Hadron densities on the light front

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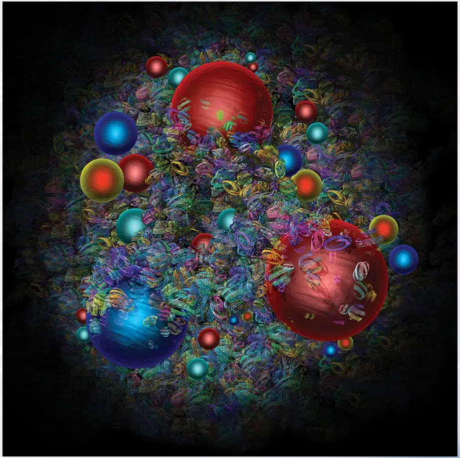
- The proton is a rich and complicated system.
- It's more than just three quarks.
  - The masses don't even add up.

$$2m_u + m_d \approx 9.4 \text{ MeV}$$
$$m_p \approx 940 \text{ MeV}$$

- Quark-gluon interactions somehow generate mass.
- How does this happen?
- …and **where is the mass inside the proton?**
  - This is what **imaging** is all about.



On right: artist's impression of the proton, CERN

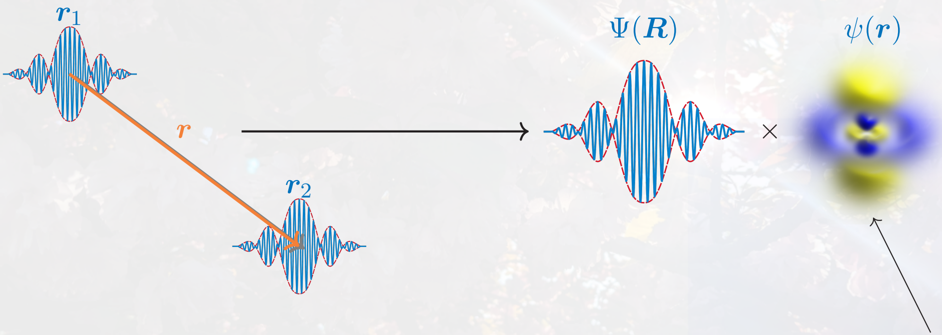
Outline

1. **The light front**
   - How do spatial densities work for quantum relativistic systems?
2. **Factorization framework**
   - How can internal structure and wave packet artifacts be separated?
3. **Spin-half systems**
   - What complications does spin introduce?
4. **The energy-momentum tensor**
   - Where is the mass inside the proton?

I. Why use the light front?

Positions and wave packets

- Quantum objects have two kinds of spatial extent:
  1. Distance between constituents
  2. Wave packet size



We want *this!*

- Reduce to overall wave packet and **internal structure**

$$\psi_{\text{total}}(\boldsymbol{r}_1, \boldsymbol{r}_2, t) = \Psi(\boldsymbol{R}, t)\psi(\boldsymbol{r}, t)$$

  - Internal structure is the interesting part

Imaging with Fourier transforms

- We measure structure via scattering
- This gives momentum info
- Get position info with **Fourier transform**:

$$\psi(\boldsymbol{r},t) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\widetilde{\psi}(\boldsymbol{p})\,\mathrm{e}^{\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{r}-Et)}$$

- Only works when wave packets factorize:

$$\psi_{\text{total}}(\boldsymbol{r}_1,\boldsymbol{r}_2,t) = \Psi(\boldsymbol{R},t)\psi(\boldsymbol{r},t)$$

- **Relativity** makes this break down

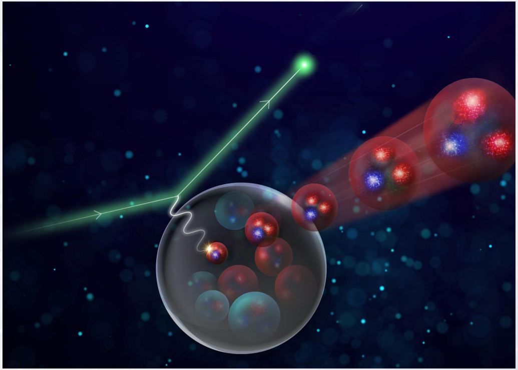
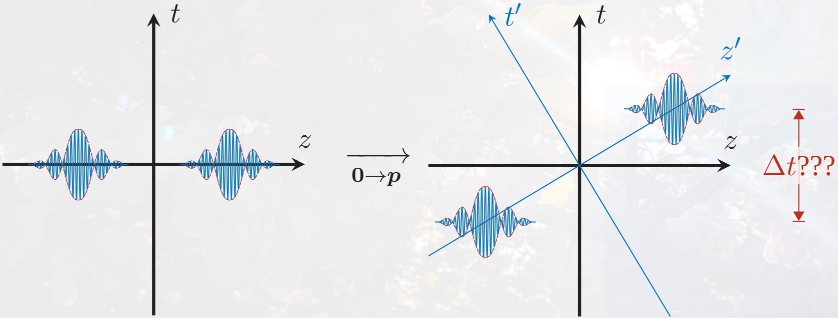


Image credit: Jefferson Lab

Relativity of simultaneity

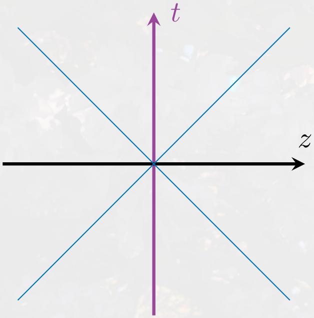
- Momentum-space wave packet contains boosted versions of composite system
  - Boosts mix up planes of simultaneity
  - Internal constituents get boosted to **different times**



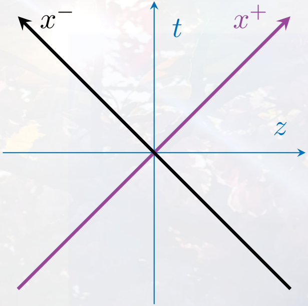
- Cannot decompose structure to overall wave packet $\otimes$ internal structure at fixed t
- **Light front coordinates** fix this by defining a **new, boost-invariant time variable**!

- Partonic images are described by **light front coordinates**.
- **Light front coordinates** are a different foliation of spacetime.
  - Involves redefining *equal-time surfaces*.

$$x^{\pm} = t \pm z \qquad \boldsymbol{x}_{\perp} = (x, y) \qquad x^{+} = t + z = \text{time}$$

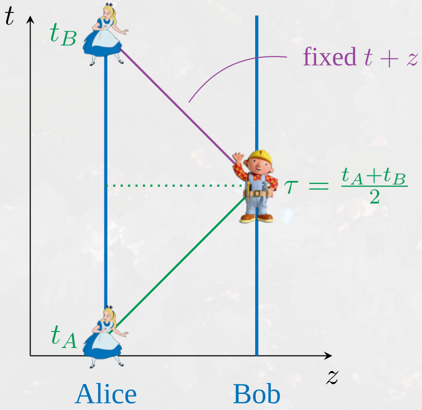


Minkowski coordinates



Light front coordinates

Synchronization vs. seeing



- Fixed t surfaces require coordination
  - Distant clocks must be synchronized
  - Must wait for light to arrive
  - Reconstruct after the fact
  - **Ruined by boosts**
- Fixed x^+ surfaces require looking
  - Look in the $+z$ direction
  - That's a fixed x^+ surface
  - **Invariant under boosts!**
  - Allow relativistic densities

Transverse boosts and Terrell rotations

- Lorentz-boosted objects *appear rotated*.
  - **Terrell rotation** (PR116, 1959)
  - Optical effect: contraction + delay

- **Light front transverse boost** *undoes* Terrell rotation:

$$B_x^{(\text{LF})} = K_x - J_y$$

  - Standard boost + counter-rotation
  - Leaves x^+ (time) invariant
  - Part of the **Galilean subgroup**

Dice images by Ute Kraus,
https://www.spacetimetravel.org/

Galilean subgroup

- Poincaré group has a $(2+1)$D **Galilean subgroup**.
  - x^+ is time and $\boldsymbol{x}_\perp$ is space under this subgroup.
  - $P^+ = E_{\boldsymbol{p}} + p_z$ is the central charge.
  - x^+ and P^+ are invariant under this subgroup!
- Light front time gives **fully relativistic** 2D picture that looks a lot like non-relativistic physics.
  - But with P^+ in place of m.

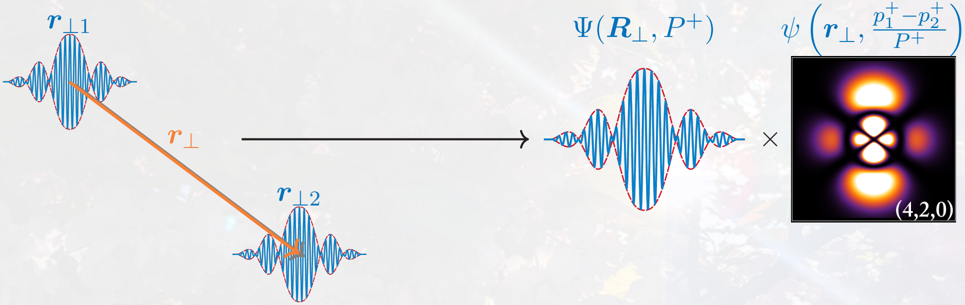
$$\frac{\mathrm{d}\boldsymbol{P}_\perp}{\mathrm{d}x^+} = P^+ \frac{\mathrm{d}^2\boldsymbol{x}_\perp}{\mathrm{d}{x^+}^2}$$

$$H = H_{\text{rest}} + \frac{\boldsymbol{P}_\perp^2}{2P^+}$$

$$\boldsymbol{v}_\perp = \frac{\boldsymbol{P}_\perp}{P^+}$$

Galilean subgroup and densities

- Wave packet separation works for *transverse* spatial coordinates



  - Works thanks to the Galilean subgroup
- Stuck with 2D spatial densities
- **Generalized parton distributions** give back a third dimension
  - But the third dimension *must be* a momentum

See AF & Miller PRD108 (2023) 034008 for formal info

II. Factorization framework

Inadequacy of localization

- For some densities, can use localized wave packets:

$$j^+_{\text{internal}}(\boldsymbol{b}_\perp) \equiv \langle \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, P^+ | \hat{J}^+(\boldsymbol{b}_\perp, x^+ = 0) | \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, P^+ \rangle$$

  - Transverse localization possible because of Galilean subgroup.
- For other densities, this doesn't work:

$$\langle \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, P^+ | T^{ij}(\boldsymbol{b}_\perp, x^+ = 0) | \boldsymbol{R}_\perp = \boldsymbol{0}_\perp, P^+ \rangle \sim \langle P^i_\perp P^j_\perp \rangle \to \infty$$

  - Consequence of uncertainty principle.
- Can instead factorize physical expectation value:

$$\langle \psi | \hat{J}^\mu(x) | \psi \rangle = \text{wave packet dependence} \otimes \text{internal structure}$$

  - Yang Li *et al*: PLB (2023), 2405.06892
  - AF & Miller: PRD108 (2023)

Electromagnetic densities

- *Physical* four-current given by quantum expectation value:

$$\langle\psi|\hat{J}^\mu(x)|\psi\rangle = \int \frac{\mathrm{d}p^+\,\mathrm{d}^2\boldsymbol{p}_\perp}{2p^+(2\pi)^3}\int \frac{\mathrm{d}p'^+\,\mathrm{d}^2\boldsymbol{p}'_\perp}{2p'^+(2\pi)^3}\langle\psi|p'\rangle\langle p'|\hat{J}^\mu(x)|p\rangle\langle p|\psi\rangle$$

  - Depends on wave packet—not entirely internal.
- Light front allows exact factorization:

$$\int \mathrm{d}x^3\,\langle\psi|\hat{j}^\mu(x)|\psi\rangle = \int \mathrm{d}^3\boldsymbol{R}\,\mathscr{P}^\mu{}_\nu(\boldsymbol{R},x^+,\psi)\,j^\nu_{\text{internal}}(\boldsymbol{x}_\perp-\boldsymbol{R}_\perp)$$

Internal density

Smearing function

invariant under LF boosts

  - Move wave packet dependence into **smearing function**.
  - Call what remains the "internal" density.
  - Only possible on light front: proof in AF & Miller, PRD107 (2023)

Factorization ambiguity

- The factorization is not unique!

$$\int \mathrm{d}x^3 \, \langle\psi|\hat{j}^\mu(x)|\psi\rangle = \int \mathrm{d}^3\boldsymbol{R}\, \mathscr{P}^\mu{}_\nu(\boldsymbol{R}, x^+, \psi)\, j^\nu_{\text{internal}}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)$$

Internal density

Smearing function

- Can shuffle terms between between smearing function & internal density.
- Cannot pick out “true” internal density.
- Separation can only be a matter of convention.

Factorization ambiguity

- The factorization is not unique!

$$\int \mathrm{d}x^3 \, \langle \psi | \hat{j}^\mu(x) | \psi \rangle = \int \mathrm{d}^3\boldsymbol{R} \, \frac{1}{2} \mathscr{P}^\mu{}_\nu(\boldsymbol{R}, x^+, \psi) 2 j^\nu_{\text{internal}}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)$$

Internal density?

Smearing function

- Can shuffle terms between between smearing function & internal density.
  - Could move a constant.
- Cannot pick out “true” internal density.
- Separation can only be a matter of convention.

Factorization ambiguity

- The factorization is not unique!

$$\int \mathrm{d}x^3 \, \langle\psi|\hat{j}^\mu(x)|\psi\rangle = \int \mathrm{d}^3\boldsymbol{R}\, \mathscr{P}^\mu{}_\sigma(\boldsymbol{R}, x^+, \psi)(\Lambda^{-1})^\sigma{}_\nu \Lambda^\nu{}_\rho j^\rho_{\text{internal}}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)$$

Internal density?

Smearing function

- Can shuffle terms between between smearing function & internal density.
  - Could move a constant.
  - Could move a Lorentz transform!
- Cannot pick out “true” internal density.
- Separation can only be a matter of convention.

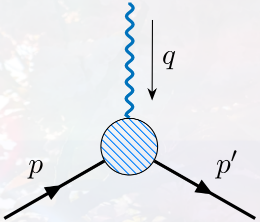
- Need momentum transfer to obtain form factors:

$$\langle p'|\hat{J}^\mu(x)|p\rangle = 2P^\mu F(q^2)\,\mathrm{e}^{\mathrm{i}q\cdot x}$$

- For a reference frame S:

$$j^\mu_S(\boldsymbol{b}_\perp) \equiv \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} F(-\boldsymbol{q}_\perp^2)\,\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

  is a valid "internal density".
  - Can always satisfy factorization formula …
  - …*provided* that $q^+ = 0$.
  - Proof in AF, in prep.
- What's a sensible convention?



$$P = \frac{1}{2}(p+p')$$

Pseudo-rest frame

- Need momentum transfer to obtain form factors:

$$\langle p' | \hat{J}^\mu(x) | p \rangle = 2P^\mu F(q^2)\, \mathrm{e}^{\mathrm{i}q\cdot x}$$

  - *Cannot* have $|p\rangle$ & $|p'\rangle$ both at rest.
- **Pseudo-rest frame**: pick frame where system is at rest *on average*.
  - And also, $q^+ = 0$.
  - Two sensible choices:

Drell-Yan frame

$$P^+ = m$$
$$\boldsymbol{P}_\perp = 0$$
$$P^- = m + \frac{\boldsymbol{q}_\perp^2}{4m}$$

- Definite P^+, indefinite P_z.
- Longitudinal velocity not zero!

2D Breit frame

$$E = m\sqrt{1 + \frac{\boldsymbol{q}_\perp^2}{4m^2}}$$
$$\boldsymbol{P} = 0$$

- Indefinite P^+.
- Longitudinal velocity is zero!

Four-current: spin zero

- **Drell-Yan frame** vs. **Breit frame**:

$$j^+_D(\boldsymbol{b}_\perp) = j^+_B(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} F(-\boldsymbol{q}^2_\perp)\,\mathrm{e}^{-\,\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

$$\boldsymbol{j}^\perp_D(\boldsymbol{b}_\perp) = \boldsymbol{j}^\perp_B(\boldsymbol{b}_\perp) = 0$$

$$j^3_D(\boldsymbol{b}_\perp) = -\int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\boldsymbol{q}^2_\perp}{8m^2} F(-\boldsymbol{q}^2_\perp)\,\mathrm{e}^{-\,\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

$$j^3_B(\boldsymbol{b}_\perp) = 0$$

- **Drell-Yan frame**: "internal" longitudinal current, due to non-zero velocity.
- **Breit frame**: No longitudinal current, as expected.
- Produce the same *physical* current, but **Breit frame** attributes longitudinal currents to wave packet dispersion.

III. Spin-half systems

Factorization formalism with spin

- For spinning systems, have **spin indices**:

$$\int \mathrm{d}x^3 \langle\psi|\hat{j}^\mu(x)|\psi\rangle = \sum_{\lambda,\lambda'} \int \mathrm{d}^3\boldsymbol{R}\, \mathscr{P}^\mu{}_\nu(\boldsymbol{R}, x^+, \psi, \lambda, \lambda') j^\nu_{\text{internal}}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp, \lambda, \lambda')$$

- Appropriate spin label is the **light front helicity**.
  - From $+$ component of Pauli-Lubanski pseudovector:

$$\lambda = \frac{W^+}{P^+} = -\frac{1}{2P^+}\epsilon^{+\nu\rho\sigma}J_{\nu\rho}P_\sigma = J_3 - \frac{(\boldsymbol{B}_\perp \times \boldsymbol{P}_\perp)\cdot\hat{z}}{P^+}$$

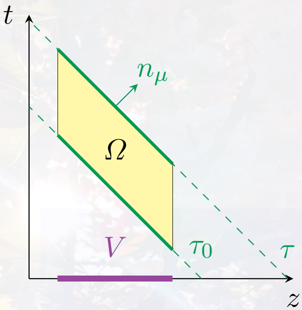
  - Invariant under light front boosts!
  - Equal to spin along z axis in rest frame.
  - Equal to helicity in infinite-momentum frame.
- Spin label appears in matrix elements:

$$\langle p', \lambda'|\hat{J}^\mu(0)|p,\lambda\rangle = \bar{u}(p',\lambda')\left\{\gamma^\mu F_1(q^2) + \frac{\mathrm{i}\sigma^{\mu q}}{2m}F_2(q^2)\right\}u(p,\lambda)$$

Charge density

- Charge density *at fixed* $x^+ = t + z$.
  - Since we're using light front synchronization.
- Charge density given by j^+.
- Temporal part of continuity equation:

$$\partial_\mu j^\mu = \frac{\partial j^+}{\partial x^+} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$$



- Simple formula due to invariance under **Galilean subgroup**:

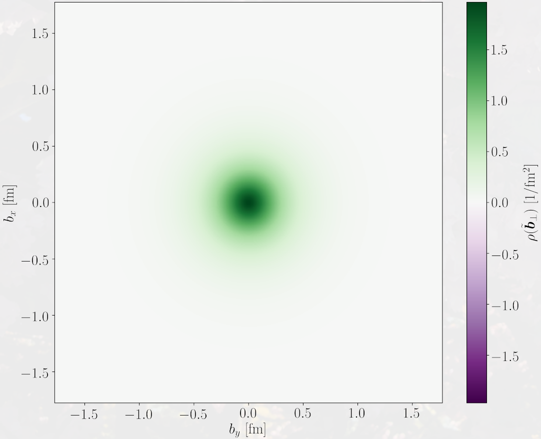
$$j^+_{\text{internal}}(\boldsymbol{b}_\perp, \hat{\boldsymbol{s}}) = \int \frac{\mathrm{d}^2 \boldsymbol{q}_\perp}{(2\pi)^2} \frac{\langle p', \hat{\boldsymbol{s}} | \hat{j}^+(0) | p, \hat{\boldsymbol{s}} \rangle}{2p^+} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

  - **Frame-independent result!**

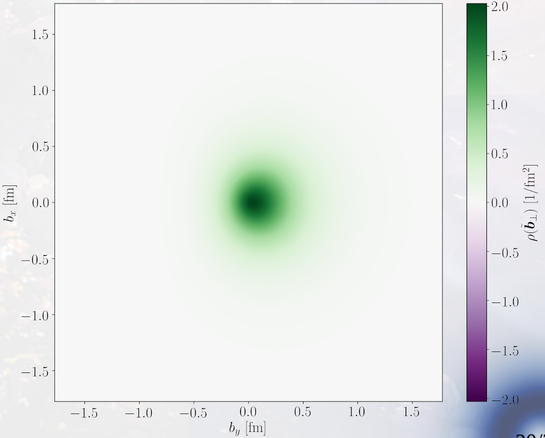
Proton charge density

$$j^{+}_{\text{internal}}(\boldsymbol{b}_\perp, \hat{\boldsymbol{s}}) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \left(F_1(-\boldsymbol{q}_\perp^2) + \frac{(\hat{\boldsymbol{s}} \times \mathrm{i}\boldsymbol{q}_\perp)\cdot\hat{z}}{2m} F_2(-\boldsymbol{q}_\perp^2) \right) \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \,,$$

Longitudinal polarization

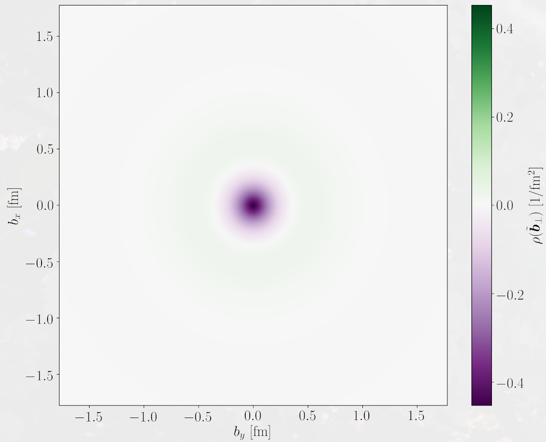


Transverse polarization

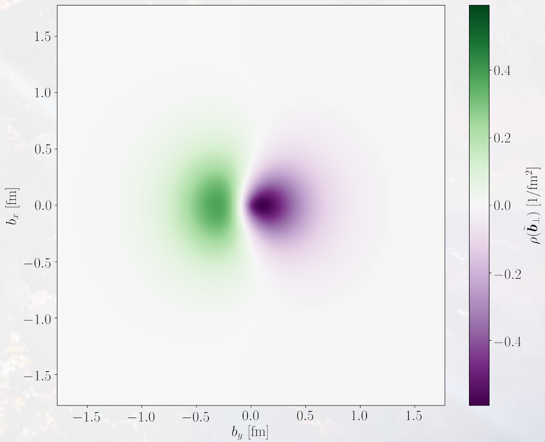


Neutron charge density

Longitudinal polarization



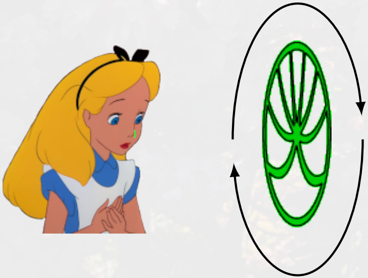
Transverse polarization



- Longitudinal polarization: negative core & diffuse positive cloud
  - Reproduces Miller, Phys. Rev. Lett. 99 (2007) 112001
- Transverse polarization: apparent electric dipole
  - Reproduces Carlson & Vanderhaegen, Phys. Rev. Lett. 100 (2008) 032004 (up to a sign)

Static wheel

Spinning wheel



- **Static wheel** has regularly-placed spokes
- **Spinning wheel** has distortions
- Spokes moving away are **redshifted**.
  - *Appear to* move slower, pile up
- Spokes moving towards are **blueshifted**.
  - *Appear to* move faster, become sparse
- These same distortions are present in the nucleon!
  - **The nucleon is a relativistic wheel!**
- Also see videos at:
  `https://www.spacetimetravel.org/rad`
  (green wheel is relevant case)

Electric currents: spin-half

- Standard form factor breakdown:

$$\langle p', \lambda' | \hat{J}^\mu(0) | p, \lambda \rangle = \bar{u}(p', \lambda') \left\{ \gamma^\mu F_1(q^2) + \frac{\mathrm{i}\sigma^{\mu q}}{2m} F_2(q^2) \right\} u(p, \lambda)$$

- **Spin-up along z axis—Drell-Yan frame vs. Breit frame:**

$$j_D^\perp(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\hat{z} \times \mathrm{i}\boldsymbol{q}_\perp}{2m} G_M(-\boldsymbol{q}_\perp^2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

$$j_B^\perp(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\hat{z} \times \mathrm{i}\boldsymbol{q}_\perp}{2m\sqrt{1+\tau}} G_M(-\boldsymbol{q}_\perp^2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

not equal!

$$j_D^3(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \tau \left(G_M(-\boldsymbol{q}_\perp^2) - \frac{1}{2} F_1(-\boldsymbol{q}_\perp^2) \right) \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

$$j_B^3(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\tau}{1+\tau} G_M(-\boldsymbol{q}_\perp^2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

neither vanishes!

where

$$G_M(q^2) = F_1(q^2) + F_2(q^2) \qquad \tau = \frac{\boldsymbol{q}_\perp^2}{4m^2}$$

Boost artifacts

Induced current

$$\boldsymbol{j}_D^{\perp}(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\hat{z}\times \mathrm{i}\boldsymbol{q}_\perp}{2m} G_M(-\boldsymbol{q}_\perp^2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp} \qquad j_D^3(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \tau\left(G_M(-\boldsymbol{q}_\perp^2) - \frac{1}{2}F_1(-\boldsymbol{q}_\perp^2)\right) \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

$$\boldsymbol{j}_B^{\perp}(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\hat{z}\times \mathrm{i}\boldsymbol{q}_\perp}{2m\sqrt{1+\tau}} G_M(-\boldsymbol{q}_\perp^2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp} \qquad j_B^3(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{\tau}{1+\tau} G_M(-\boldsymbol{q}_\perp^2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

Redshift factors

- Boosts leave light front *densities* invariant, but not light front *currents*.
- Initial & final states **not at rest**—$\boldsymbol{q}_\perp \neq 0$.
- **Redshifts from longitudinal boosts in Breit frame.**
- **Induced currents from Terrell counter-rotations in Drell-Yan frame.**
- Boost artifacts don't explain why $j^3 \neq 0$—similar cause to charge density modulations?
- **Same physics (or artifacts) are present in energy densities!**

IV. The energy-momentum tensor

- The energy-momentum tensor describes **density** and **flow** of energy & momentum.

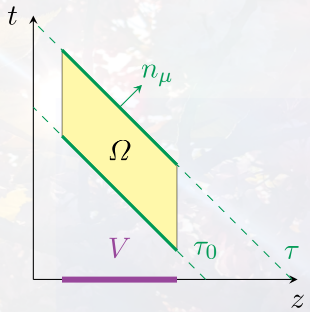
Energy density

Momentum densities

$$T^{\tilde{\mu}\nu}(x) = \begin{bmatrix} T^{+0}(x) & T^{+1}(x) & T^{+2}(x) & T^{+3}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

Energy fluxes

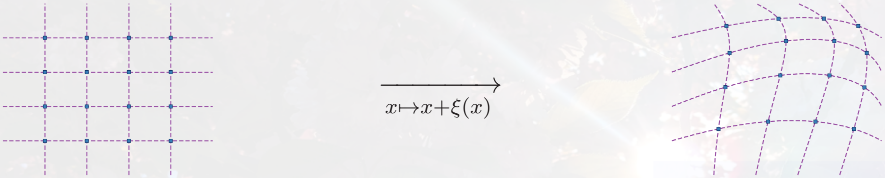
Stress tensor



Static densities: $n_\mu T^{\mu\nu}(x)$

Noether's theorems and spacetime distortions

- Conserved current from *local* spacetime translations (**Noether's second theorem**):



  - **Noether's theorems**: symmetries imply conservation laws
  - *Local* translation: move spacetime differently everywhere

- The **energy-momentum tensor** is a response to these deformations

$$\Delta S_{\mathrm{QCD}} = \int \mathrm{d}^4 x \, T^{\mu\nu}_{\mathrm{QCD}}(x) \partial_\mu \xi_\nu(x)$$

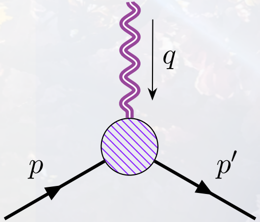
  - Conserved if the action is invariant
  - Basically, equivalent to doing a gravitational gauge transform.

Gravitational form factors

- The energy-momentum tensor is parametrized using **gravitational form factors**
  - It's just a name.
  - The energy-momentum tensor is the source of gravitation.
  - But we don't really use gravitation to measure them.
- Spin-zero example:

$$\langle p'|\hat{T}^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A(q^2) + \frac{1}{2}(q^\mu q^\nu - q^2 g^{\mu\nu})D(q^2) + 2m^2 g^{\mu\nu}\bar{c}(q^2)$$

  - $A(q^2)$ encodes momentum density
  - $D(q^2)$ encodes stress distributions (anisotropic pressures)
  - $\bar{c}(q^2) = 0$ by energy/momentum conservation



Possible energy densities: spin zero

- **Drell-Yan frame** energy density:

$$t^{+0}(\boldsymbol{b}_\perp) = m \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \left\{ A(-\boldsymbol{q}_\perp^2) + \bar{c}(-\boldsymbol{q}_\perp^2) + \tau\Big(A(-\boldsymbol{q}_\perp^2) + D(-\boldsymbol{q}_\perp^2) \Big) \right\} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

- **Breit frame** energy density:

$$t^{+0}(\boldsymbol{b}_\perp) = m \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{1}{\sqrt{1+\tau}} \left\{ A(-\boldsymbol{q}_\perp^2) + \bar{c}(-\boldsymbol{q}_\perp^2) + \tau\Big(A(-\boldsymbol{q}_\perp^2) + D(-\boldsymbol{q}_\perp^2) \Big) \right\} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

  - 2D projection of instant form density!
  - See Polyakov & Schweitzer, IJMPA (2018)
- Both seem sensible, & differ only by a boost factor.

Possible energy densities: spin-half

- Spin-half form factor breakdown:

$$\langle p',\lambda'|\hat{T}^{\mu\nu}(0)|p,\lambda\rangle = \bar{u}(p',\lambda')\Big\{P^\mu P^\nu A(q^2) + \frac{\mathrm{i}P^{\{\mu}\sigma^{\nu\}q}}{4m}J(q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{4m}D(q^2) + m g^{\mu\nu}\bar{c}(q^2) + \gamma^{[\mu}P^{\nu]}S(q^2)\Big\}u(p,\lambda)$$

  - Drop $S(q^2)$ (spin form factor) for symmetric EMT.
- **Drell-Yan frame** energy density (**light front helicity state**):

$$t^{+0}(\boldsymbol{b}_\perp) = m\int\frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2}\left\{A(\tau)+\bar{c}(\tau)+\tau\left(\frac{1}{2}A(\tau)-J(\tau)+S(\tau)+D(\tau)\right)\right\}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

  - See Lorcé/Moutarde/Trawińsky, EPJC (2019)
- **Breit frame** energy density (**light front helicity state**):

$$t^{+0}(\boldsymbol{b}_\perp) = m\int\frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2}\frac{1}{\sqrt{1+\tau}}\Big\{A(\tau)+\bar{c}(\tau)+\tau\Big(A(\tau)-J(\tau)+S(\tau)+D(\tau)\Big)\Big\}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

  - **Not** 2D projection of instant form density!
  - Differs by $\tau\big(J(q^2)+S(q^2)\big)$—similar to j^3 term.

Artifacts and physics in energy density

- Boost artifacts present in both energy densities.

Induced by boost $|\mathbf{0}_\perp\rangle \to \left|\pm\frac{1}{2}\boldsymbol{q}_\perp\right\rangle$.

- **Drell-Yan frame** energy density (**light front helicity state**):

$$t^{+0}(\boldsymbol{b}_\perp) = m \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \left\{ A(\tau) + \bar{c}(\tau) + \tau\left(A(\tau) - \frac{1}{2}A(\tau) - 2J(\tau) + \Big(J(\tau) + S(\tau)\Big) + D(\tau)\right)\right\} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

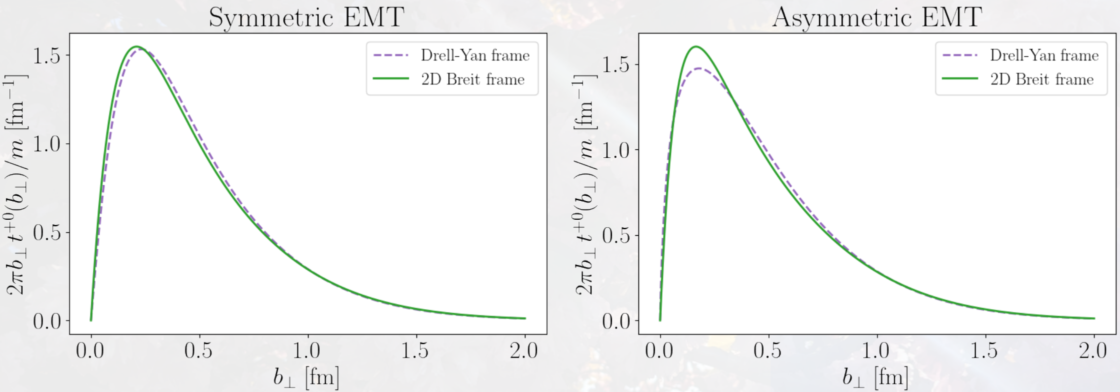
- **Breit frame** energy density (**light front helicity state**):

$$t^{+0}(\boldsymbol{b}_\perp) = m \int \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{1}{\sqrt{1+\tau}} \left\{ A(\tau) + \bar{c}(\tau) + \tau\Big(A(\tau) - 2J(\tau) + \Big(J(\tau) + S(\tau)\Big) + D(\tau)\Big)\right\} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$

Redshift   Relativistic optical effect

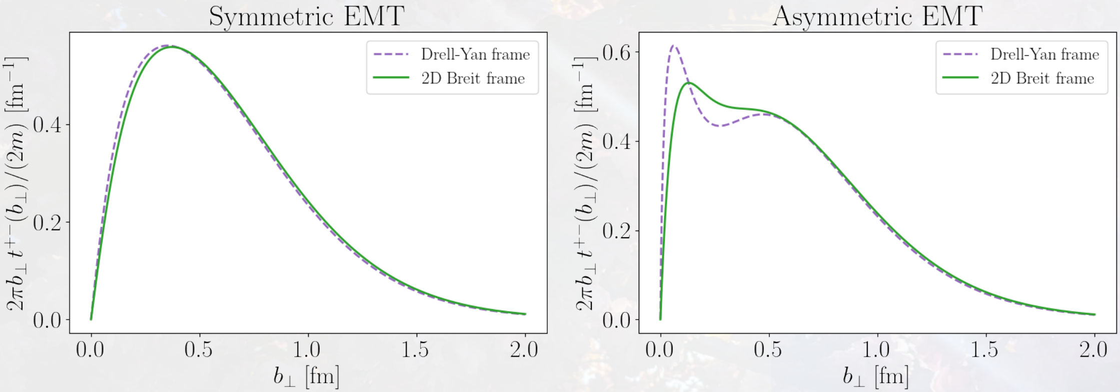
- **Relativistic optical effects (not boost artifacts) present, too.**
  - Similar to non-zero j^3.
  - Related to relativistic wheel—consequence of fixed x^+.
  - Formally related to Melosh rotation; see Chen & Lorcé, PRD (2022)

Spin-half energy density: numerical results



- **Energy**: P^0 generates x^+ evolution *at fixed* x^3 (see AF & Miller, PRD (2023)).
- Using Mamo-Zahed model for GFFs.
  - Plus Lorcé/Moutarde/Trawiński, EPJC (2019) for $S(q^2)$.
- Results all look qualitatively similar.

Spin-half P^- density: numerical results



- P^- often considered the "light front energy."
  - Is x^+ translation generator at fixed $x-$.
  - (Though P^0 generates x^+ translations at fixed x^3—see AF & Miller, PRD (2023).)
- Asym. Drell-Yan density echoes Lorcé/Moutarde/Trawiński, EPJC (2019)

Conclusions & summary

- The light front allows exact factorization of physical densities:

$$\int \mathrm{d}x^3 \, \langle\psi|\hat{j}^\mu(x)|\psi\rangle = \int \mathrm{d}^3\boldsymbol{R}\, \mathscr{P}^\mu{}_\nu(\boldsymbol{R}, x^+, \psi) j^\nu_{\text{internal}}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)$$

Internal density

Smearing function

invariant under LF boosts

- Multiple factorizations are possible—matter of choice/convention.
  - Drell-Yan frame & Breit frame both sensible.
- Strange optical effects in spinning targets.
  - Angular modulations—proton is a relativistic wheel!
  - Mysterious longitudinal current.

Thank you for your time!