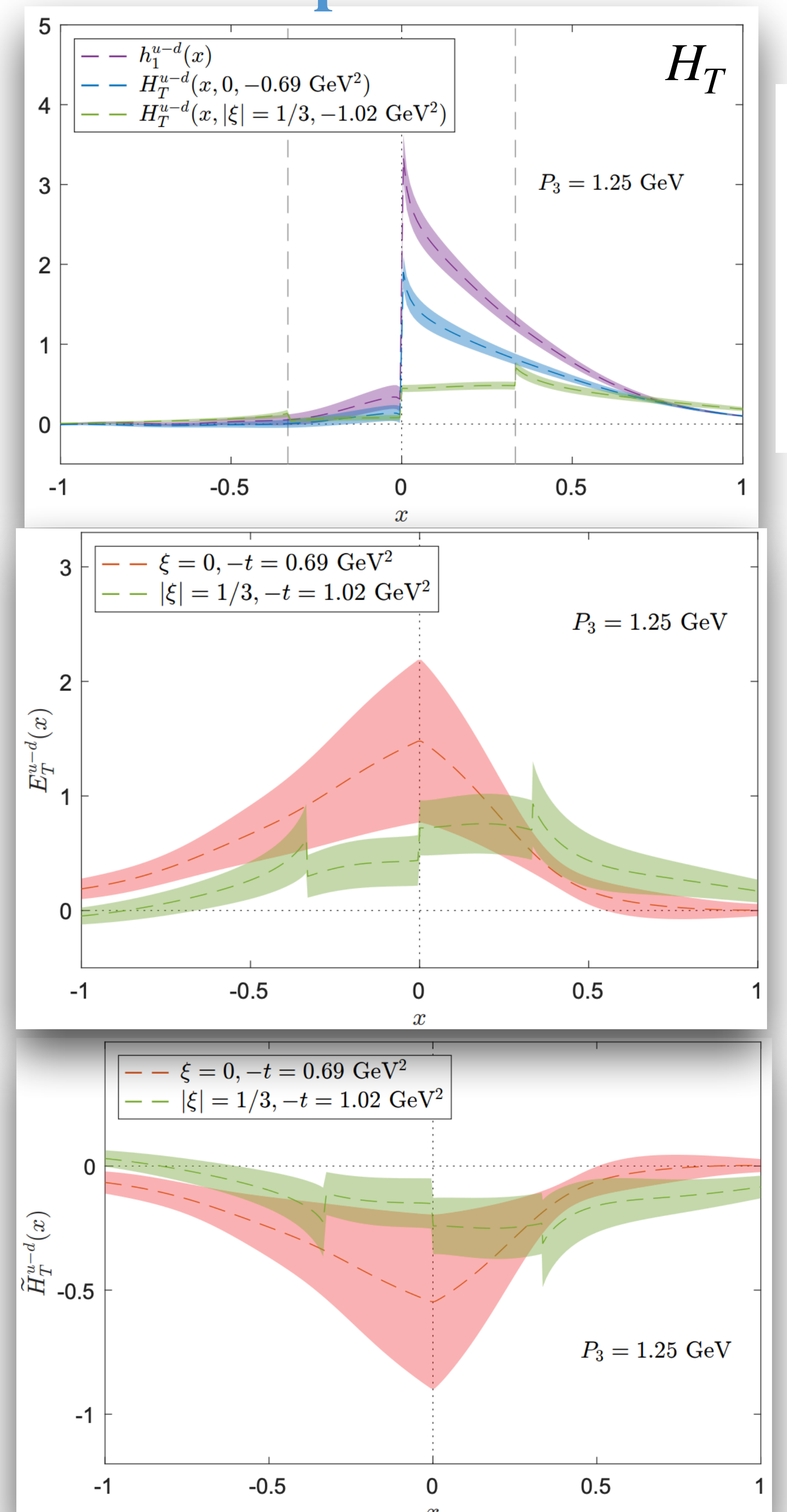


Lorentz covariant parametrization

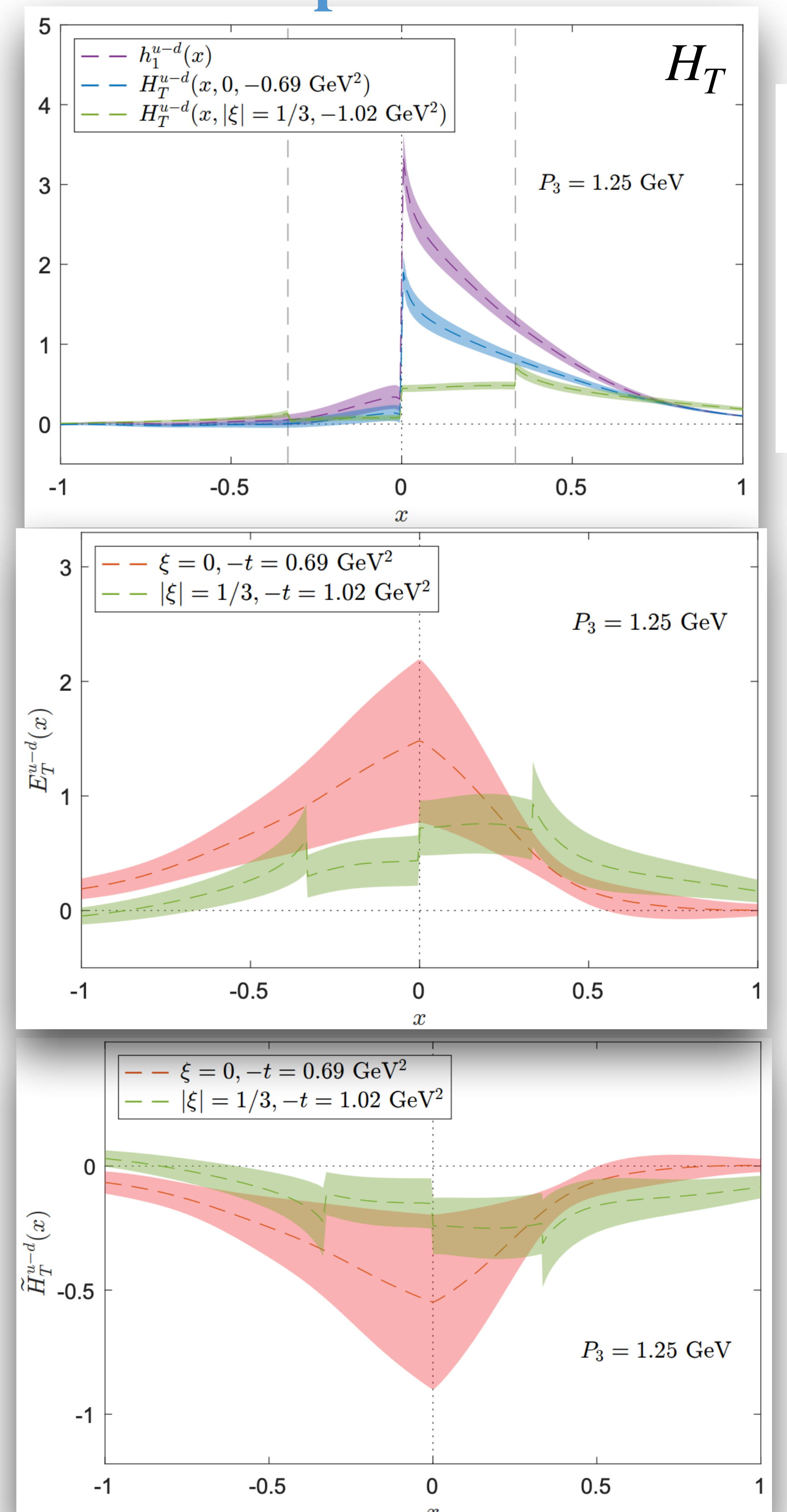
$$
\begin{aligned}
F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = {} & P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + M z^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 \\
& + M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10} \\
& + i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}
\end{aligned}
$$

$$
\begin{aligned}
\Pi^s_{01}(\Gamma_0) &= K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_2}{16m^2} + A_{T12}\frac{\left(P_3^2 - E(E+m)\right)z\Delta_2}{8m^2}\right) \\
\Pi^s_{01}(\Gamma_1) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2 + 4P_3^2 + \Delta_1^2 - \Delta_2^2\right)}{16m^2}\right. \\
&\qquad \left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right) \\
\Pi^s_{01}(\Gamma_2) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right) \\
\Pi^s_{01}(\Gamma_3) &= i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right) \\
\Pi^s_{02}(\Gamma_0) &= K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_1}{16m^2} + A_{T12}\frac{\left(E(E+m) - P_3^2\right)z\Delta_1}{8m^2}\right) \\
\Pi^s_{02}(\Gamma_1) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right) \\
\Pi^s_{02}(\Gamma_2) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m) - \Delta_1^2\right)}{8m^2}\right. \\
&\qquad \left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right) \\
\Pi^s_{02}(\Gamma_3) &= i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)
\end{aligned}
$$

[C. Alexandrou et al., PRD 105, 034501 (2022)]

Transversity GPDs

Standard parametrization



Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + M z^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5$$
$$+ M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

$$\Pi^s_{01}(\Gamma_0) = K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{(P_3^2+E(E+m))\,z\Delta_2}{16m^2} + A_{T12}\frac{(P_3^2-E(E+m))\,z\Delta_2}{8m^2}\right)$$

$$\Pi^s_{01}(\Gamma_1) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2+4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2+4P_3^2+\Delta_1^2-\Delta_2^2\right)}{16m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m)-\Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_2) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_3) = i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right)$$

$$\Pi^s_{02}(\Gamma_0) = K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{(P_3^2+E(E+m))\,z\Delta_1}{16m^2} + A_{T12}\frac{(E(E+m)-P_3^2)\,z\Delta_1}{8m^2}\right)$$

$$\Pi^s_{02}(\Gamma_1) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_2) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2+4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m)-\Delta_1^2\right)}{8m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m)-\Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right)$$

Ongoing work

$$\Pi^s_{02}(\Gamma_3) = i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)$$

[C. Alexandrou et al., PRD 105, 034501 (2022)]

How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

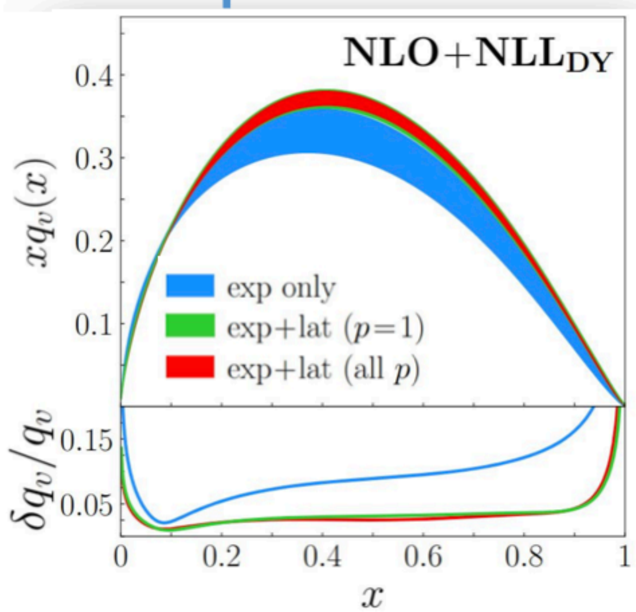
1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. Lattice QCD calculations of GPDs and related structures

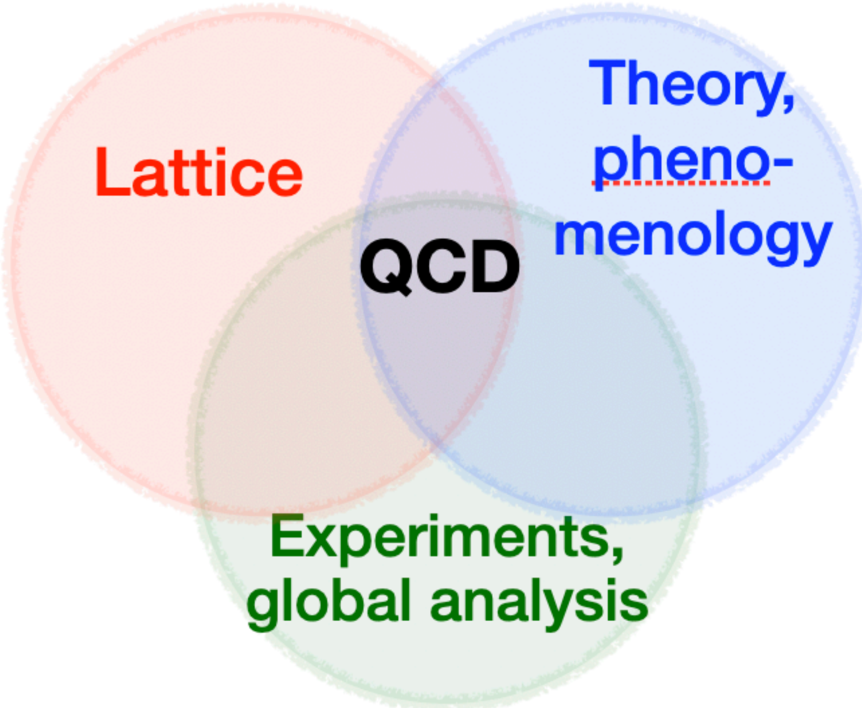
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergies: constraints & predictive power of lattice QCD

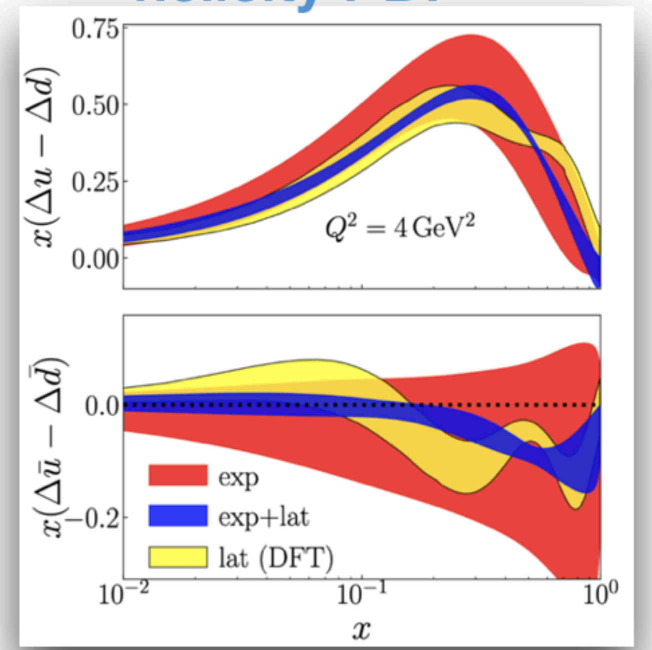
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

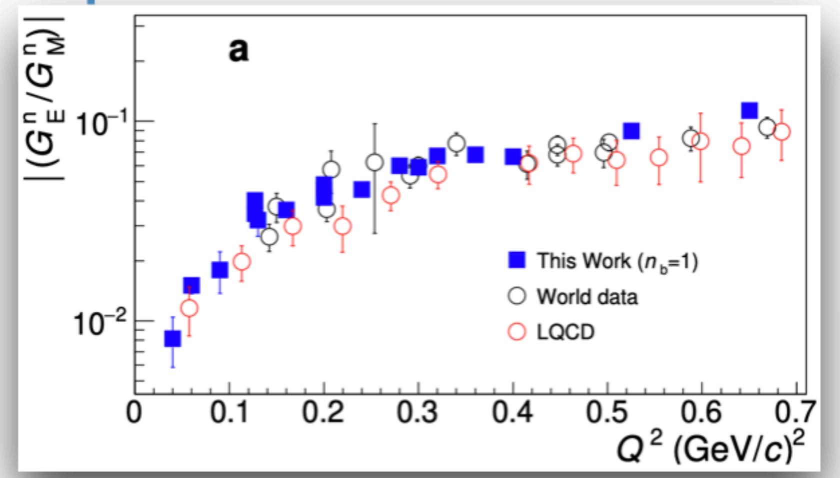


helicity PDF



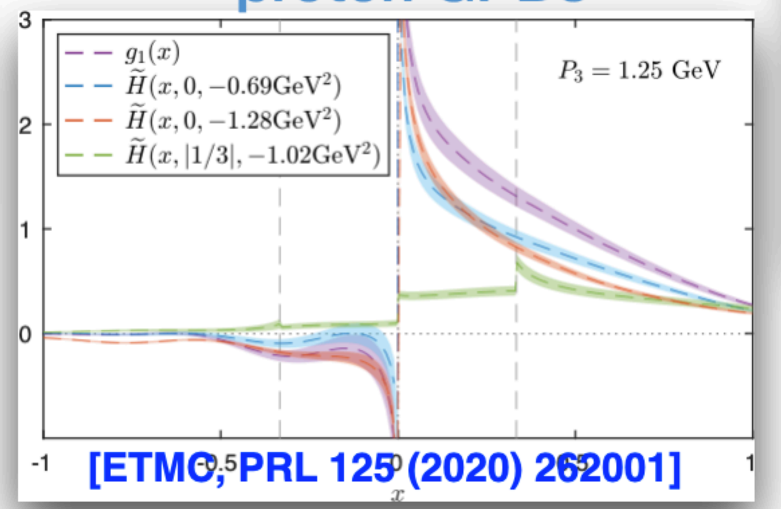
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



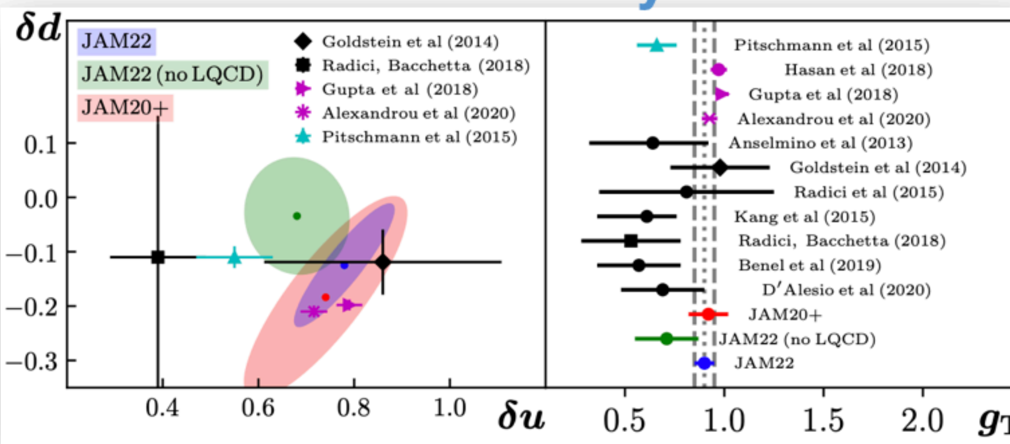
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

And many more!

Composition of QGT Collaboration

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★ ~35 students and postdocs

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★ Three bridge faculty positions will be created in nuclear theory

Stony Brook & Temple: Faculty positions in Fall 2024

QGT-related Publications

1. “Gluon helicity in the nucleon from lattice QCD and machine learning”, Khan, Liu, Sabbir Sufian, *Physical Review D*, Accepted, 2023.

2. "Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO", Bhattacharya Cichy, Constantinou, Gao, Metz, Miller, Mukherjee, Petreczky, Steffens, Zhao, *Physical Review D*, DOI: 10.1103/PhysRevD.108.014507.

3. "Generalized parton distributions through universal moment parameterization: non-zero skewness case", Guo, Ji, Santiago, Shiells, Yang, *Journal of High Energy Physics*, DOI: 10.1007/JHEP05(2023)150.

4. "Hadronic structure on the light-front VI. Generalized parton distributions of unpolarized hadrons", Shuryak, Zahed, *Physical Review D*, DOI: 10.1103/PhysRevD.107.094005.

5. "Chiral-even axial twist-3 GPDs of the proton from lattice QCD", Bhattacharya, Cichy, Constantinou, Dodson, Metz, Scapellato, Fernanda Steffens, *Physical Review D*, DOI: 10.1103/PhysRevD.108.054501.

6. "Shedding light on shadow generalized parton distributions", Moffat, Freese, Cloët, Donohoe, Gamberg, Melnitchouk, Metz, Prokudin, Sato, *Physical Review D*, DOI: 10.1103/PhysRevD.108.036027.

7. "Colloquium: Gravitational Form Factors of the Proton", Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, Shanahan, *Physical Review D*, Under Review.

8. "Synchronization effects on rest frame energy and momentum densities in the proton", Freese, Miller, eprint: 2307.11165.

9. "Proton's gluon GPDs at large skewness and gravitational form factors from near threshold heavy quarkonium photo-production", Guo, Ji, Yuan, e-Print: 2308.13006.

10. "Parton Distributions from Boosted Fields in the Coulomb Gauge", Gao, Liu, Zhao, e-Print: 2306.14960.

11. "Exactly solvable models of nonlinear extensions of the Schrödinger equation", Dodge, Schweitzer, e-Print: 2304.01183.

12. "Role of strange quarks in the D-term and cosmological constant term of the proton", Won, Kim, Kim, e-Print: 2307.00740.

13. "Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays", Ma, Feng, Gorchtein, Jin, Liu, Seng, Wang, Zhang, e-Print: 2308.16755.

Summary

★ Definitions of quasi-GPDs on Euclidean lattice: intrinsically frame dependent. Historically used symmetric frame is computationally very expensive

★ Novel Lorentz covariant decomposition has great advantages:
- access to symmetric-frame GPDs from matrix elements in any frame
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Numerical results demonstrate the validity of the approach

★ Future calculations have the potential to transform the field of GPDs

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV.

★ Synergy with phenomenology is an exciting prospect!

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Thank you

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