Trace anomaly and the $\bar{C}_{q,g}$ gravitational form factors

Yoshitaka Hatta

BNL/RIKEN BNL

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The trace anomaly

QCD Lagrangian has scale invariance classically. Broken by quantum anomaly

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^2 + m(1+\gamma_m(g))\bar{q}q$$

Fundamentally important in QCD. Trace anomaly is the origin of hadron masses

$$\langle P|T^{\mu}_{\mu}|P\rangle = 2M^2$$

Anomaly consists of two parts:

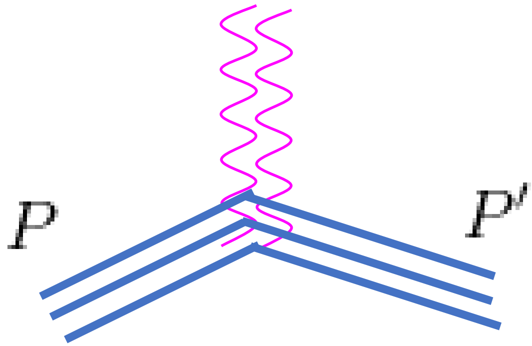
Quark condensate (`sigma term')

$$\langle P|m\bar{q}q|P\rangle$$

Gluon condensate

$$\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$$

Nucleon gravitational form factors



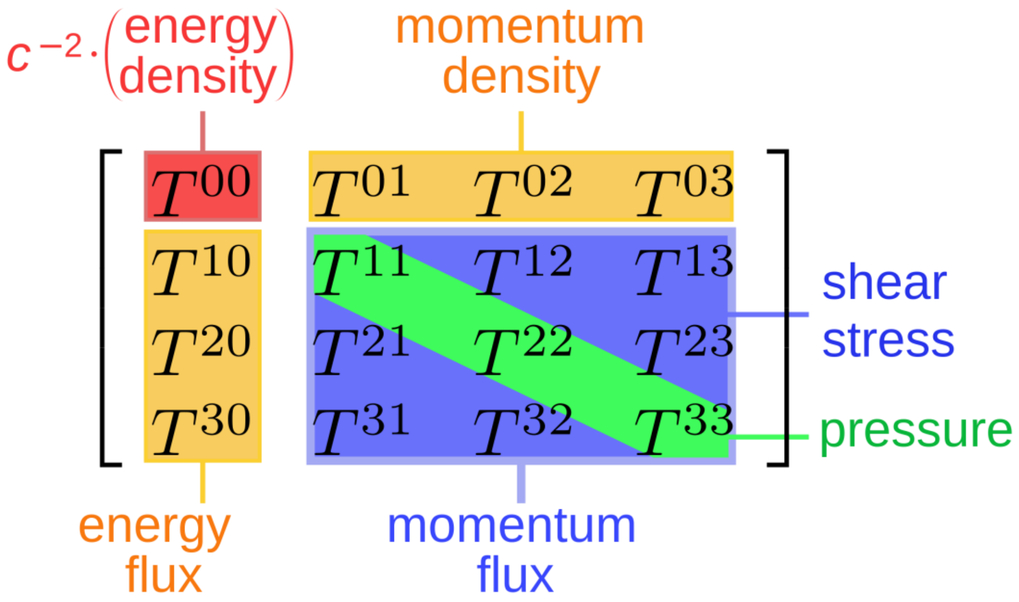
Off-forward matrix element of the QCD energy momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{\eta^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta} + \bar{\psi}i\gamma^{(\mu}D^{\nu)}\psi$$

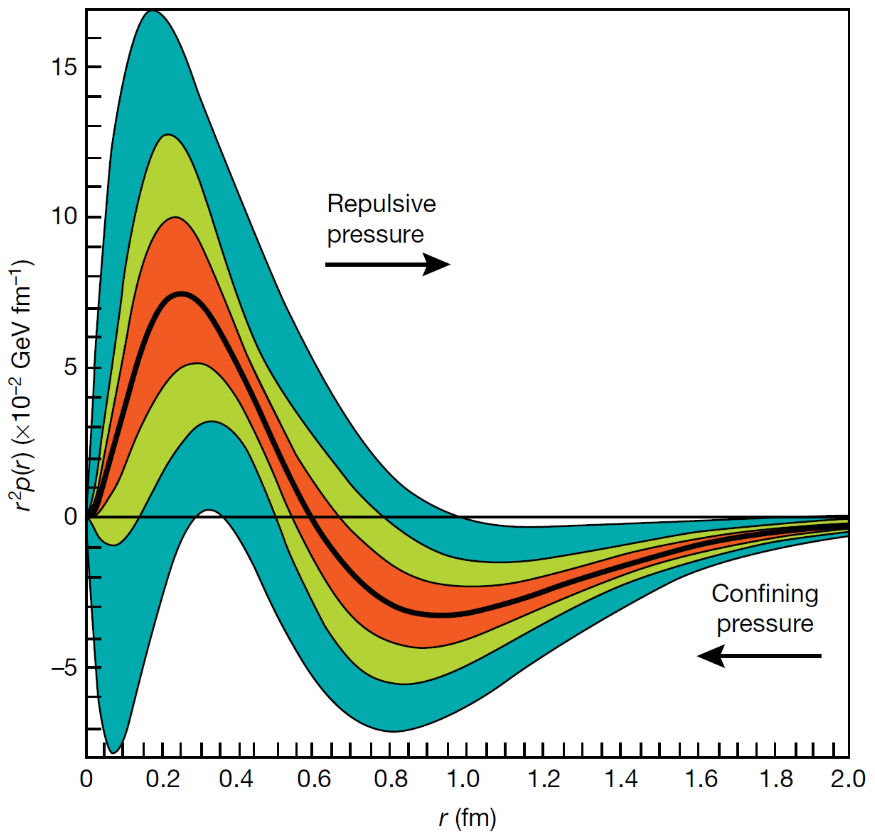
$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\left[A(t)\gamma^{(\mu}\bar{P}^{\nu)} + B(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}\right]u(P)$$

Form factors associated with graviton exchange

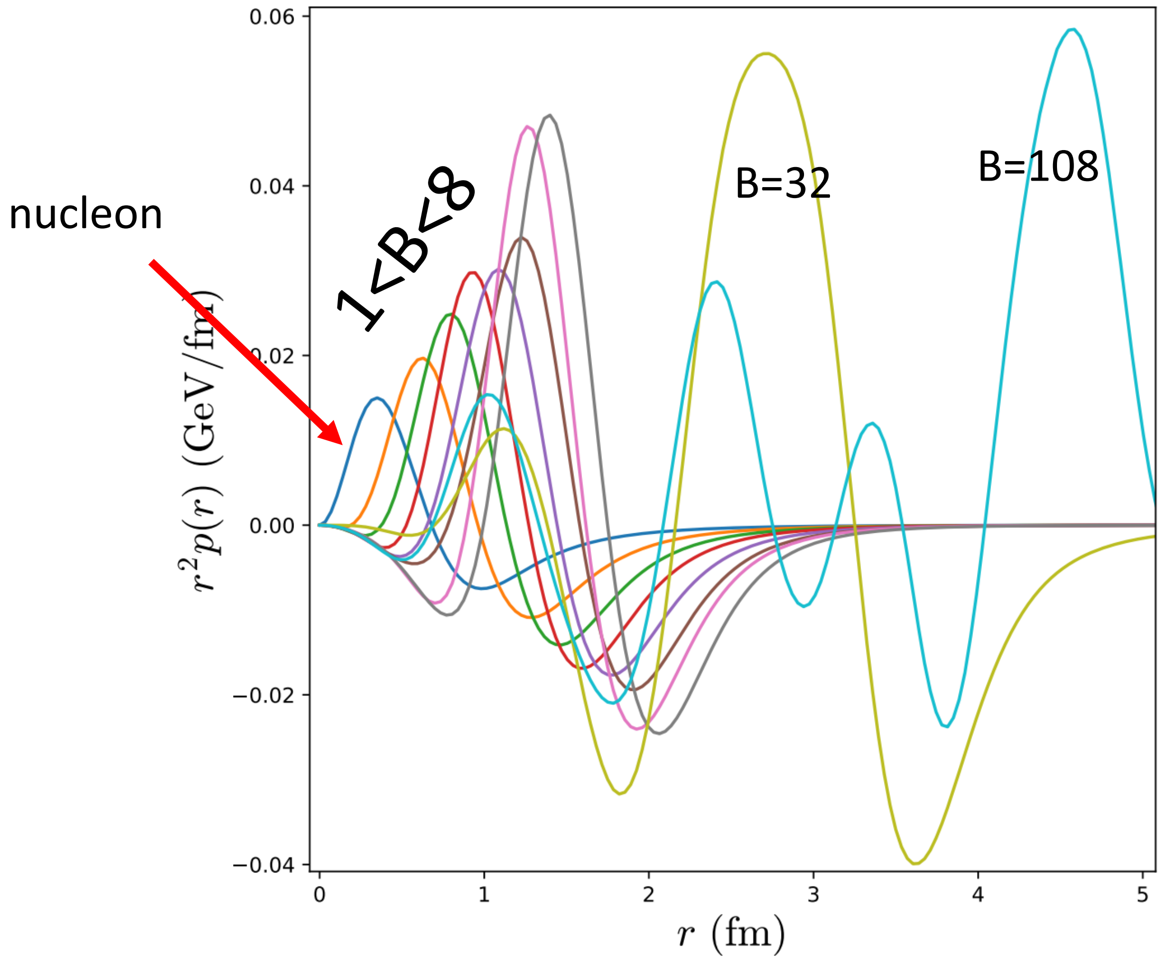
For a spin-1/2 hadron, there are 3 independent form factors.



`Pressure' inside nucleon and nuclei



Burkert, Elouadrhiri, Girod (2018)



Martin-Caro, Huidobro, YH, 2312.12984

Trace anomaly and GFFs

Form factor of the trace related to GFFs.

$$\langle P'|T^\mu_\mu|P\rangle = M\left(A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t\right)\bar{u}(P')u(P)$$

In the forward limit, $A(0) = 1$ and the other two terms vanish.

$$\langle P|T^\mu_\mu|P\rangle = 2M^2$$

A, B, D form factors cannot separately probe the quark and gluon condensate, crucial ingredients of proton mass decomposition.

$$\langle P'|F^{\mu\nu}F_{\mu\nu}|P\rangle \qquad \langle P'|m\bar{q}q|P\rangle$$

GFFs: Quark and gluon components

Energy momentum tensor consists of quark and gluon parts

$$T^{\mu\nu} = \underbrace{-F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{\eta^{\mu\nu}}{4}F^2}_{T_g^{\mu\nu}} + \underbrace{i\bar{q}\gamma^{(\mu}D^{\nu)}q}_{T_q^{\mu\nu}}$$

Introduce GFFs separately for the quark and gluon parts Ji (1996)

$$\langle P'|T_{q,g}^{\mu\nu}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

The fourth form factor $\bar{C}_{q,g}$

Non-conservation of the quark/gluon parts

$$\langle \partial_\mu T^{\mu\nu}_{q,g} \rangle \sim \Delta^\nu \bar{C}_{q,g}$$

$\bar{C}_q + \bar{C}_g = 0$ because the total EMT is conserved.

$$\langle P | (T_{q,g})^\mu_\mu | P \rangle = 2M^2 (A_{q,g} + 4\bar{C}_{q,g})$$

Introducing $\bar{C}_{q,g}$ is equivalent to computing $(T_q)^\mu_\mu$ and $(T_g)^\mu_\mu$ separately.

A delicate problem in quantum field theory.

Renormalization of $\bar{C}_{q,g}$: a first look

Polyakov, Son (2018)

RG equation for $\bar{C}_{q,g}$

$$\frac{\partial}{\partial \ln \mu}\bar{C}_q^R = -\frac{\alpha_s}{4\pi}\left(\frac{16}{3}C_F + \frac{4n_f}{3}\right)\bar{C}_q^R + \mathcal{O}(m) + \mathcal{O}(\alpha_s^2)$$

1-loop anomalous dimension

This implies, in the chiral limit,

$$\bar{C}_{q,g}^R(\mu \to \infty) \to 0 \qquad ??$$

$(T_q)^\mu_\mu$, $(T_g)^\mu_\mu$ in $\overline{\rm MS}$ at one-loop

YH, Rajan, Tanaka (2018)

$$\eta_{\mu\nu}T^{\mu\nu}_{gR} = \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}(F^2)_R + \frac{14C_F}{3}(m\bar{\psi}\psi)_R\right),$$

$$\eta_{\mu\nu}T^{\mu\nu}_{qR} = (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}(F^2)_R + \frac{4C_F}{3}(m\bar{\psi}\psi)_R\right)$$

n_f term in the 1-loop beta function

Can be systematically extended to n-loops. We need n-loop beta function and n-loop anomalous dimensions of the twist-two operators.

Result in $\overline{\mathrm{MS}}$ at two-loops

YH, Rajan, Tanaka (2018)

$$\eta_{\mu\nu}\left(T_g^{\mu\nu}\right)_R = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F\left(m\bar{\psi}\psi\right)_R - \frac{11}{6}C_A\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R + \left(\frac{28C_An_f}{27} - \frac{17C_A^2}{3} + \frac{5C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$

n_f terms contributes to the gluon part

$$\eta_{\mu\nu}\left(T_q^{\mu\nu}\right)_R = \left(m\bar{\psi}\psi\right)_R + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F\left(m\bar{\psi}\psi\right)_R + \frac{1}{3}n_f\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R + \left(\frac{17C_An_f}{27} + \frac{49C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$

Result in $\overline{\text{MS}}$ at three-loops

Tanaka, (2019)

$$
\begin{aligned}
\eta_{\mu\nu}\left(T_g^{\mu\nu}\right)_R =\ & \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F\left(m\bar{\psi}\psi\right)_R-\frac{11}{6}C_A\left(F^2\right)_R\right)+\left(\frac{\alpha_s}{4\pi}\right)^2 \\
& \times\left[\left(C_F\left(\frac{812C_A}{27}-\frac{22n_f}{27}\right)+\frac{85C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R+\left(\frac{28C_An_f}{27}-\frac{17C_A^2}{3}+\frac{5C_Fn_f}{54}\right)\left(F^2\right)_R\right] \\
& +\left(\frac{\alpha_s}{4\pi}\right)^3\bigg[\bigg\{n_f\bigg(\left(\frac{368\zeta(3)}{9}-\frac{25229}{729}\right)C_F^2-\frac{2}{243}(4968\zeta(3)+1423)C_AC_F\bigg) \\
& +\left(\frac{32\zeta(3)}{3}-\frac{91753}{1458}\right)C_AC_F^2+\left(\frac{294929}{1458}-\frac{32\zeta(3)}{9}\right)C_A^2C_F-\frac{554}{243}C_Fn_f^2 \\
& +\left(\frac{95041}{729}-\frac{64\zeta(3)}{9}\right)C_F^3\bigg\}\left(m\bar{\psi}\psi\right)_R \\
& +\bigg\{n_f\bigg(\left(\frac{1123}{162}-\frac{52\zeta(3)}{9}\right)C_AC_F+\left(4\zeta(3)+\frac{293}{36}\right)C_A^2+\frac{16}{729}(81\zeta(3)-98)C_F^2\bigg)+n_f^2\left(\frac{655C_A}{2916}-\frac{361C_F}{729}\right)-\frac{2857C_A^3}{108}\bigg\}\left(F^2\right)_R\bigg]
\end{aligned}
$$

$$
\begin{aligned}
\eta_{\mu\nu}\left(T_q^{\mu\nu}\right)_R =\ & \left(m\bar{\psi}\psi\right)_R+\frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F\left(m\bar{\psi}\psi\right)_R+\frac{1}{3}n_f\left(F^2\right)_R\right)+\left(\frac{\alpha_s}{4\pi}\right)^2 \\
& \times\left[\left(C_F\left(\frac{61C_A}{27}-\frac{68n_f}{27}\right)-\frac{4C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R+\left(\frac{17C_An_f}{27}+\frac{49C_Fn_f}{54}\right)\left(F^2\right)_R\right] \\
& +\left(\frac{\alpha_s}{4\pi}\right)^3\bigg[\bigg\{n_f\bigg(\left(\frac{64\zeta(3)}{9}-\frac{8305}{729}\right)C_F^2-\frac{2}{243}(864\zeta(3)+1079)C_AC_F\bigg) \\
& -\frac{8}{729}(972\zeta(3)+143)C_AC_F^2+\left(\frac{32\zeta(3)}{9}+\frac{6611}{729}\right)C_A^2C_F-\frac{76}{243}C_Fn_f^2 \\
& +\frac{8}{729}(648\zeta(3)-125)C_F^3\bigg\}\left(m\bar{\psi}\psi\right)_R \\
& +\bigg\{n_f\bigg(\left(\frac{52\zeta(3)}{9}-\frac{401}{324}\right)C_AC_F+\left(\frac{134}{27}-4\zeta(3)\right)C_A^2+\left(\frac{2407}{1458}-\frac{16\zeta(3)}{9}\right)C_F^2\bigg) \\
& +n_f^2\left(-\frac{697C_A}{729}-\frac{169C_F}{1458}\right)\bigg\}\left(F^2\right)_R\bigg],
\end{aligned}
$$

Result in $\overline{\mathrm{MS}}$ at four-loops

Ahmed, Chen, Czakon (2022)

$$\left\langle \mathrm{Tr}\left([\Theta_g]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}} \left(-0.437676\,\alpha_s - 0.261512\,\alpha_s^2 - 0.183827\,\alpha_s^3 - 0.256096\,\alpha_s^4\right)$$
$$+ \langle [O_m]_R\rangle_{\mathrm{P}} \left(0.495149\,\alpha_s + 0.776587\,\alpha_s^2 + 0.865492\,\alpha_s^3 + 0.974674\,\alpha_s^4\right) ,$$

$$\left\langle \mathrm{Tr}\left([\Theta_q]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}} \left(0.079578\,\alpha_s + 0.058870\,\alpha_s^2 + 0.021604\,\alpha_s^3 + 0.013675\,\alpha_s^4\right)$$
$$+ \langle [O_m]_R\rangle_{\mathrm{P}} \left(1 + 0.141471\,\alpha_s - 0.008235\,\alpha_s^2 - 0.064351\,\alpha_s^3 - 0.065869\,\alpha_s^4\right)$$

Very precise relation between $\bar{C}_{q,g}$ and $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$!

Measuring $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ is equivalent to measuring $\bar{C}_{q,g}$

Renormalization of $\bar{C}_{q,g}$: the real thing

Return to

$$\frac{\partial}{\partial \ln \mu}\bar{C}_q^R = -\frac{\alpha_s}{4\pi}\left(\frac{16}{3}C_F + \frac{4n_f}{3}\right)\bar{C}_q^R + \mathcal{O}(m) + \mathcal{O}(\alpha_s^2)$$

Correct result at $\mathcal{O}(\alpha_s)$

$$\frac{\partial \bar{C}_q^R}{\partial \ln \mu} = -\frac{\alpha_s}{4\pi}\left(\frac{16C_F}{3} + \frac{4n_f}{3}\right)\bar{C}_q^R + \frac{\alpha_s}{4\pi}\left[\frac{4C_F}{3}\frac{\langle P|(m\bar{\psi}\psi)_R|P\rangle}{2M^2} + \frac{n_f}{3}\left(\frac{\langle P|(m\bar{\psi}\psi)_R|P\rangle}{2M^2} - 1\right)\right]$$

↑ $\mathcal{O}(m)$

↑ Naively $\mathcal{O}(\alpha_s^2)$, but promoted to $\mathcal{O}(\alpha_s)$ due to trace anomaly!

$$\alpha_s F^2 \sim \mathcal{O}(1)$$

$$\bar{C}_q^R(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}$$
$$-\frac{4C_F A_q^R(\mu_0)+n_f\left(A_q^R(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}}$$
$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f\left(-\frac{34C_A}{27}-\frac{49C_F}{27}\right)}{4\beta_0}+\frac{\beta_1 n_f}{6\beta_0^2}\right.$$
$$\left.+\frac{1}{4}\left(\frac{n_f\left(\frac{34C_A}{27}+\frac{157C_F}{27}\right)}{\beta_0}+\frac{4C_F}{3}-\frac{2\beta_1 n_f}{3\beta_0^2}\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}\right]+\cdots,$$
$$\simeq -0.146-0.25\left(A_q^R(\mu_0)-0.36\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}}-0.01\alpha_s(\mu)$$
$$+\left(0.306+0.08\alpha_s(\mu)\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2},$$

Asymptotic value
in the chiral limit
$(n_f = 3)$

Accessing the gluon condensate $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ in experiments

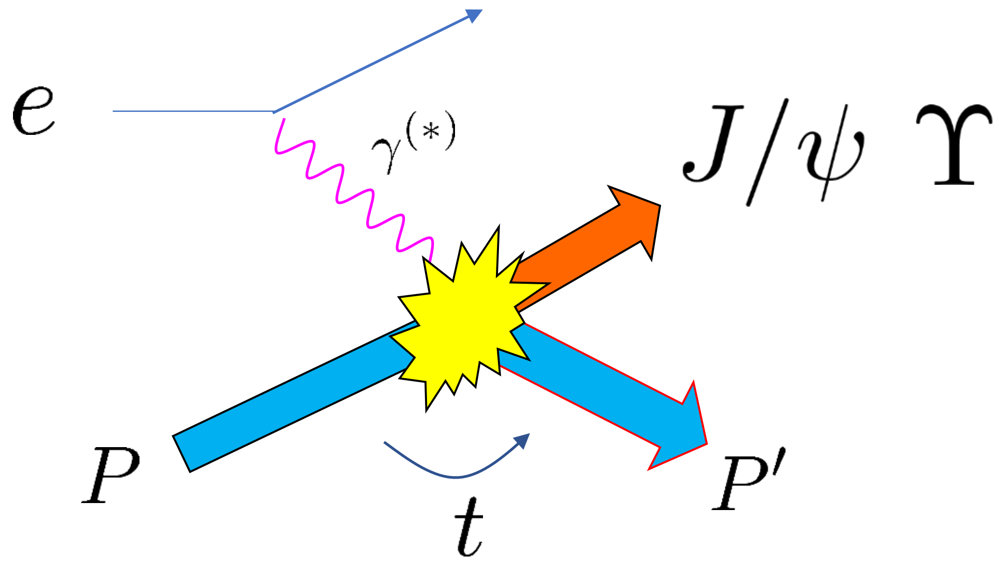
The operator $F^{\mu\nu}F_{\mu\nu}$ is twist-four,
highly suppressed in high energy scattering.

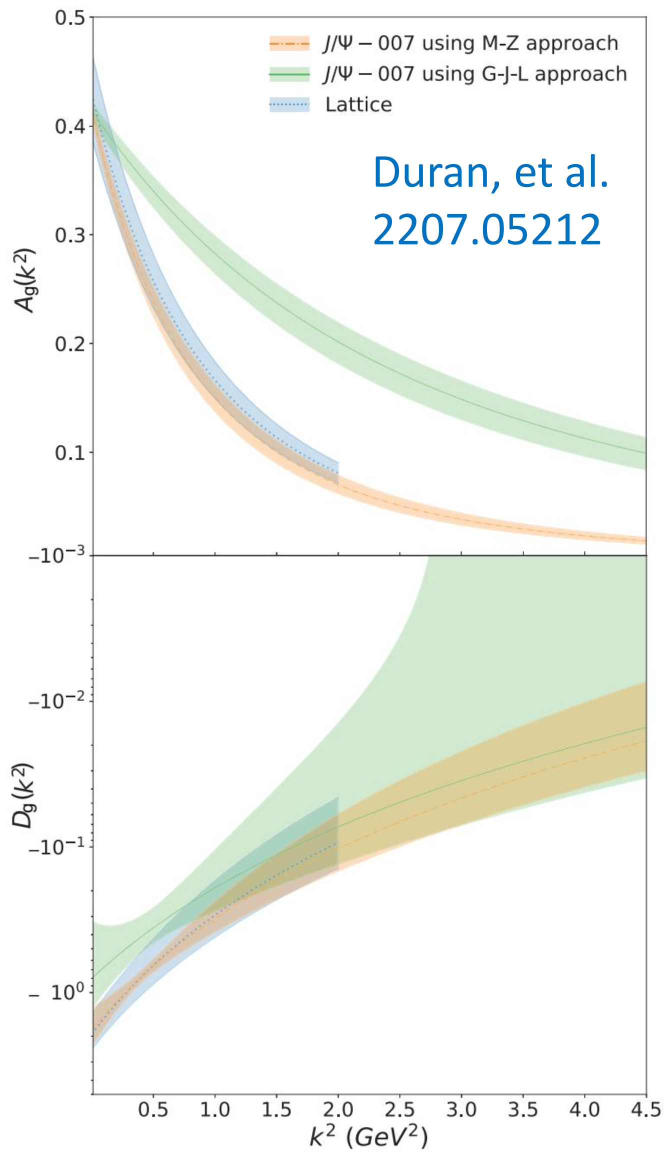
Instead, we should look at low-energy scattering.

Purely gluonic operator. Use quarkonium as a probe.

→ J/ψ photo- or electro-production near threshold.

Quarkonium photo-(electro-)production near threshold





Ongoing experiments at JLab, future measurement at EIC?

Originally proposed by Kharzeev, Satz, Syamtomov, Zinovev (1997) to probe the gluon condensate. $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$

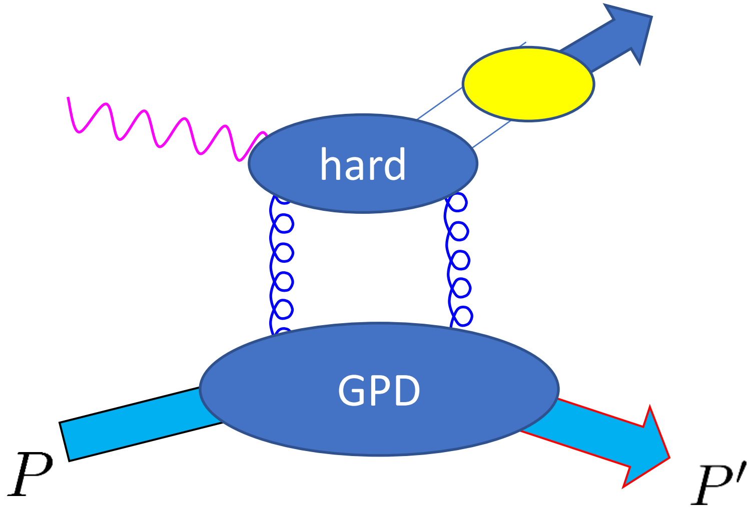
One can also study gluon GFFs in this process

YH, Yang (2018); Mamo, Zahed (2019~)

GPD factorization

Quarkonium photo-production in the limit $M_{QQ} \to \infty$
Computed to NLO in the GPD framework.

Ivanov, Schafer, Szymanowski, Krasnikov (2004)



Amplitude proportional to Compton form factor

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

Gluon GPD

Skewness $\xi = \frac{P^+ - P'^+}{P + +P'^+}$

Connection to GFFs?

Skewness

Threshold production characterized by large values of skewness

YH, Strikman (2021)
Guo, Ji, Liu (2021)

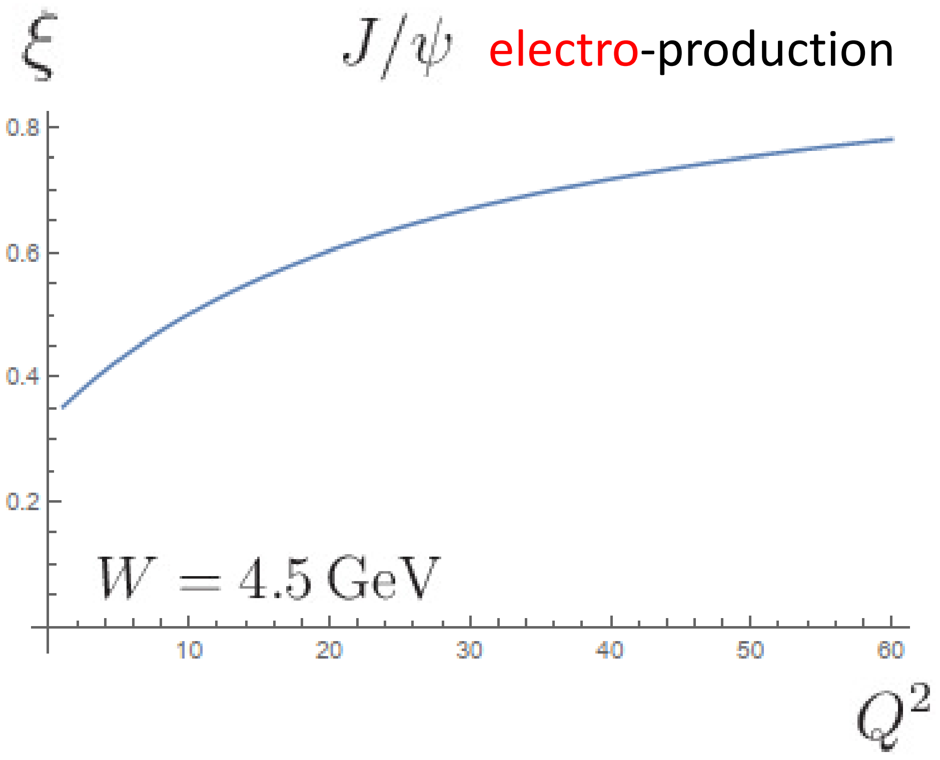
$\xi \approx 1$ in the ideal limit $Q^2 \to \infty$ or $m_V \to \infty$

Enhanced sensitivity to the energy momentum tensor.

$\xi < 1$ in practice, but still promising. Guo, Ji, Yuan (2023)

But sensitive only to $\langle T^{++} \rangle$

Access to A_g, D_g form factors, but no sensitivity to gluon condensate in the GPD approach.

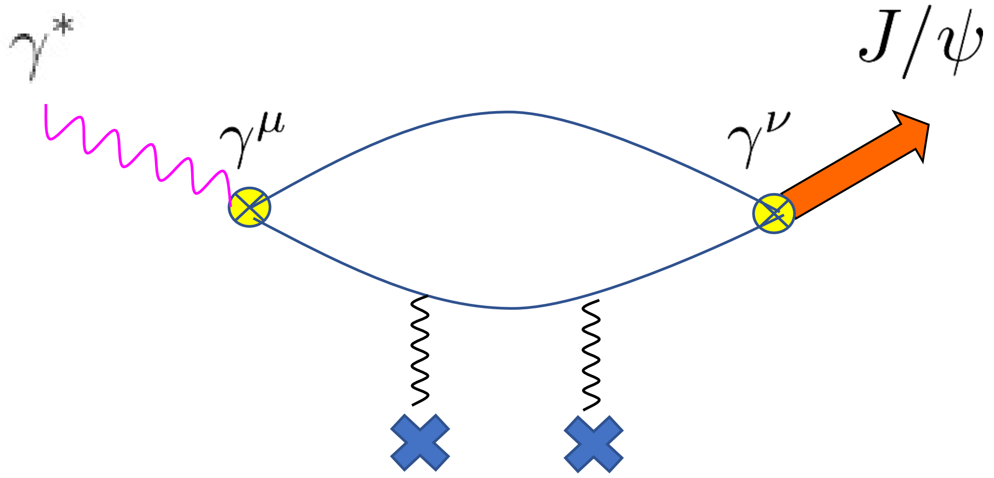


Electro-production near threshold

Boussarie, YH (2020)

Amplitude dominantly real near threshold
Dominated by EMT (local operator) when $\xi \approx 1$

→ OPE situation



$$i\int d^4r e^{ir\cdot q}\bar{c}\gamma^\mu c(0)\bar{c}\gamma^\nu c(-r)$$

$$\approx -\frac{\alpha_s(\mu_R)}{3\pi q^2}\left[2\ln(-q^2/\mu_R^2)\left\{\left(g^{\mu\alpha}-\frac{q^\mu q^\alpha}{q^2}\right)\left(g^{\nu\beta}-\frac{q^\nu q^\beta}{q^2}\right)+\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\right\}\hat{T}^g_{\alpha\beta}(0)\right.$$

$$\left.-2\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\hat{T}^g_{\alpha\beta}(0)+3\frac{q_\alpha q_\beta}{q^2}F^{\mu\alpha}F^{\nu\beta}(0)\right],$$

twist-2, sensitive to A_g, D_g

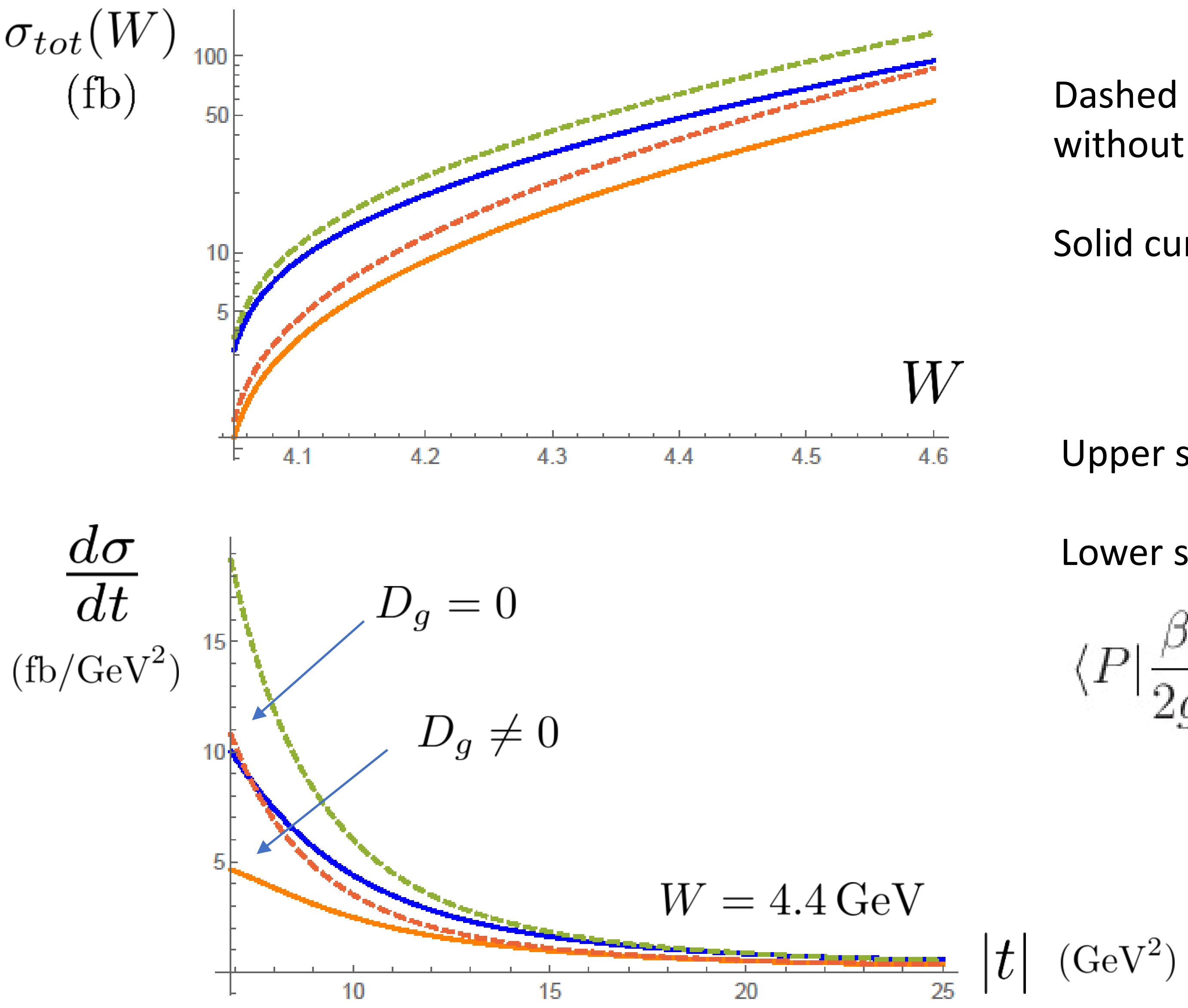
twist-2 & 4, sensitive to gluon condensate

Two-gluon operator with 4 open indices

$$
\begin{aligned}
\langle p'| - F_a^{\mu\alpha} F_a^{\nu\beta} |p\rangle \; = \; & \frac{A}{2}\bar{u}(p')(g^{\mu\nu}\gamma^{(\alpha}P^{\beta)} - g^{\mu\beta}\gamma^{(\alpha}P^{\nu)} - g^{\alpha\nu}\gamma^{(\mu}P^{\beta)} + g^{\alpha\beta}\gamma^{(\mu}P^{\nu)})u(p) \\
& + \frac{B}{4m_N}\bar{u}(p')\left(g^{\mu\nu}i\sigma^{(\alpha\lambda}\Delta_\lambda P^{\beta)} - g^{\mu\beta}i\sigma^{(\alpha\lambda}\Delta_\lambda P^{\nu)} - g^{\alpha\nu}i\sigma^{(\mu\lambda}\Delta_\lambda P^{\beta)} + g^{\alpha\beta}i\sigma^{(\mu\lambda}\Delta_\lambda P^{\nu)}\right)u(p) \\
& + \frac{D}{8m_N}\bar{u}(p')\left(g^{\mu\nu}\Delta^\alpha\Delta^\beta - g^{\alpha\nu}\Delta^\mu\Delta^\beta + g^{\alpha\beta}\Delta^\mu\Delta^\nu - g^{\mu\beta}\Delta^\alpha\Delta^\nu\right)u(p) \\
& + \frac{W}{3}m_N\bar{u}(p')(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\alpha\nu})u(p) \\
& + \frac{X}{2m_N^2}\bar{u}(p')\left((\gamma^\mu\Delta^\alpha - \gamma^\alpha\Delta^\mu)(P^\nu\Delta^\beta - P^\beta\Delta^\nu) + (P^\mu\Delta^\alpha - P^\alpha\Delta^\mu)(\gamma^\nu\Delta^\beta - \gamma^\beta\Delta^\nu)\right)u(p) \\
& + \frac{Y}{m_N^3}\bar{u}(p')(P^\mu\Delta^\alpha - P^\alpha\Delta^\mu)(P^\nu\Delta^\beta - P^\beta\Delta^\nu)u(p) \\
& + \frac{Z}{4m_N}\bar{u}(p')\left(i\sigma^{\mu\alpha}(P^\nu\Delta^\beta - P^\beta\Delta^\nu) + i\sigma^{\nu\beta}(P^\mu\Delta^\alpha - P^\alpha\Delta^\mu)\right)u(p), \qquad 30)
\end{aligned}
$$

Related to $\bar{C}_{q,g}$ → related to gluon condensate $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$

J/ψ $\quad Q^2 = 64\,\mathrm{GeV}^2 \quad \sqrt{S_{ep}} = 20\,\mathrm{GeV}$ (plots revised from 2004.12715)



Dashed curves:
without gluon D-term

Solid curves: with gluon D-term

Upper solid $\quad b = 1$

Lower solid $\quad b = 0$

$$\langle P | \frac{\beta}{2g} F^2 | P \rangle = 2M^2(1-b)$$

Summary

- $\bar{C}_{q,g}$ form factor key to understand the nucleon mass decomposition. Very precise (4-loop) relation to gluon condensate $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$

- Threshold quarkonium production at $\xi \approx 1$ promising for A_g, D_g

- Leading twist GPD approach cannot probe $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$.

- Estimated the impact of $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$, can be numerically significant.

  Remember $\alpha_s F^2 \sim \mathcal{O}(1)$