Transverse Momentum Distributions from Lattice QCD

SoLID Opportunities and Challenges of Nuclear Physics at the Luminosity Frontier
Argonne National Laboratory, Lemont, IL, USA
June 18, 2024

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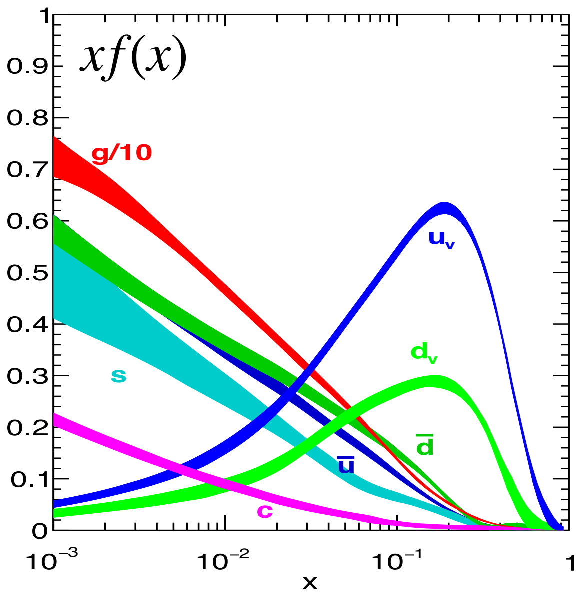
Argonne NATIONAL LABORATORY

Outline

- **Overview of TMD physics**
- **Large-Momentum Effective Theory (LaMET)**
  - Theoretical framework for lattice calculation
  - Collins-Soper kernel
  - Soft function and TMD PDF
- **New approach without Wilson lines**
  - Coulomb-gauge quasi-TMD
  - Exploratory calculation of the Collins-Soper kernel
  - Preliminary results of helicity TMD PDF
- **Summary**

3D imaging of the proton

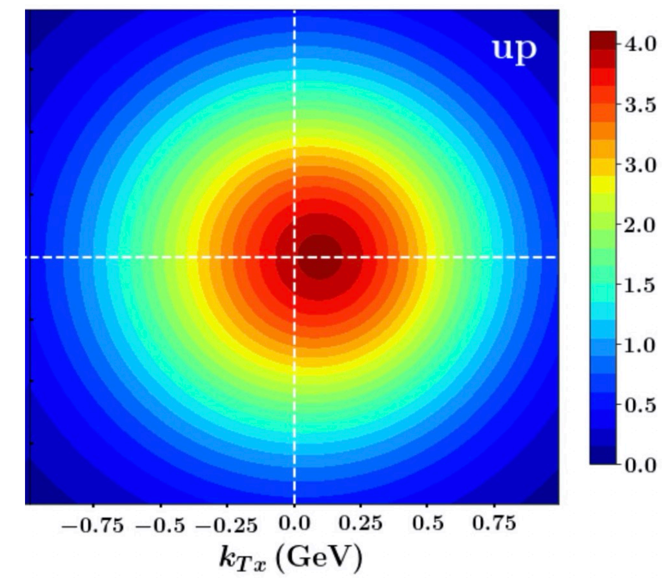
PDFs



NNPDF, EPJ C77 (2017)

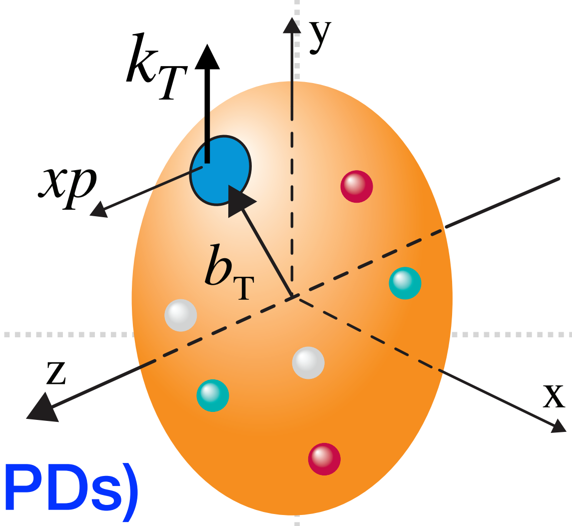
$\int d^2\vec{b}_T$

Transvers momentum distributions (TMDs)



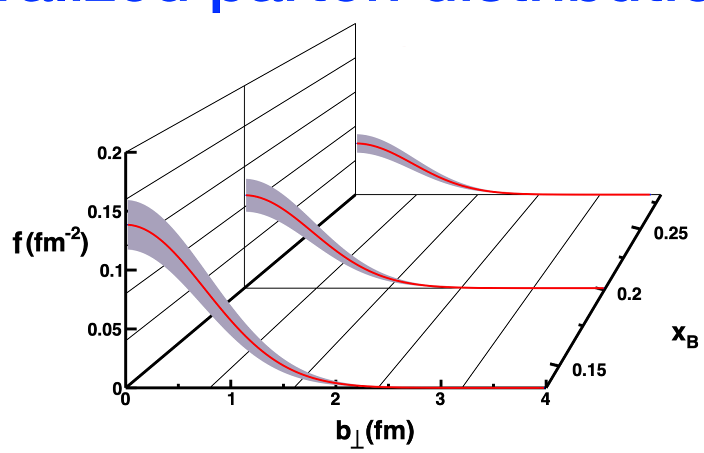
Cammarota, et al. (JAM), PRD **102** (2020).

$\int d^2\vec{k}_T$



$\int d^2\vec{b}_T$

Generalized parton distributions (GPDs)



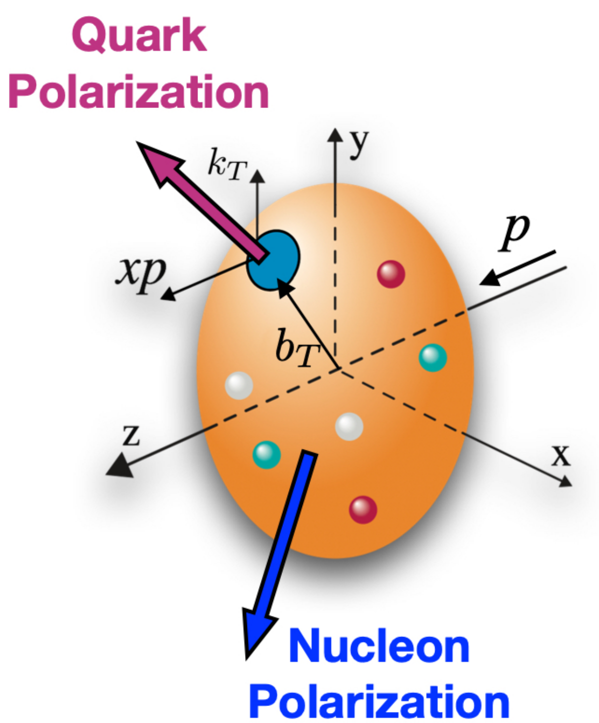
W. Armstrong et al., arXiv: 1708.00888.

$\int d^2\vec{k}_T$

Wigner distributions/Generalized TMDs

$$W(x, \vec{k}_T, \vec{b}_T)$$

TMDs of different spin structures



Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization		Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

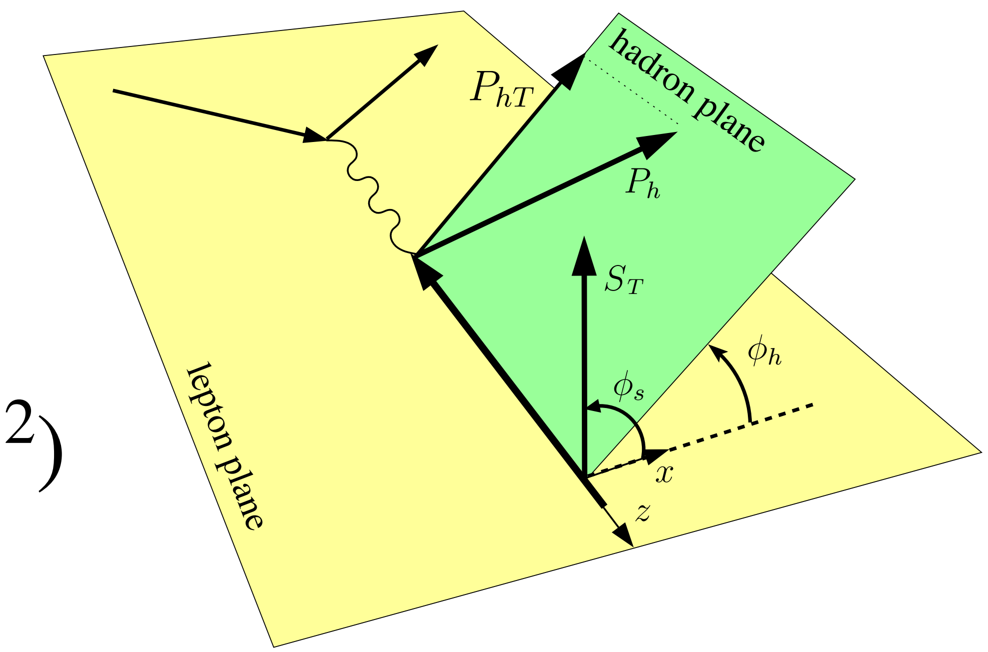
TMD Handbook, TMD Topical Collaboration, arXiv: 2304.03302.

TMDs from global analyses

e.g., semi-inclusive DIS: $l + p \longrightarrow l + h(P_h) + X\,, \qquad P_{hT} \ll Q$

$$\frac{d\sigma^W}{dxdydz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T\, e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2)\; D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2)$$



Kang, Prokudin, Sun and Yuan, PRD **93** (2016)

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f^{\text{pert}}_{i/p}(x, b^*(b_T), \mu, \zeta)$$

$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2} \;\; f^{\text{NP}}_{i/p}(x, b_T)$$

$g_K(b_T)/2$ → **Collins-Soper kernel (Non-perturbative part)**

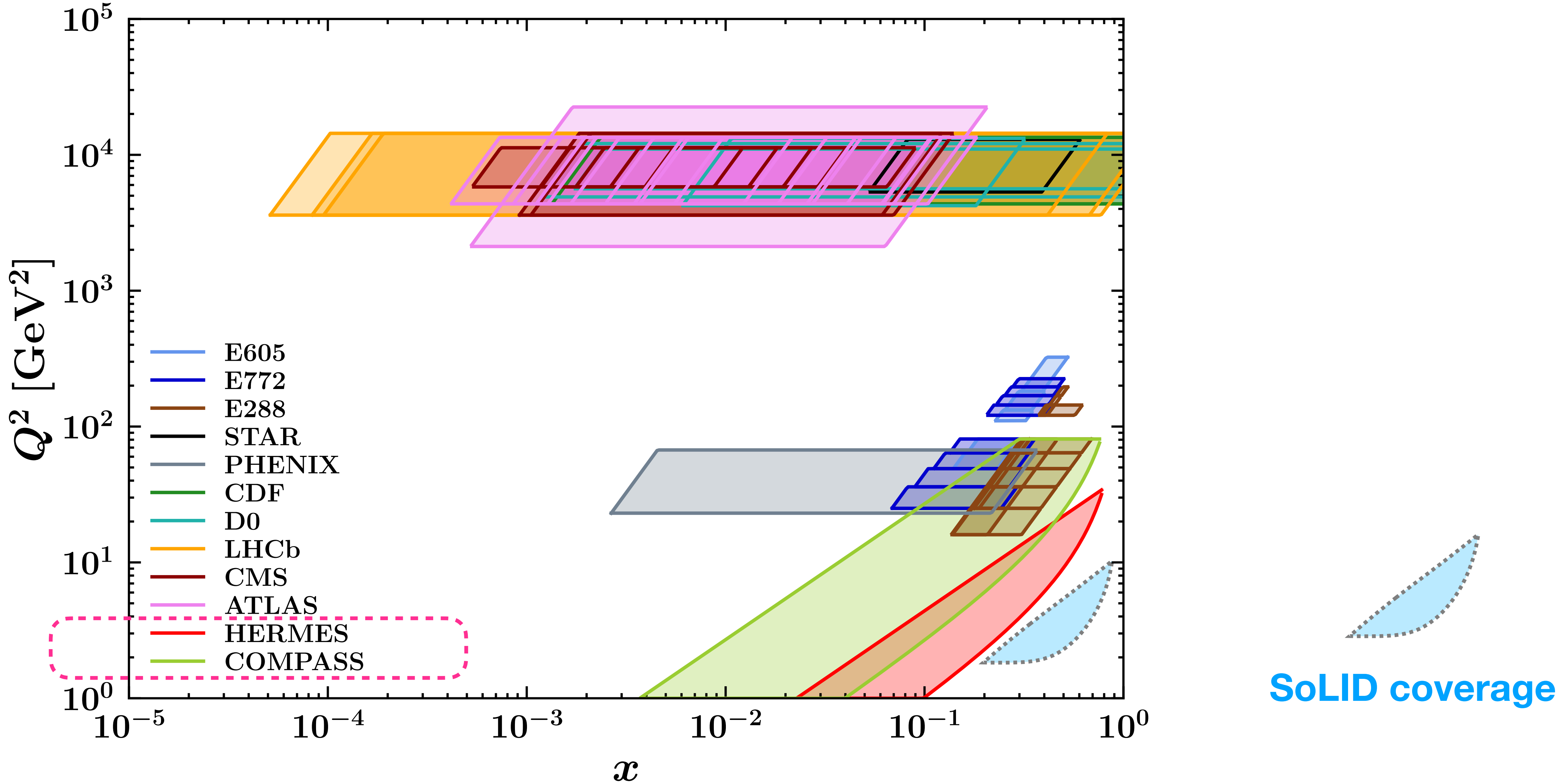
$f^{\text{NP}}_{i/p}(x, b_T)$ → **Intrinsic TMD**

$Q_0 \sim 1$ GeV

Non-perturbative when $b_T \sim 1/\Lambda_{\text{QCD}}$!

TMDs from global analyses

- Kinematic coverage of Drell-Yan and SIDIS experiments



MAP Collaboration, JHEP **10** (2022), arXiv: 2405.13833.

A wide range of (x, Q^2) is essential for extracting the intrinsic TMDs while sufficiently suppressing the power corrections.

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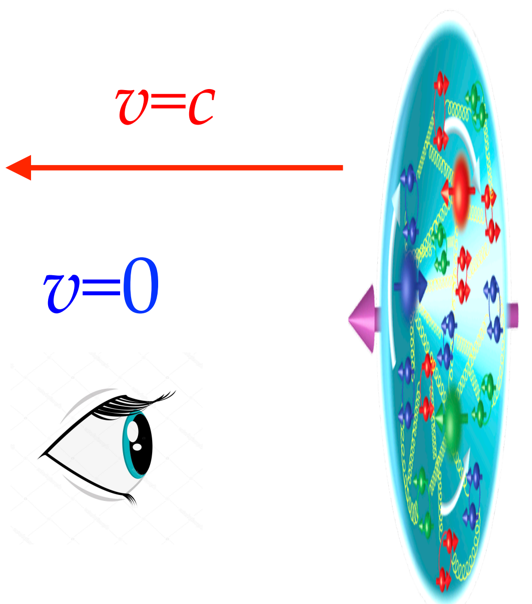
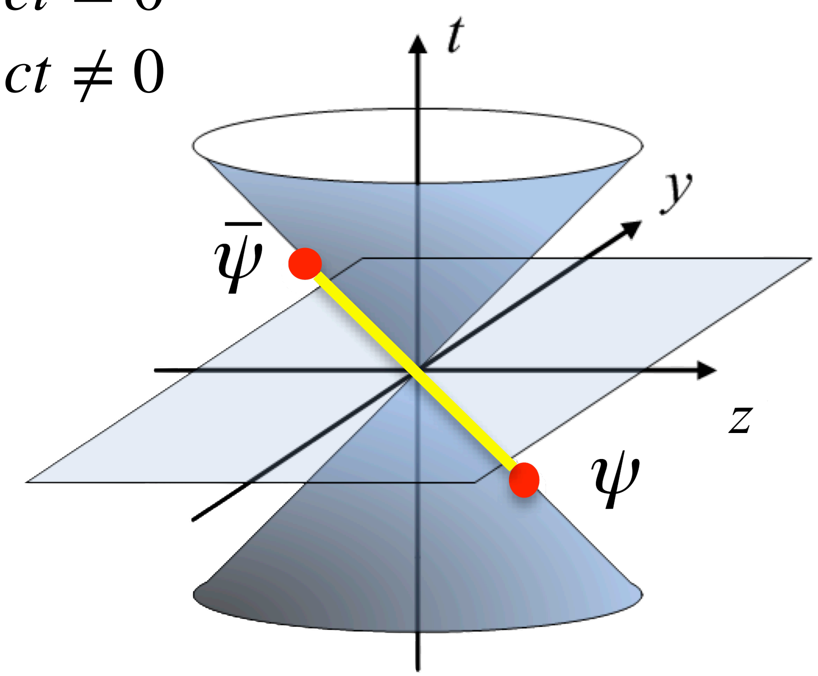
Simulating partons on the Euclidean lattice?

PDFs can be defined from light-cone correlations

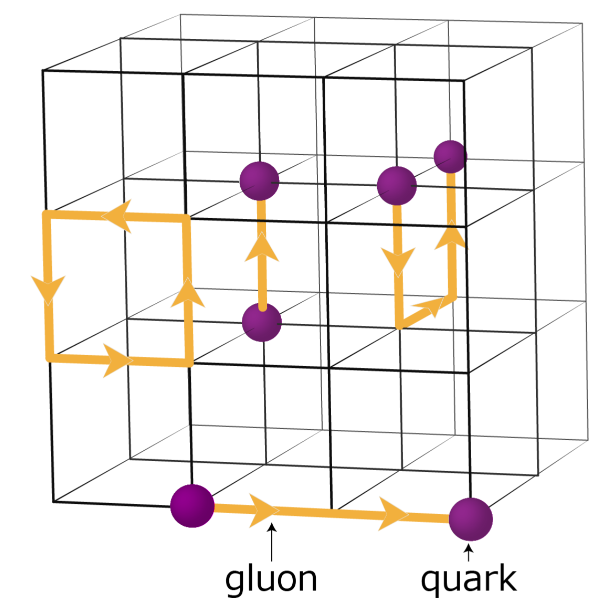


$z + ct = 0$

$z - ct \neq 0$



$t \to i\tau$



$$f(x) = \int \frac{d\xi^-}{2\pi}\, e^{-ix\lambda} \langle P \,|\, \bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0) \,|\, P \rangle$$

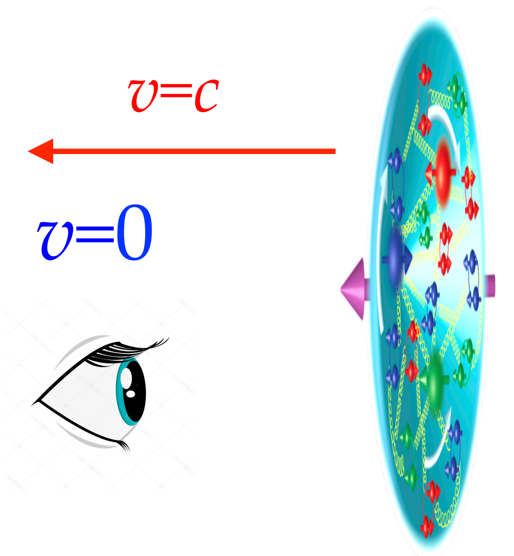
$$O(i\tau) \overset{?}{\to} O(t)$$

$\xi^- = (t - z)/\sqrt{2}$ $\quad$ $W(\xi^-,0)$: Wilson line that ensures SU(3) gauge invariance

Time-dependence makes it impossible to calculate the PDFs directly on the Euclidean lattice. 🙁

Large-Momentum Effective Theory (LaMET)

Revisit Feynman's parton picture in the infinite momentum frame



Simulating $\langle P=\infty \,|\, O(t=0) \,|\, P=\infty \rangle$? ✘

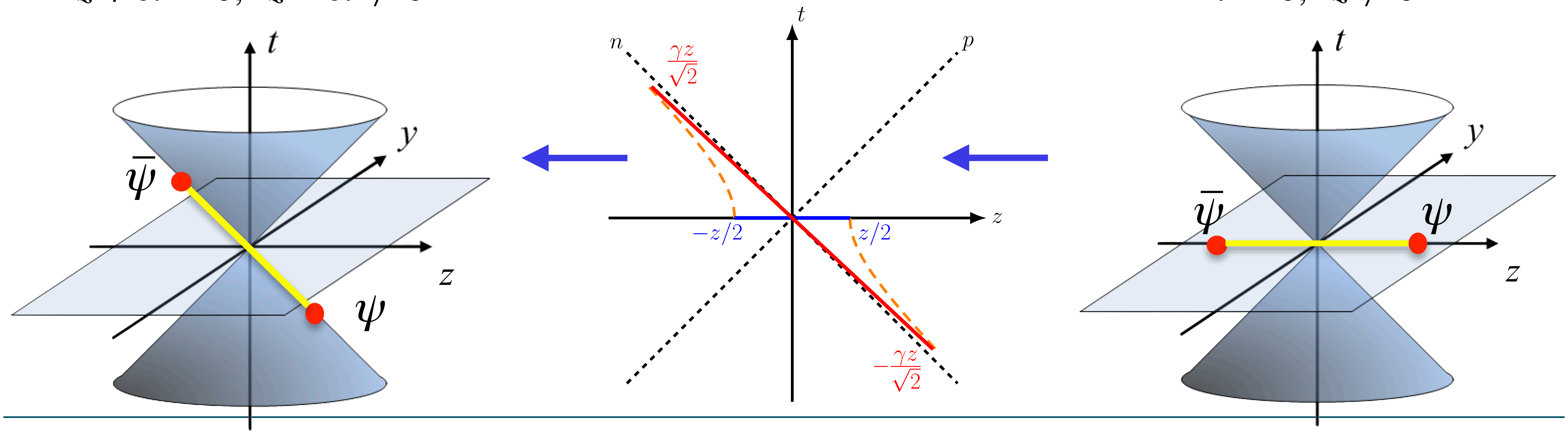
$P \ll \dfrac{2\pi}{a}$!

Nevertheless, it is possible to simulate a proton at large *P*:

$z + ct = 0, \;\; z - ct \neq 0$

X. Ji, PRL **110** (2013)

$t = 0, \;\; z \neq 0$

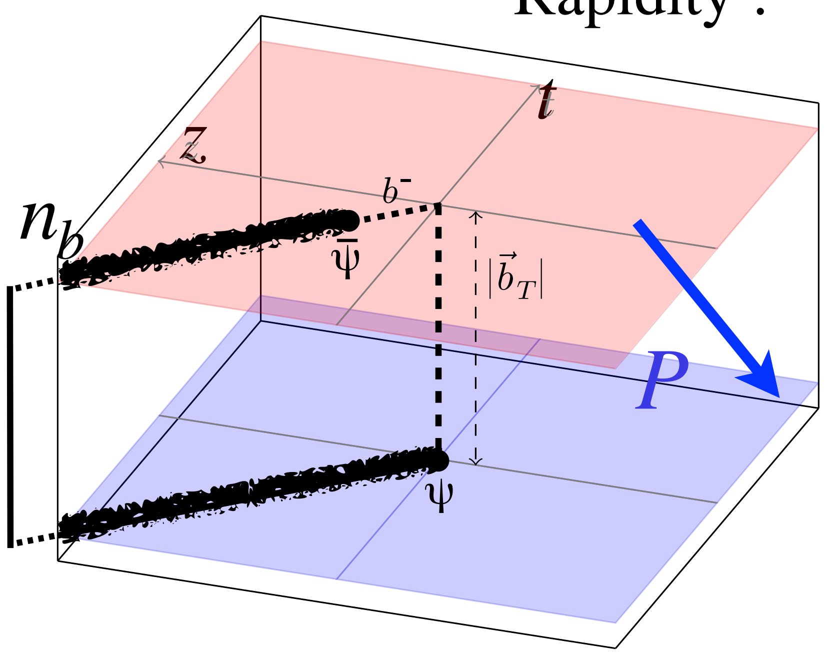


Transverse Momentum Distributions (TMDs)

- Beam function:

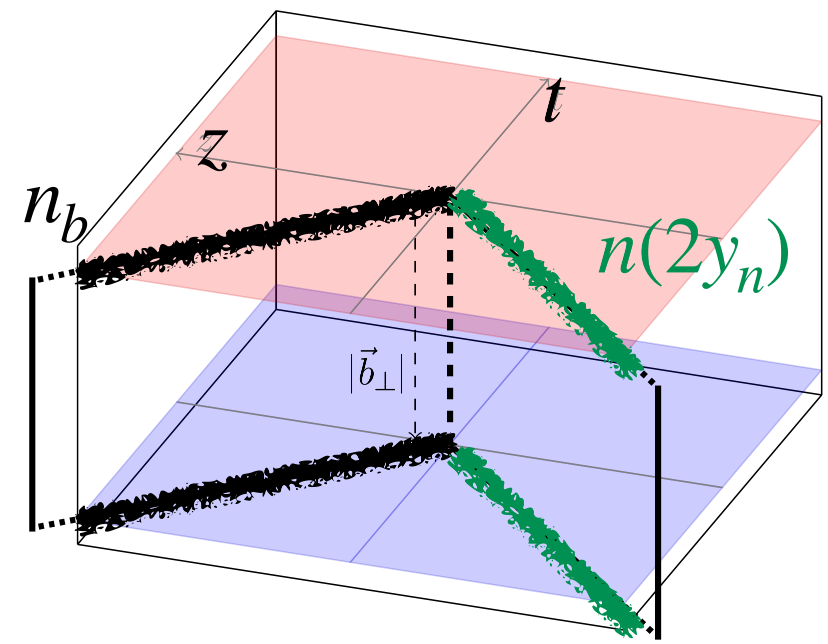
$$n_b^2 = 0$$

$$\text{Rapidity}: \quad y_B = \frac{1}{2}\ln\left|\frac{n_b^+}{n_b^-}\right| = -\infty$$



Hadronic matrix element

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\rm UV} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

Rapidity divergence regulator

Transverse Momentum Distributions (TMDs)

- Beam function:

- Soft function :

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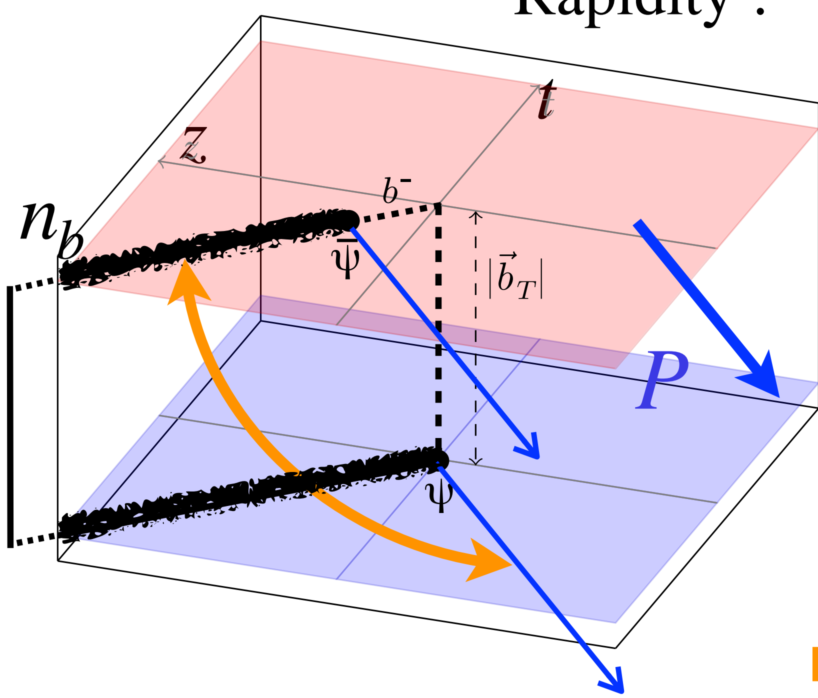
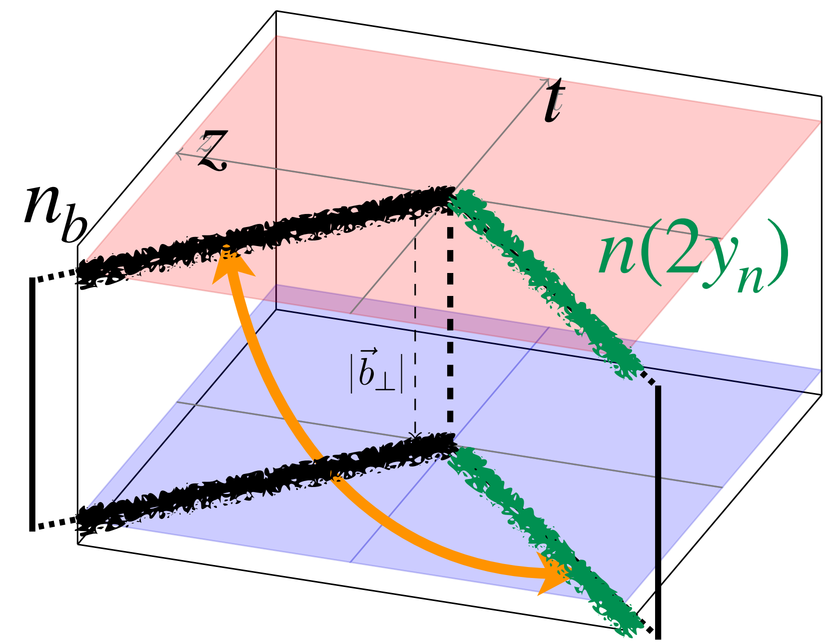


Hadronic matrix element

Rapidity divergences



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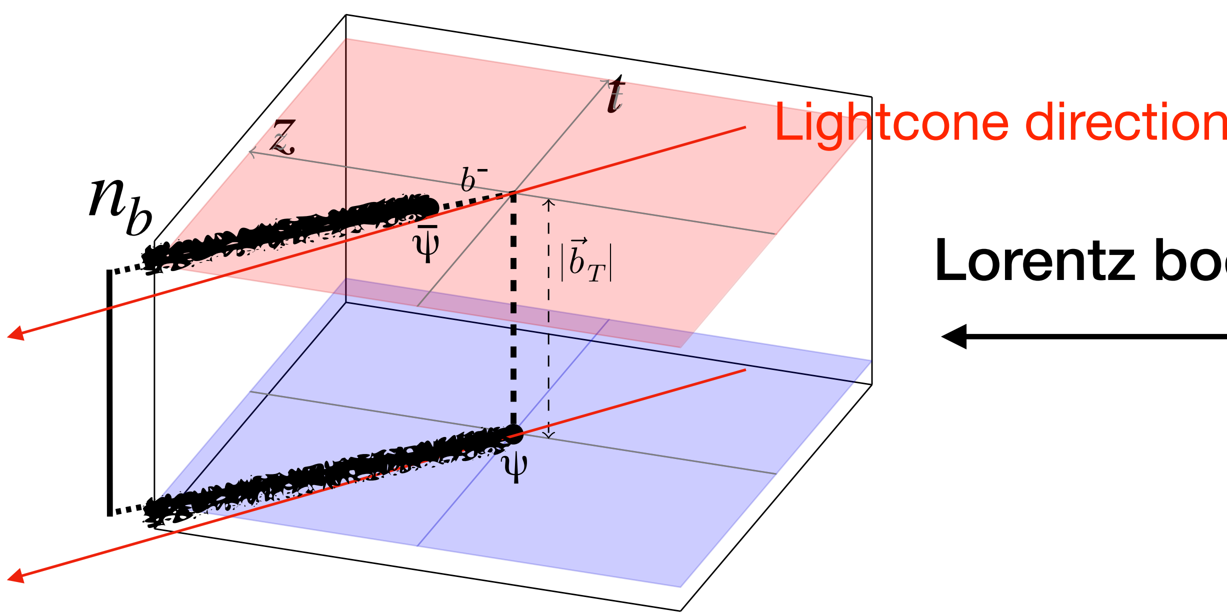
Collins-Soper scale: $\zeta = 2(xP^+ e^{-y_n})^2$

Rapidity divergence regulator

TMDs from LaMET

- Beam function (Collins's scheme):

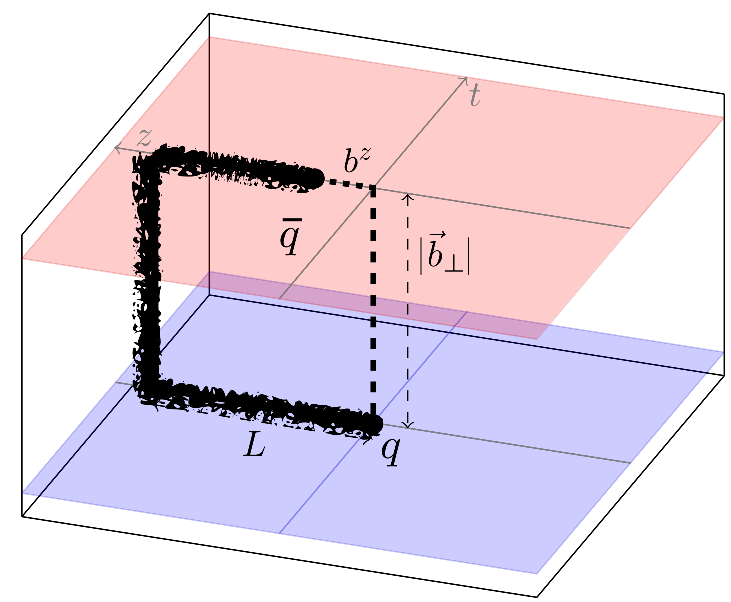


$$n_b^\mu(y_B) = (n_b^+, n_b^-, \vec{0}_\perp) = (-e^{2y_B}, 1, \vec{0}_\perp)$$

Spacelike but close-to-light-cone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹️

Lorentz boost and $L \to \infty$

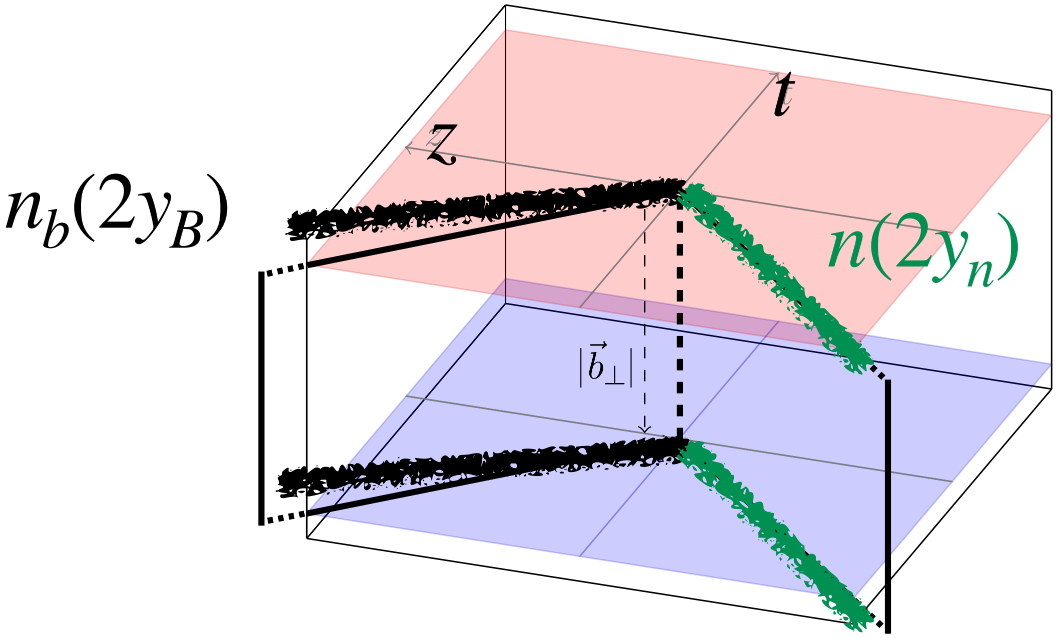
- Quasi beam function :



Equal-time Wilson lines, directly calculable on the lattice🙂

Ebert, Schindler, Stewart and YZ, JHEP 04 (2022).

Soft factor



Collins-Soper kernel

$$\xrightarrow{y_n-y_B\to\infty} S_r(b_T,\mu)\; e^{-2(y_n-y_B)\gamma_\zeta(b_T,\mu)}$$

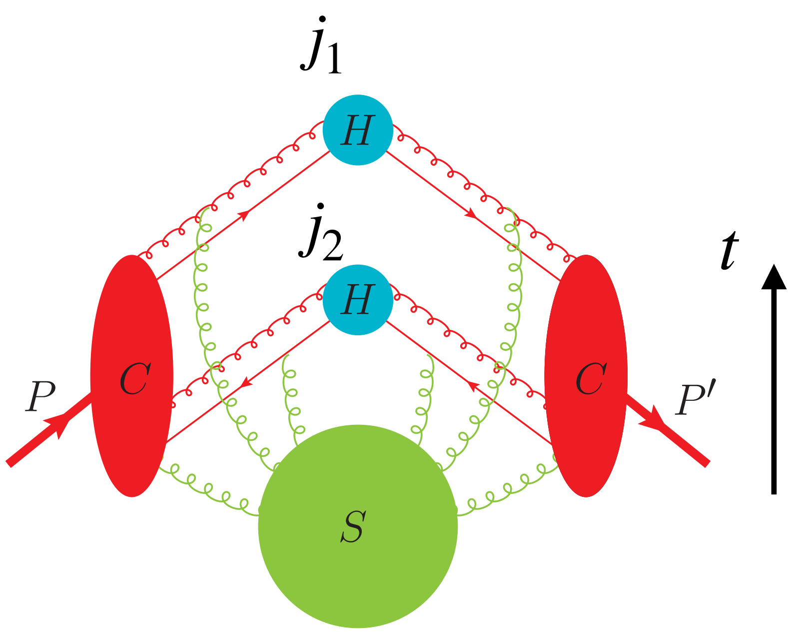
Reduced soft factor

Light-meson form factor:

$$F(b_T,P^z) = \langle \pi(-P)\,|\,j_1(b_T)j_2(0)\,|\,\pi(P)\rangle$$

$$\overset{P^z\gg m_N}{=} S_r(b_T,\mu)\int dx dx'\; H(x,x',\mu)$$

$$\times\, \Phi^\dagger(x,b_T,P^z,\mu)\Phi(x',b_T,P^z,\mu)$$



$\Phi(x,b_T,P^z,\mu)$: **quasi-TMD wave function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \ \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta) + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T) \ln \frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta) \left\{ 1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right] \right\}$$

✳ Collins-Soper kernel;

$$\gamma_\zeta(\mu, b_T) = \frac{d}{d \ln \tilde{P}^z} \ln \frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

✳ Flavor separation; $\frac{f_{i/p}^{[s]}(x, \mathbf{b}_T)}{f_{j/p}^{[s']}(x, \mathbf{b}_T)} = \frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T)}{\tilde{f}_{j/p}^{\text{naive}[s']}(x, \mathbf{b}_T)}$

✳ Spin-dependence, e.g., Sivers function (single-spin asymmetry);

✳ Full TMD kinematic dependence.

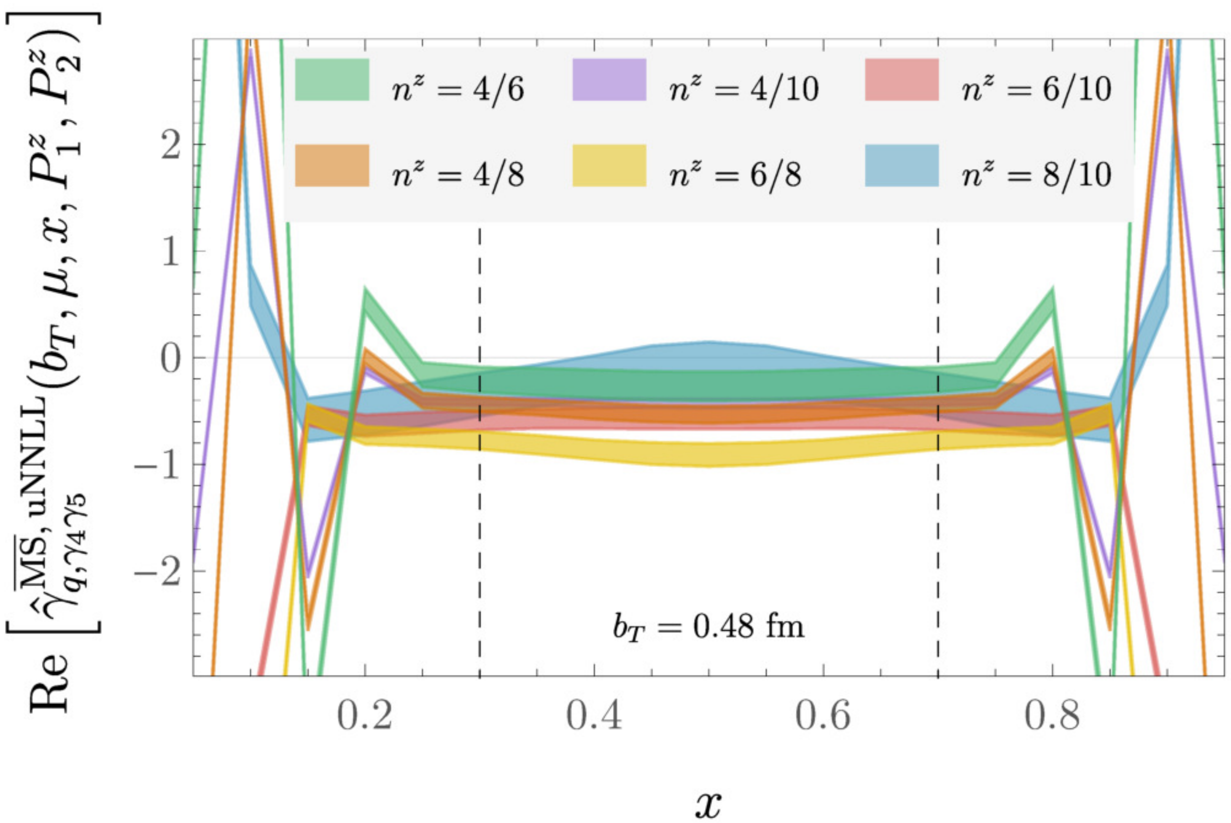
✳ Twist-3 PDFs from small b_T expansion of TMDs. Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

✳ Higher-twist TMDs. Rodini and Vladimirov, JHEP 08 (2022).

State-of-the-art determination of the Collins-Soper kernel

$$\gamma_\zeta(\mu, b_T) = \frac{d}{d \ln \tilde{P}^z} \ln \frac{\tilde{f}_{i/p}^{\,\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- Physical quark masses, large Lorentz boosts
- Continuum limit with a = 0.15, 0.12, 0.09 fm
- Controlled renormalization and Fourier transform
- Next-to-next-to-leading logarithmic (NNLL) order



Almost flat at moderate x, an important indicator of validity of factorization

- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, Phys.Rev.D 108 (2023);
- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, Phys.Rev.Lett. 132 (2024).

State-of-the-art determination of the Collins-Soper kernel

$$\gamma_\zeta(\mu, b_T) = \frac{d}{d \ln \tilde{P}^z} \ln \frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

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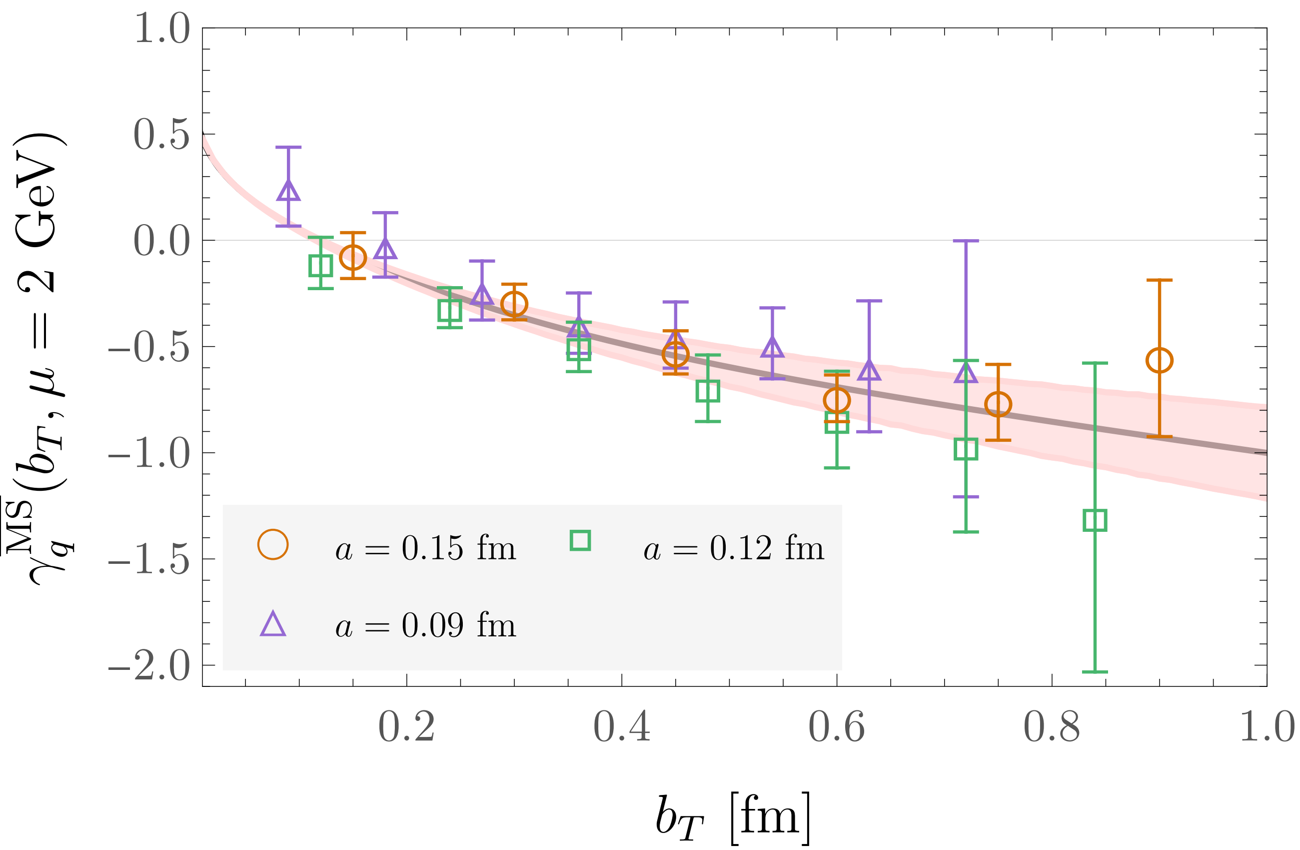
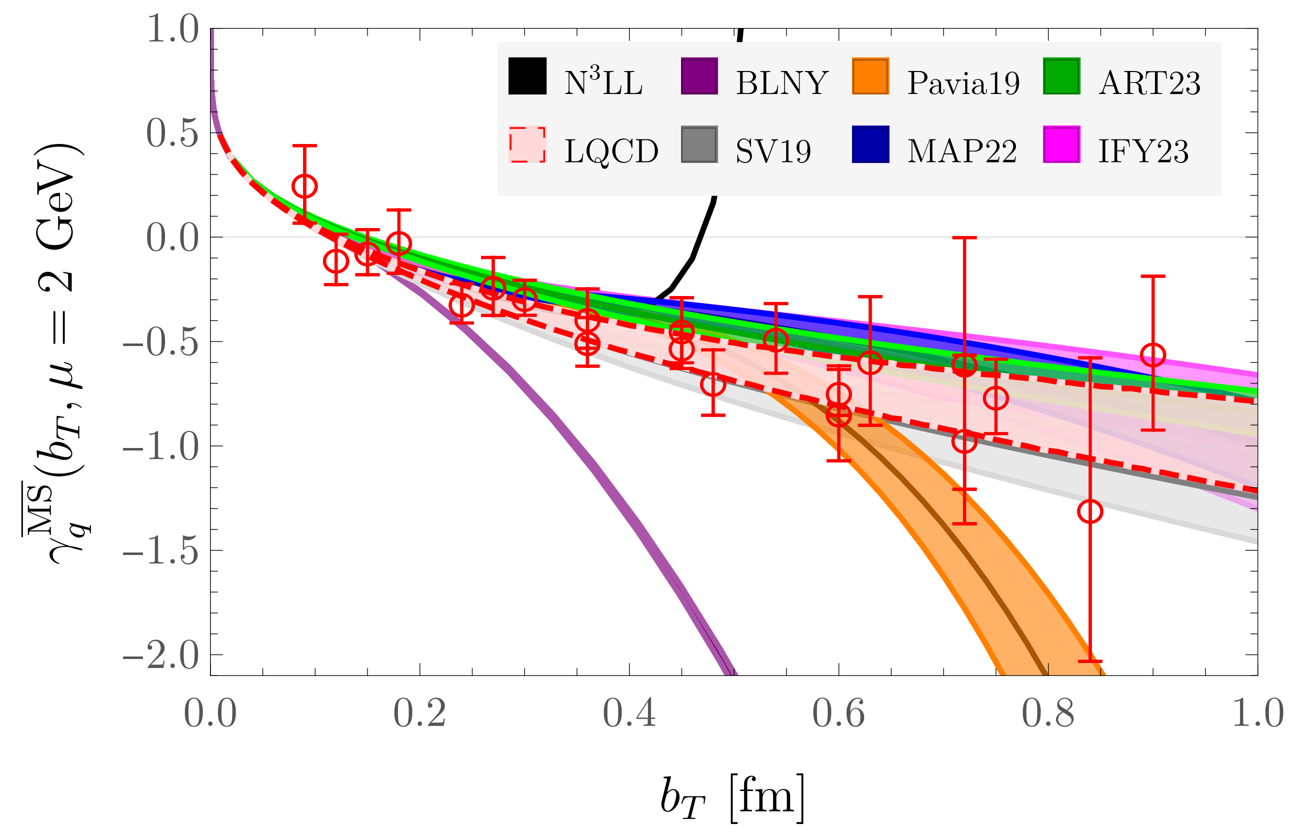
Nice agreement with phenomenology ☺

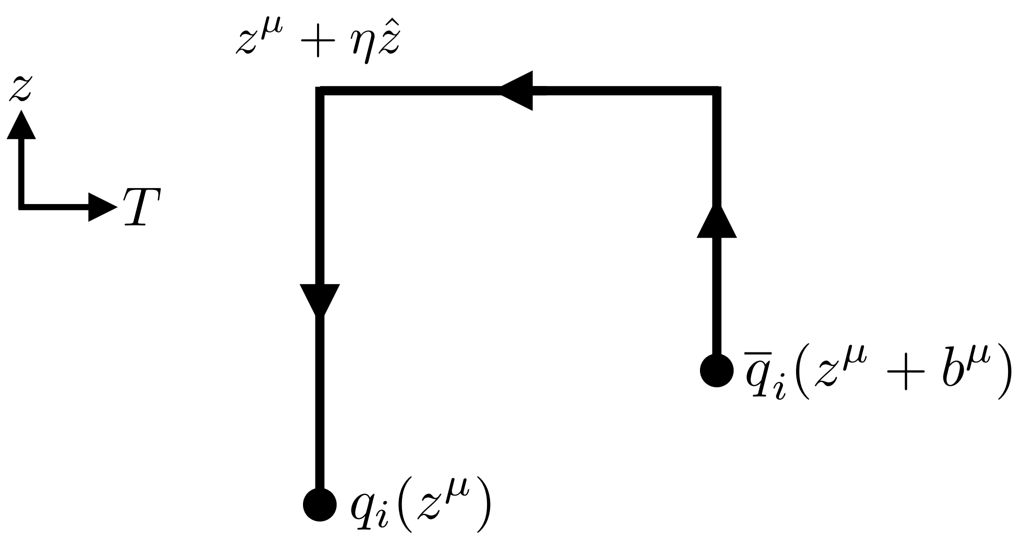




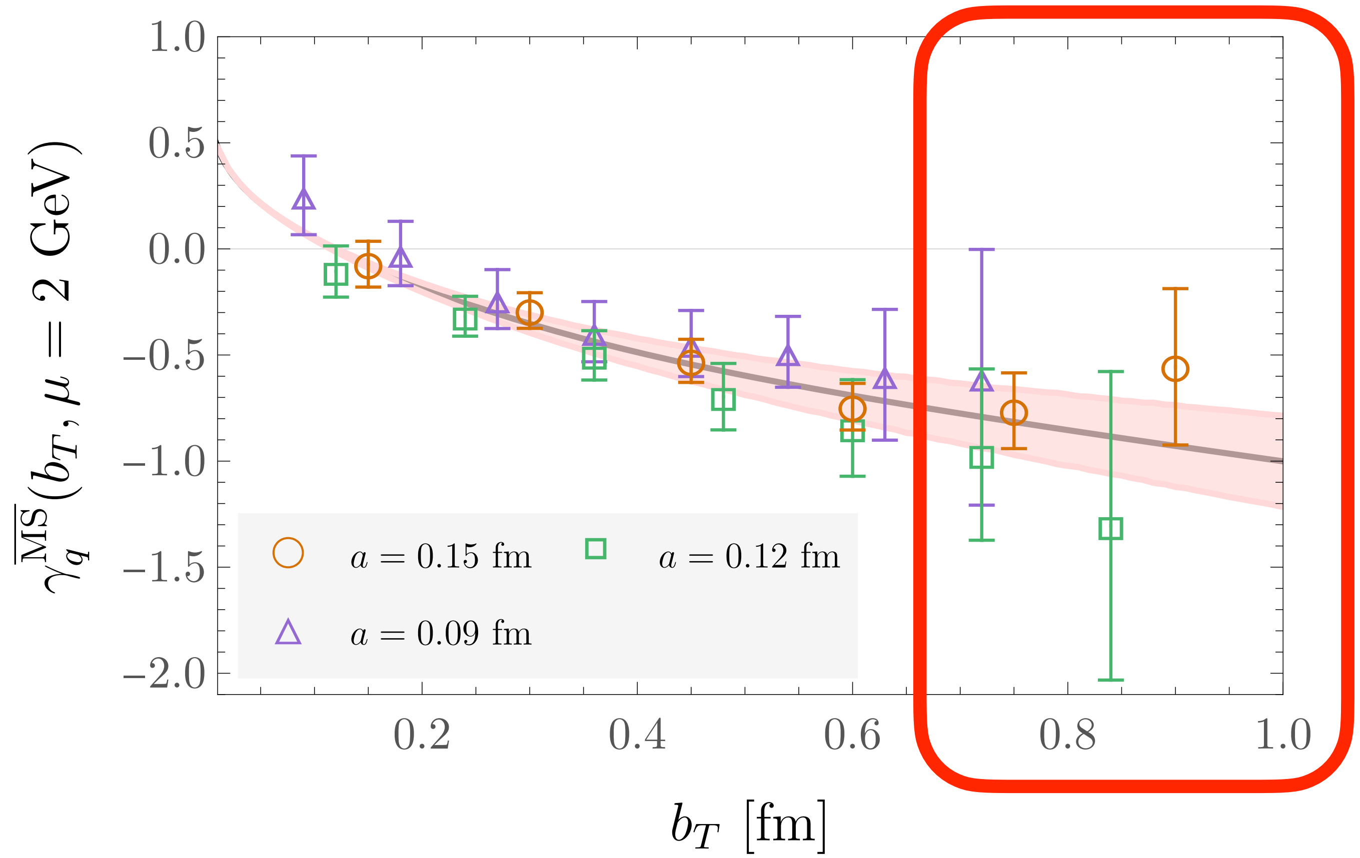
- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, Phys.Rev.D 108 (2023);
- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, Phys.Rev.Lett. 132 (2024).

Systematics in lattice calculation

Staple-shaped Wilson line

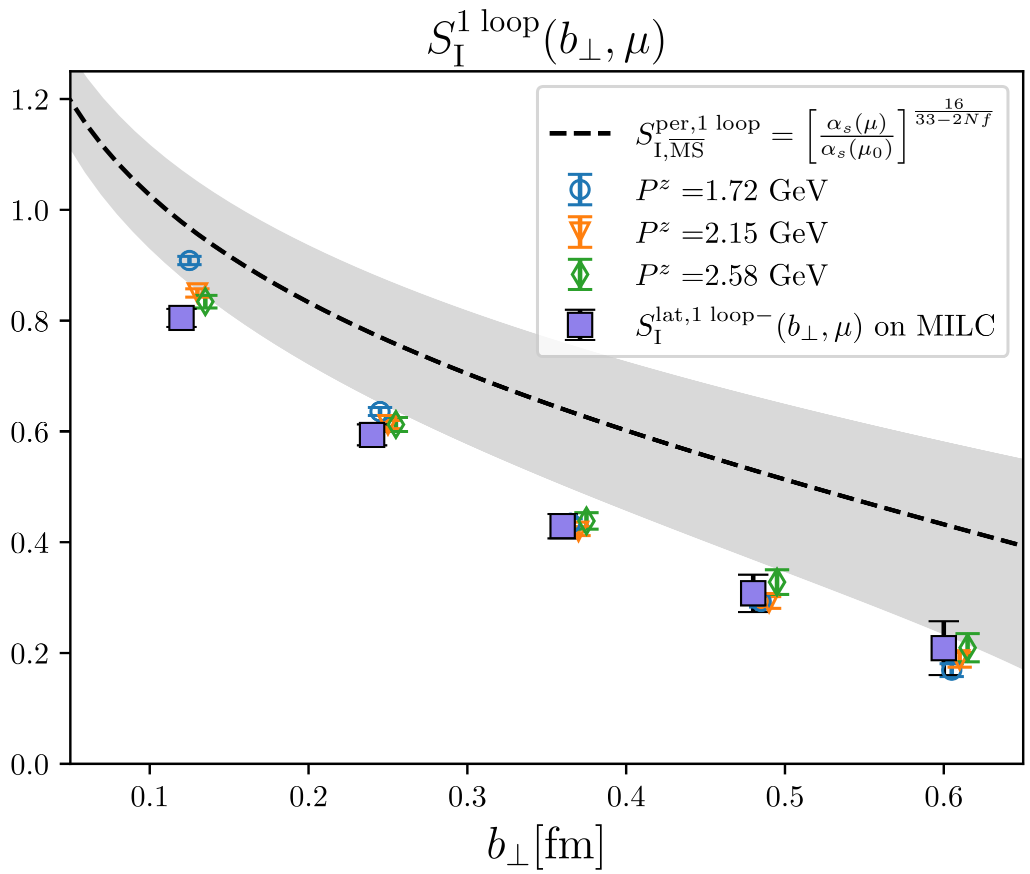


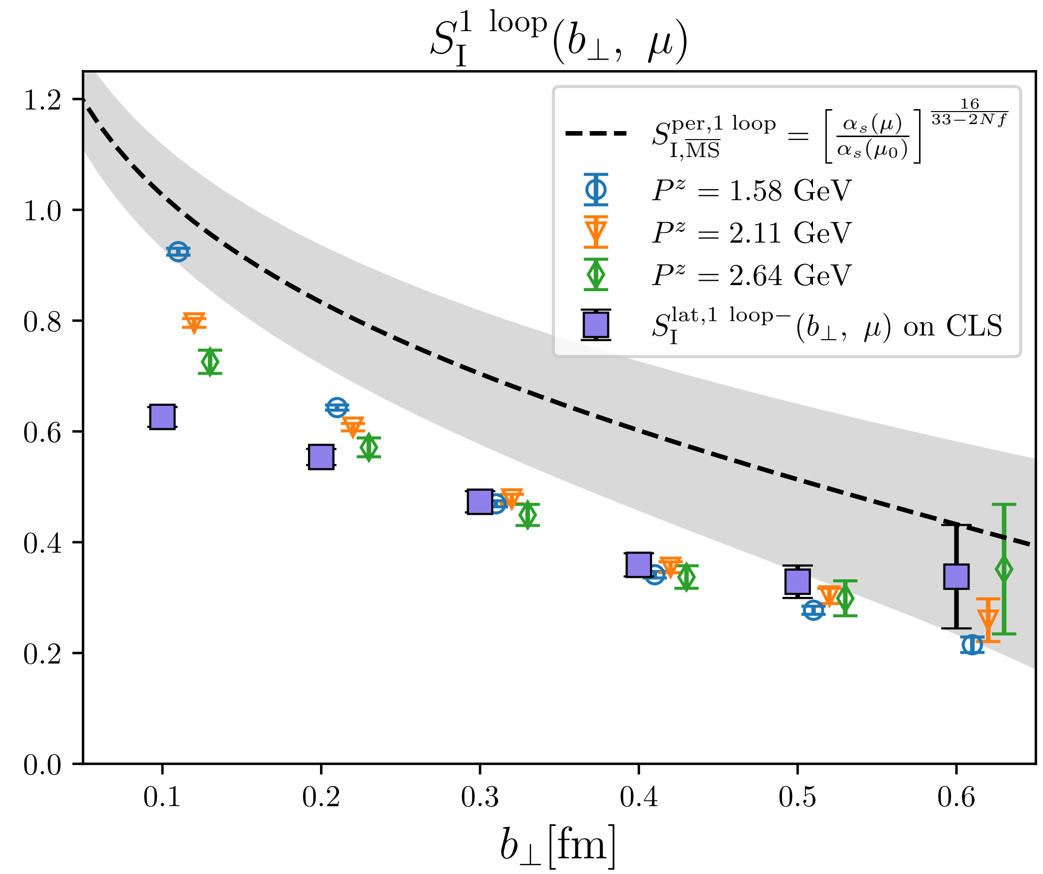
$$\eta \gg \{b^z, b_T\},\ xP^z \gg 1/b_T$$



- A large staple (gauge link) induces large statistical noises, which becomes worse at larger b_T;
- Complex operator mixings induced by the staple geometry; $$O_\Gamma = \sum_{\Gamma'} Z_{\Gamma\Gamma'} O_{\Gamma'}$$
- Additional power correction of order b^z/b_T, or equivalently $1/(xP^z b_T)$, from the staple self energy, which has not been handled by renormalization so far.

Lattice calculation of the soft function at NLO





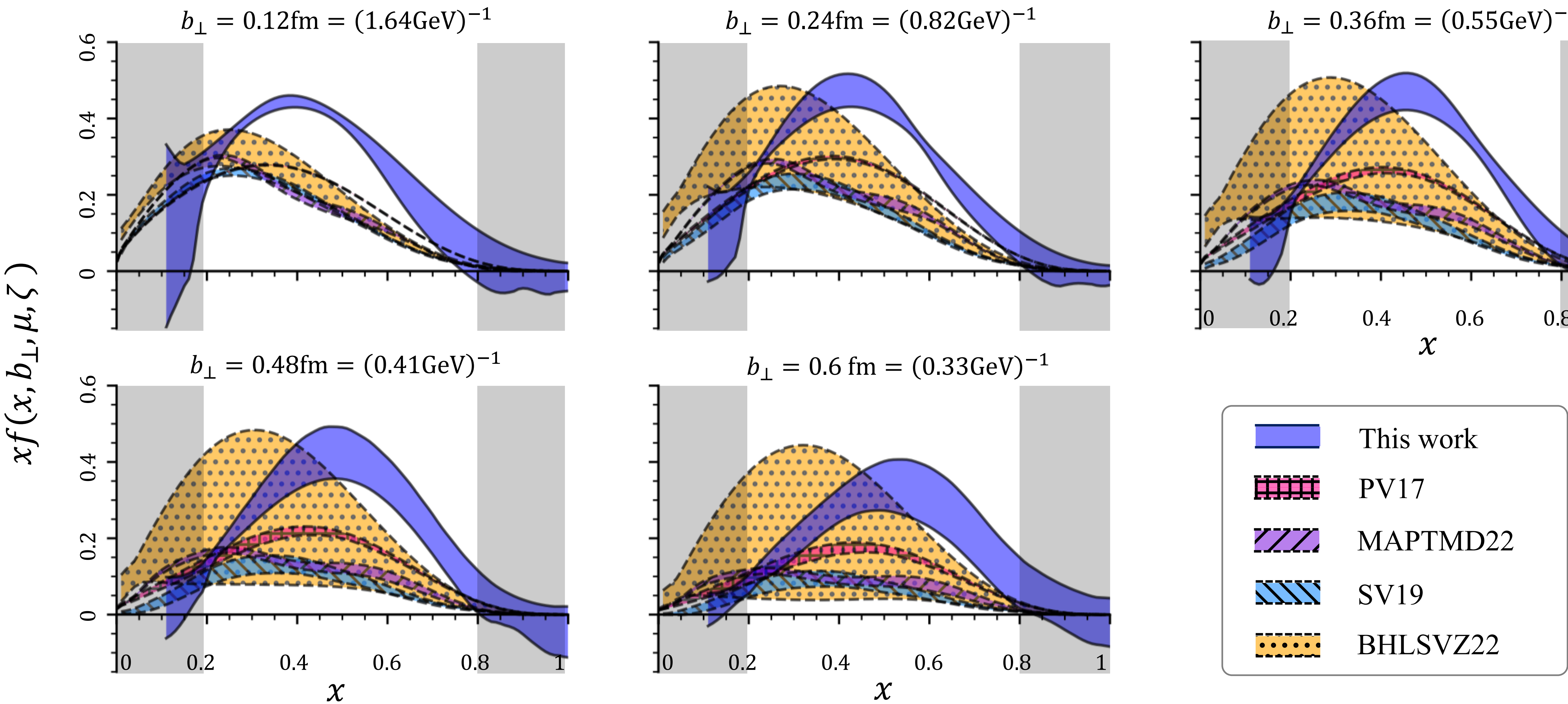
M.-H. Chu, et al. (LPC), JHEP 08 (2023).

$$a = 0.121\ \mathrm{fm},\quad m_\pi = 670\ \mathrm{MeV},\quad P^z_{\mathrm{max}} = 2.58\ \mathrm{GeV}$$

$$a = 0.098\ \mathrm{fm},\quad m_\pi = 662\ \mathrm{MeV},\quad P^z_{\mathrm{max}} = 2.64\ \mathrm{GeV}$$

(x, b_T) dependence of the unpolarized proton TMD

J.-C. He, M.-H. Chu, J. Hua et al., (LPC), arXiv: 2211.02340.



$$a = 0.12 \text{ fm},$$
$$m_\pi = \{310,\ 220\} \text{ MeV},$$
$$P^z_{\max} = 2.58 \text{ GeV}$$

SV19: Scimemi and Vladimirov, JHEP **06** (2020)
Pavia19: Bacchetta et al., JHEP **07** (2020).
MAPTMD22: Bacchetta et al., JHEP **10** (2022).
BHLSVZ22: Bury et al., JHEP **10** (2022).

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Universality in LaMET

Gauge-invariant bilinear

$$\bar{\psi}(z)\Gamma W[z,0]\psi(0)$$

- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Current-current correlator

$$J^{\mu}(z)J^{\nu}(0)$$

- K. Liu and S. Dong, PRL **72** (1994);
- Detmold and Lin, PRD **73** (2006);
- Braun and Müller, EPJC **55** (2008);
- A Chambers et al. (QCDSF), PRL **118** (2017)
- Ma and Qiu, PRL **120** (2018).

Free bilinear in a physical gauge

$$\bar{\psi}(z)\Gamma\psi(0)\Big|_{G(A)=0} \qquad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}$$

Light-cone bilinear

$$\bar{\psi}(\xi^-)\gamma^+ W[\xi^-,0]\psi(0)$$

Or

$$\bar{\psi}(\xi^-)\gamma^+\psi(0)\Big|_{A^+=0}$$

For axial gauges, the linear divergences still appear in the quark self-energy 🙁

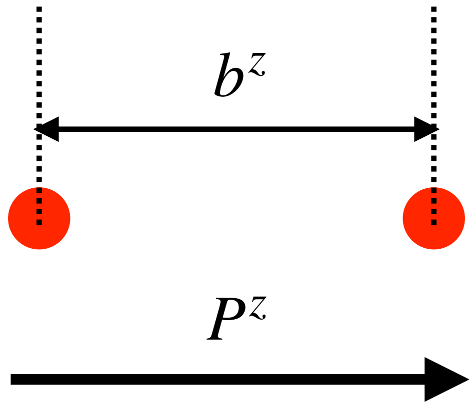
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Quasi-TMD in the Coulomb gauge

$$\tilde{h}(\vec{b}, \vec{P}, \mu) = \frac{1}{2P^t}\langle P | \bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0} |P\rangle$$

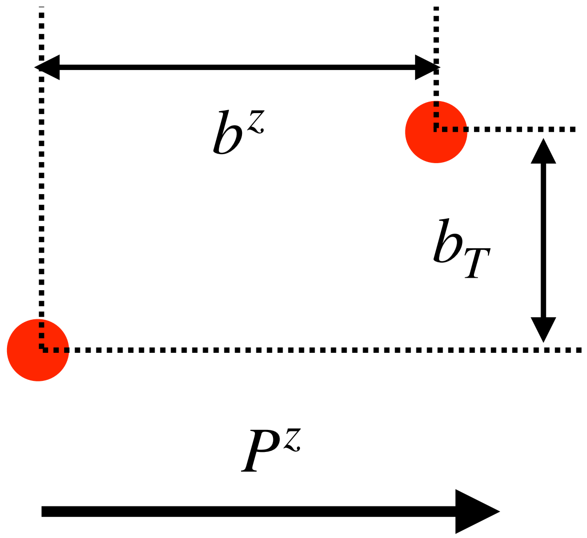
$$\tilde{f}(x, b_T, P^z, \mu) = P^z \int_{-\infty}^{\infty} \frac{db^z}{2\pi}\, e^{ixP^zb^z}\, \tilde{h}(b^z, b_T, P^z, \mu)$$

Quasi-PDF



X. Gao, W.-Y. Liu and YZ, Phys.Rev.D 109 (2024)

Quasi-TMD



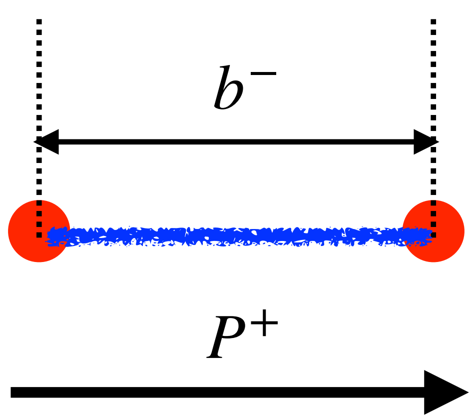
YZ, arXiv: 2311.01391.

Quasi-TMD in the Coulomb gauge

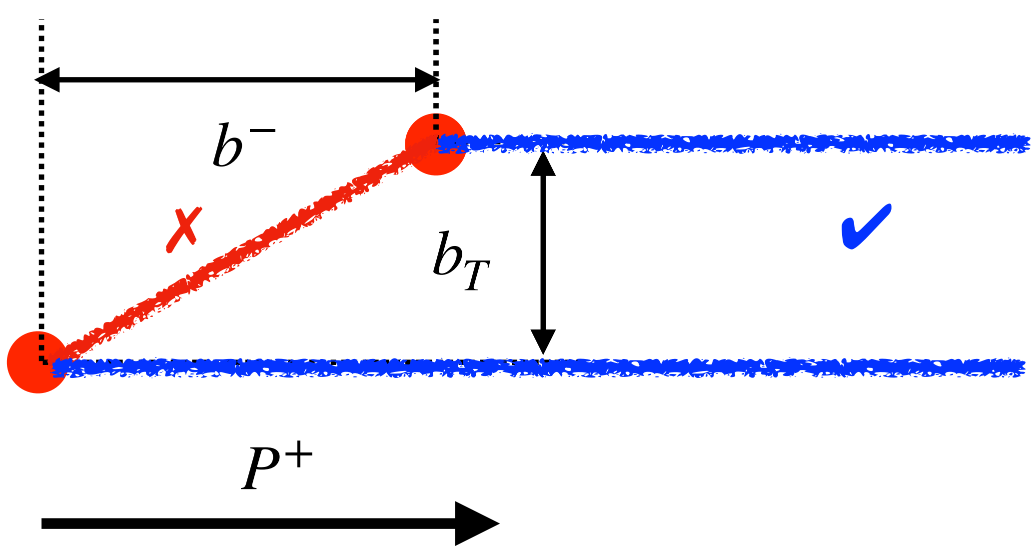
$$\tilde{h}(\vec{b}, \vec{P}, \mu) = \frac{1}{2P^t}\langle P\,|\,\bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|P\rangle$$

$$\tilde{B}(x, b_T, P^z, \mu) = P^z\int_{-\infty}^{\infty}\frac{db^z}{2\pi}\,e^{ixP^zb^z}\,\tilde{h}(b^z, b_T, P^z, \mu)$$

PDF



TMD



Parton distributions probe the correlation of energetic quarks and gluons dressed in the gauge background, which can be formulated by fixing a physical gauge condition.

$$G(A) = 0, \quad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}, A^+$$

Factorization formula

$$\frac{\tilde{B}(x, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, y_n)} = |C(xP^+/\mu)|^2 f(x, b_\perp, \mu, \zeta) + O(\lambda^2)$$

Verified at 1-loop!

Collins-Soper scale

$$\zeta = 2(xP^+)^2 e^{-2y_n}$$

Quasi soft factor

$$\tilde{S}_C(b_\perp, \mu, y_n) \equiv \frac{S_C^0(b_\perp, \ldots)}{S(b_\perp, \ldots, y_n)}$$

Coulomb-gauge zero-bin contribution

Soft function for physical TMD PDF

$$S_C^0 = \frac{1}{N_c}\langle 0 | T\left[S_n^\dagger(b_\perp)(U_C^s)^\dagger(b_\perp)U_C^s(0)S_n(0)\right]|0\rangle$$

Calculation from the same light-meson form factor:

$$F(b_\perp, P^z) = \langle \pi(-P) | j_1(b_\perp) j_2(0) | \pi(P) \rangle$$

$$= \int dx_1 dx_2 \; H_F(x_1, x_2, P^z, \mu) \frac{\phi(x_1, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, 0)} \frac{\phi(x_2, b_\perp, \mu, P^z)]}{\tilde{S}_C(b_\perp, \mu, 0)}$$

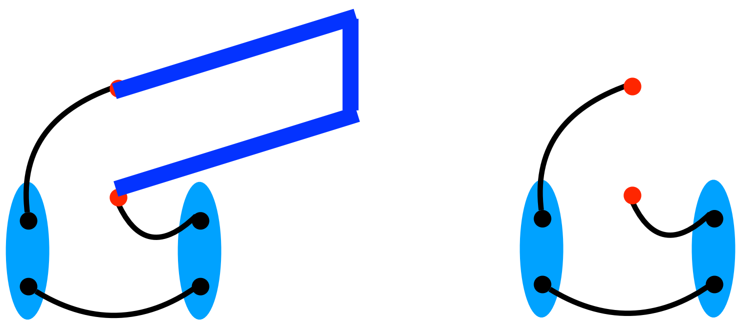
Hard kernel: known at 1-loop

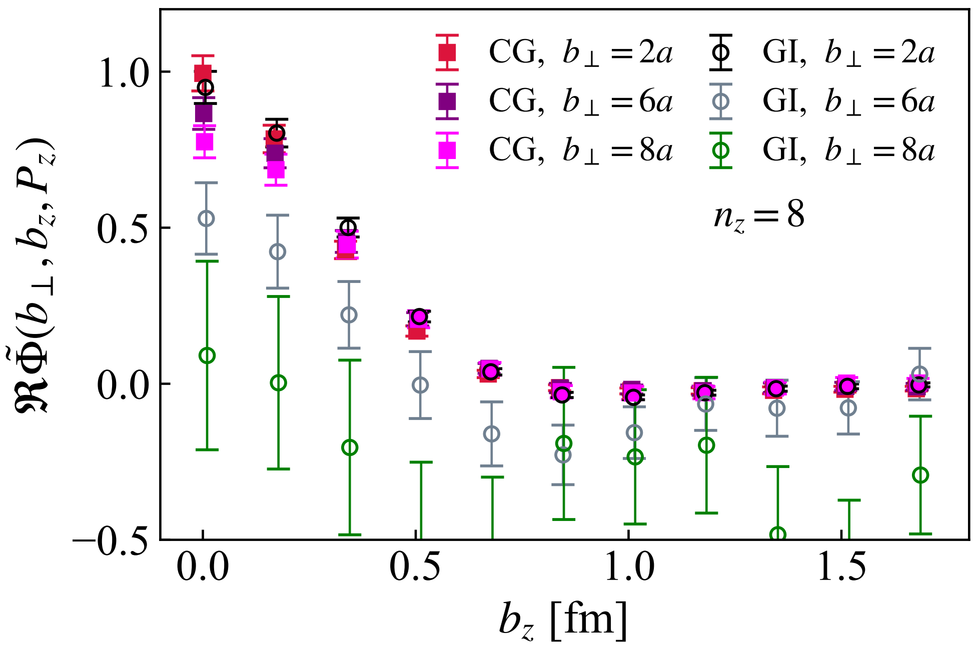
✔ $\phi(x, b_T, \mu, P^z)$: Coulomb gauge quasi-TMD wave function

$$\phi^* = \phi$$

Advantages

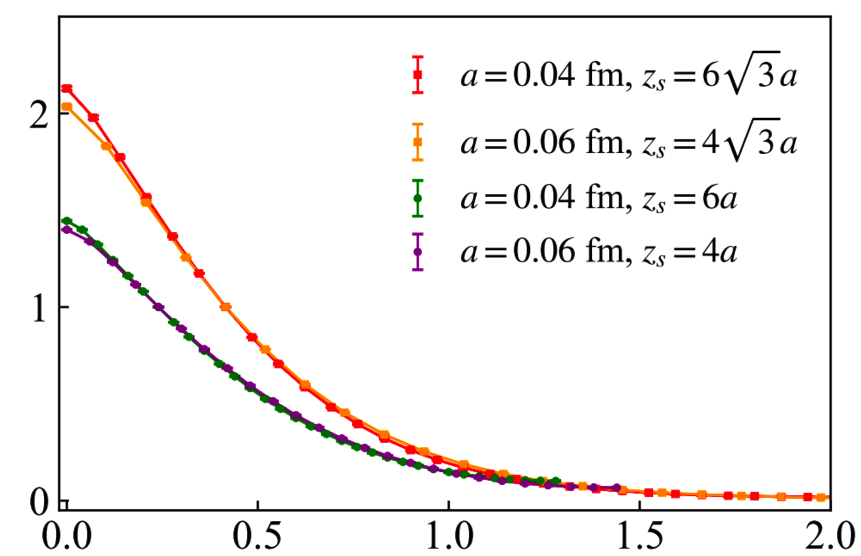
- Significantly improved statistical precision, access to larger b_T;





D. Bollweg, X. Gao, S. Mukherjee and YZ, Phys.Lett.B **852** (2024)

- Absence of linear power divergence and multiplicative renormalization;



X. Gao, W.-Y. Liu and YZ, Phys.Rev.D 109 (2024)

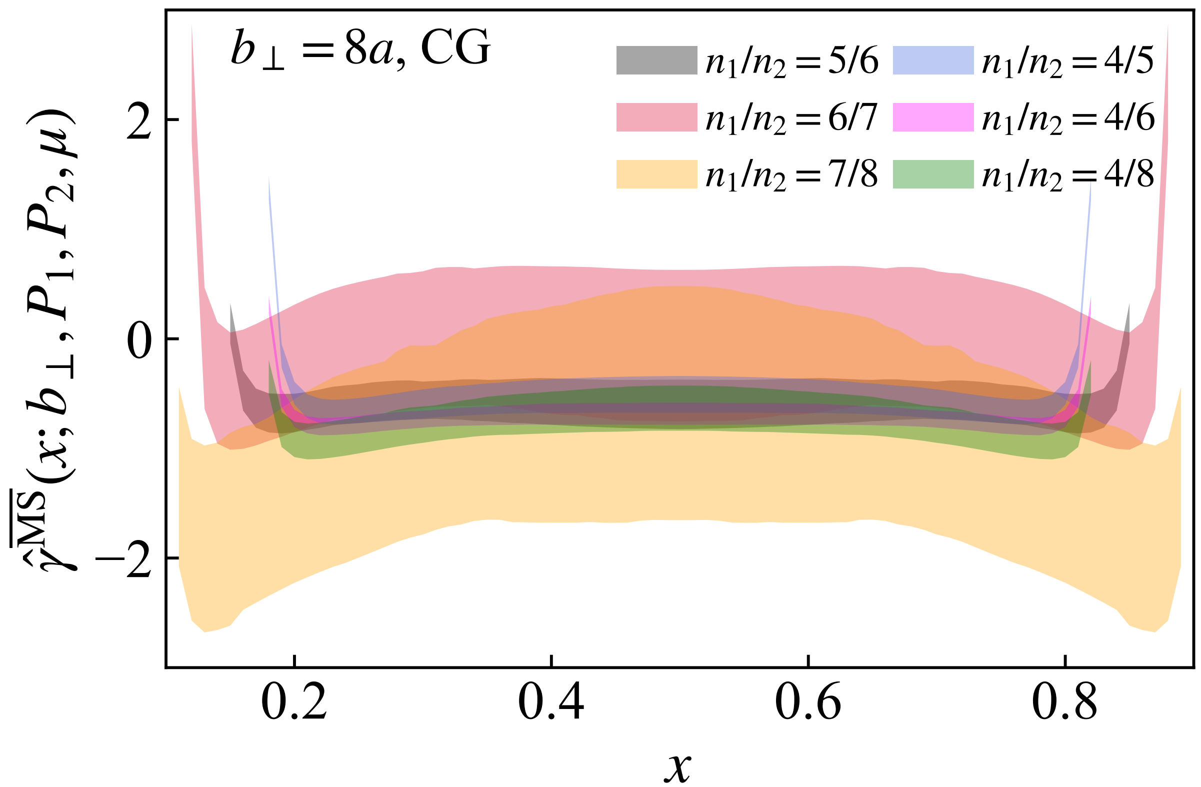
- Access to larger off-axis momenta.

$$\vec{P} = (0, P^z, P^z), \qquad \vec{b} = (b_\perp, b^z, b^z)$$

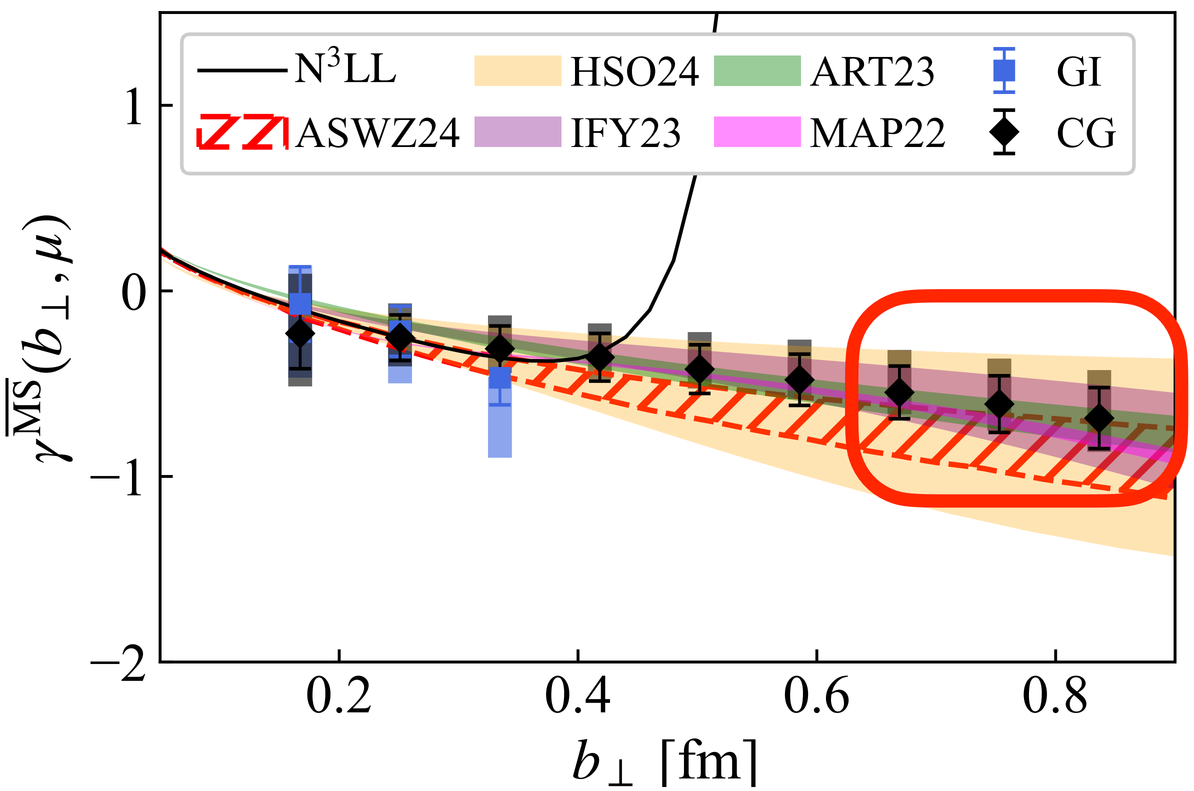
Exploratory calculation of the Collins-Soper kernel

- Nf=2+1 (chiral) domain-wall fermion configurations
- a=0.0836 fm, m_π=140 MeV, P^z_{max}=1.85 GeV.

Nice plateau in x and convergence in Pz



Agreement with earlier method and recent global fits



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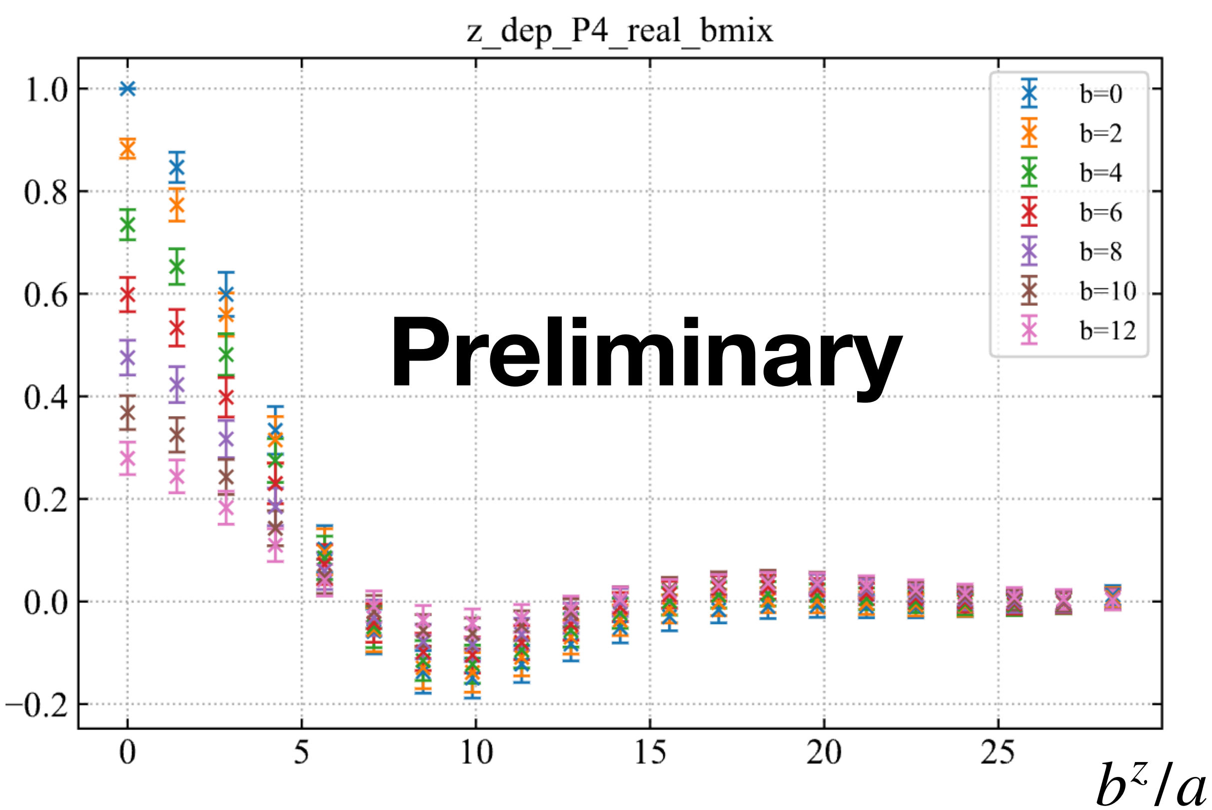
Accessibility to deeper non-perturbative region!

Spin-dependent proton TMDs

Preliminary results with the Coulomb-gauge method are encouraging:

u-d, unpolarized



u-d, helicity

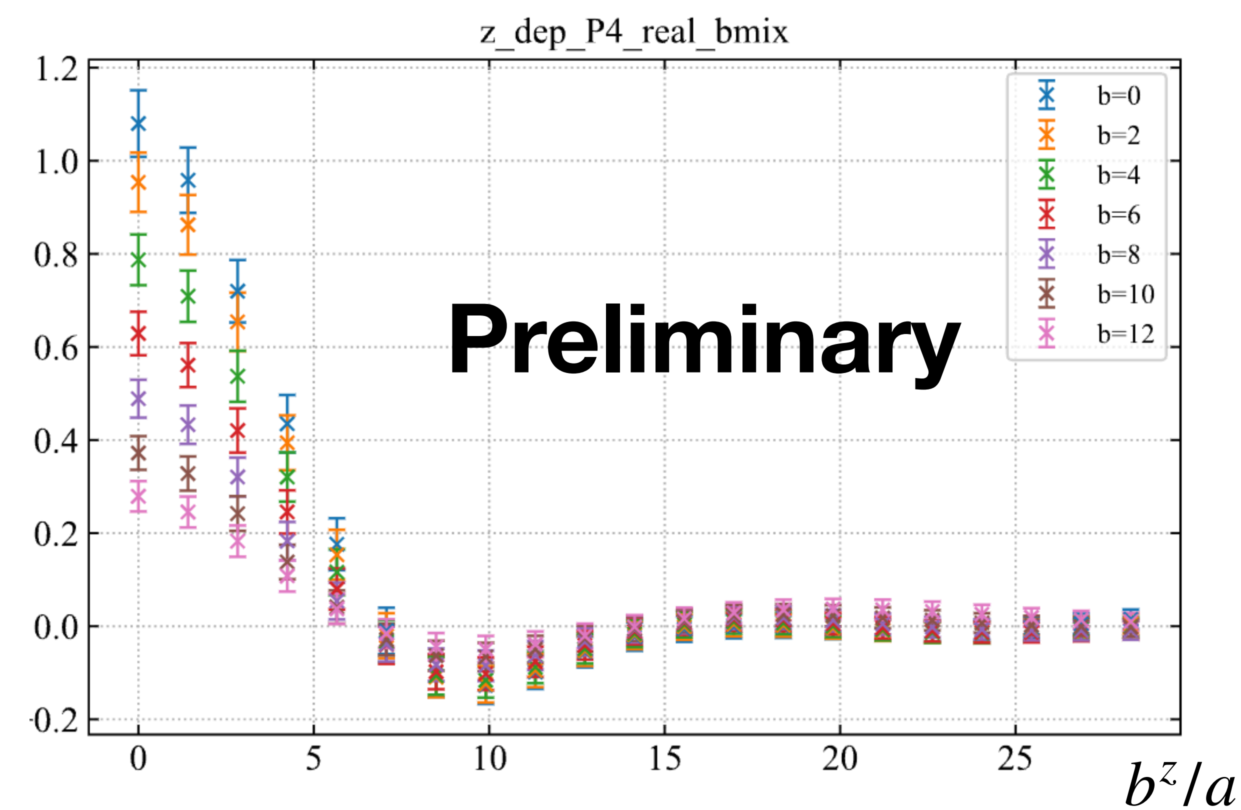


Image courtesy of Jinchen He and Xiang Gao.

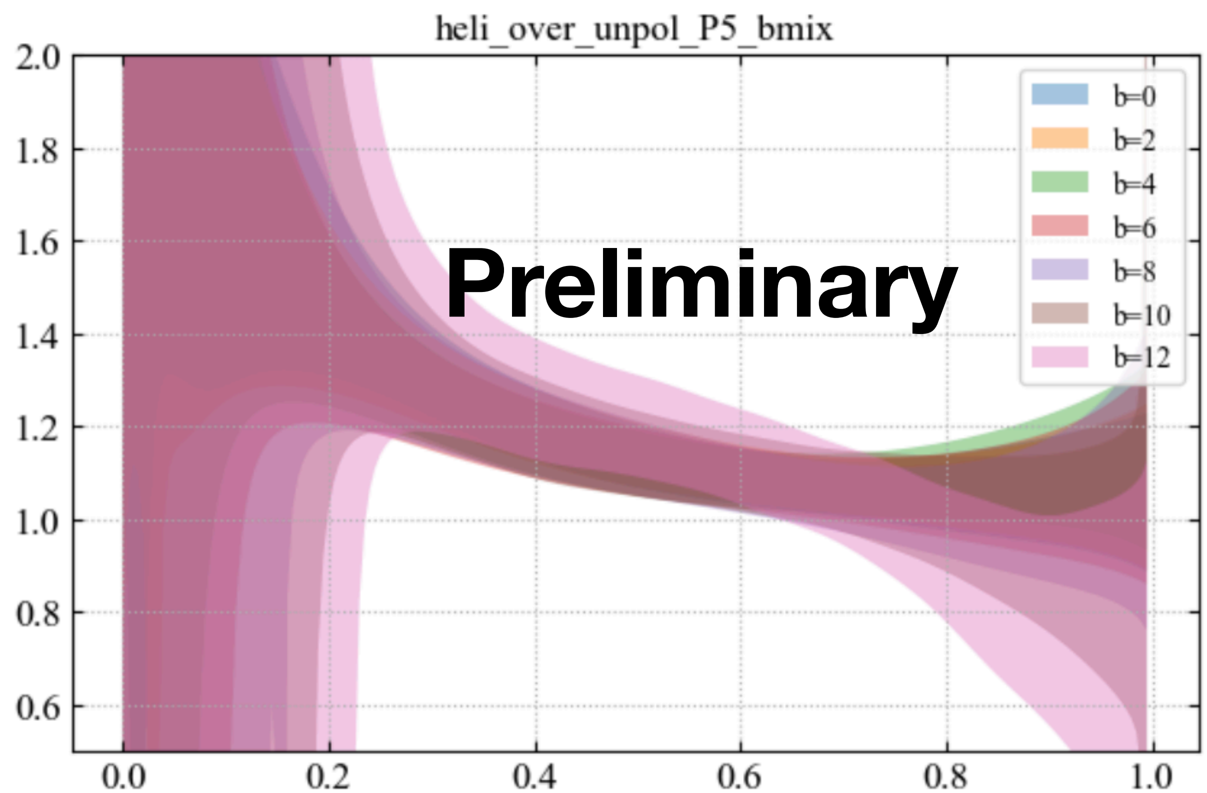
- Nf=2+1 HISQ fermion configurations with Wilson Clover fermion
- a=0.06 fm, m_π=300 MeV, $P^z{}_{\mathrm{max}}$=3.0 GeV.

Spin-dependent proton TMDs

No mixing between different spin structures or flavors:

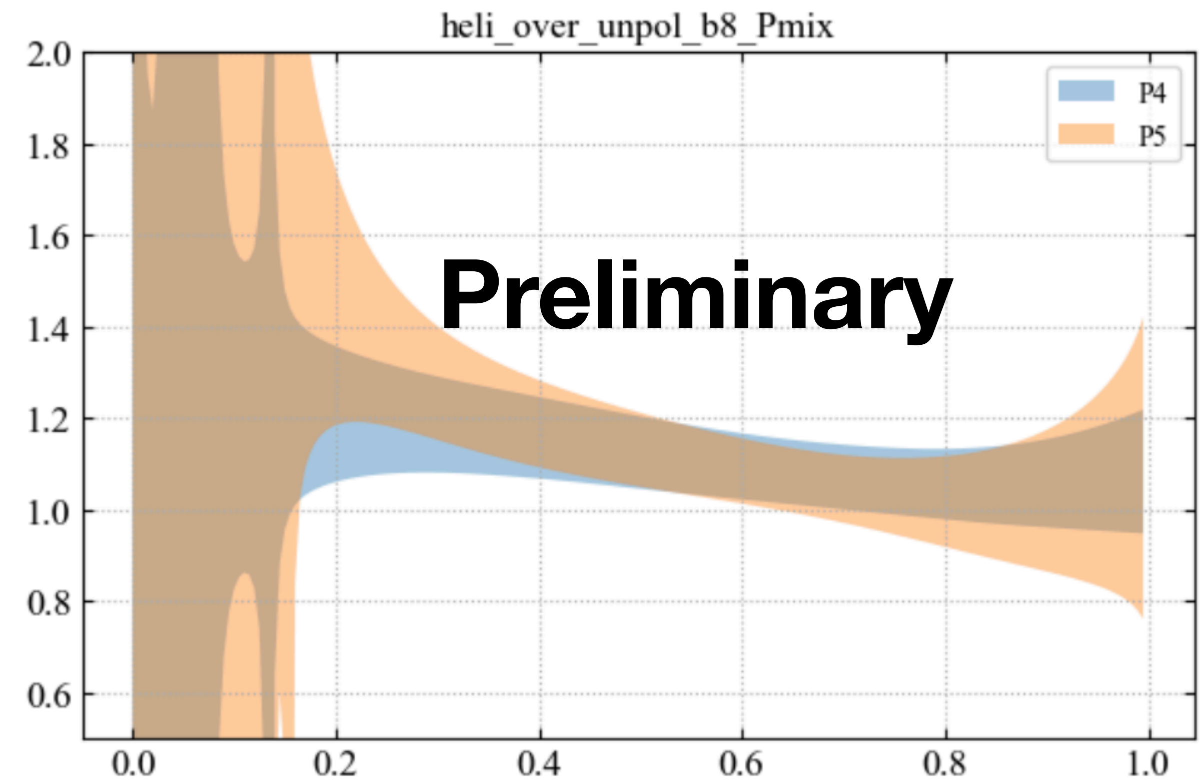
$$\frac{f_{i/p}^{[s]}(x, \mathbf{b}_T)}{f_{j/p}^{[s']}(x, \mathbf{b}_T)} = \frac{\tilde{B}_{i/p}^{[s]}(x, \mathbf{b}_T, P^z)}{\tilde{B}_{j/p}^{[s']}(x, \mathbf{b}_T, P^z)}$$

$$\frac{\Delta u(x, b_T) - \Delta d(x, b_T)}{u(x, b_T) - d(x, b_T)}$$



The b^T dependence is very weak.
Implication of flavor independence of intrinsic non-perturbative part?

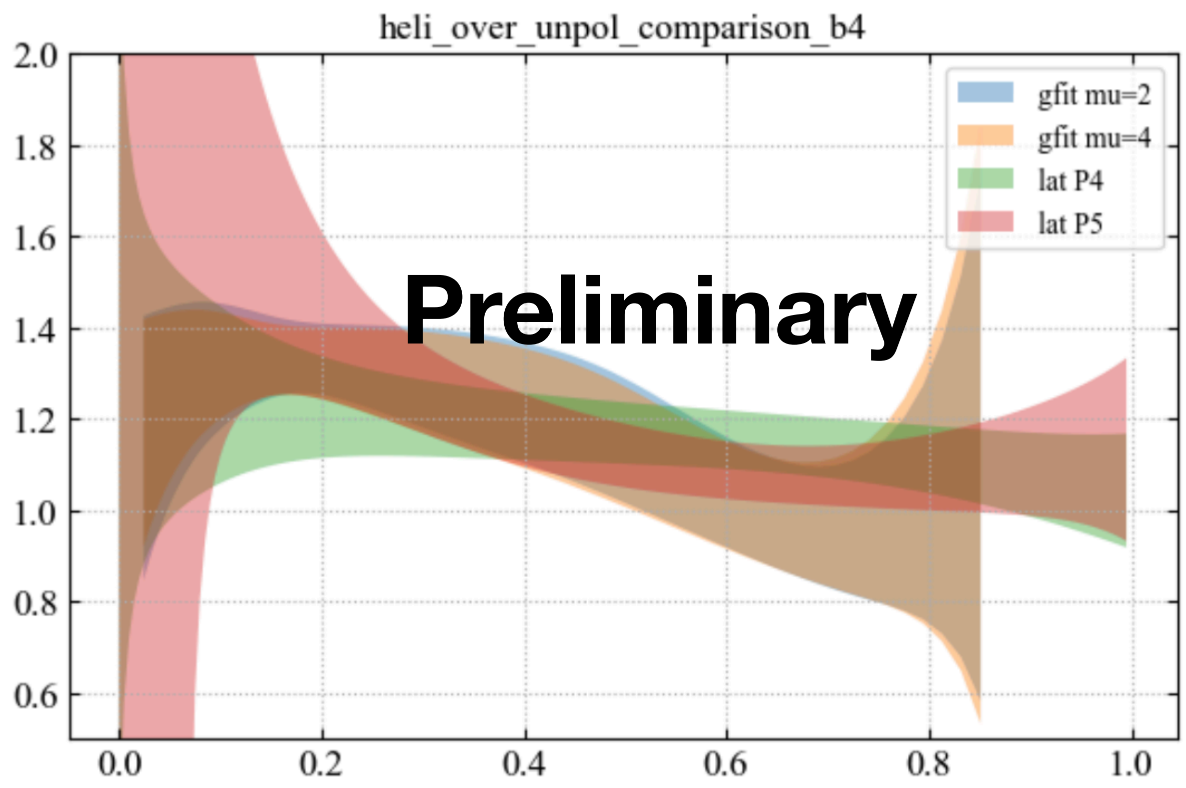
$$\frac{\Delta u(x, b_T) - \Delta d(x, b_T)}{u(x, b_T) - d(x, b_T)}$$

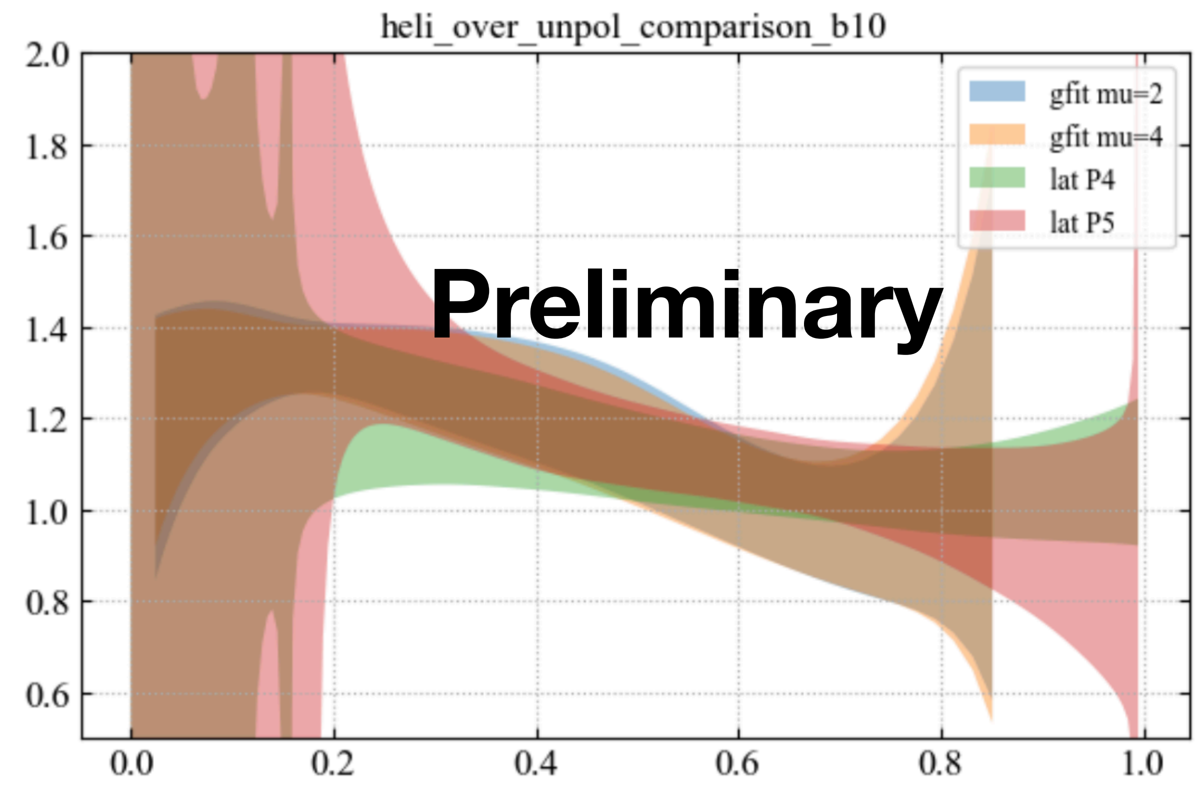


The Pz dependence appears saturated at moderate x.

Spin-dependent proton TMDs

Comparison with collinear PDF (nnpdf) ratio: $\dfrac{\Delta u(x) - \Delta d(x)}{u(x) - d(x)}$





- **Consistent with the PDF ratio within the moderate x region;**
- **The helicity TMD PDF is about a factor of 1.0 to 1.4 to the unpolarized case.**

Summary

- The LaMET framework has enabled lattice QCD calculation of the TMDs;
- A state-of-the-art lattice calculation has been carried out for the lattice calculation of the Collins-Soper kernel, which has achieved sufficient precision to differentiate phenomenological fits;
- The CG approach does not require the use of Wilson lines, which can significantly reduce the statistical error, simplify the renormalization and access higher off-axis momenta;
- Exploratory calculation of the Collins-Soper kernel with the CG approach shows efficiency of the method. Preliminary results of the proton helicity TMDs have achieved comparable statistical precision as global fits of the PDF.
- The new approach has the potential to enable precise lattice calculation of TMD physics in the deep non-perturbative region.

Gauge fixing and Gribov copies

- Find the gauge transformation Ω of link variables $U_i(t,\vec{x})$ that minimizes:

$$F[U^\Omega] = \frac{1}{9V}\sum_{\vec{x}}\sum_{i=1,2,3}\left[-\textbf{re Tr}\ U_i^\Omega(t,\vec{x})\right]$$

Precision ~ 10^{-7}

- Gribov copies correspond to different local extrema.
- Gauge-variant correlations may differ in different Gribov copies.
- However, the copies should contribute to the statistical noise if the algorithm randomly select the copies.

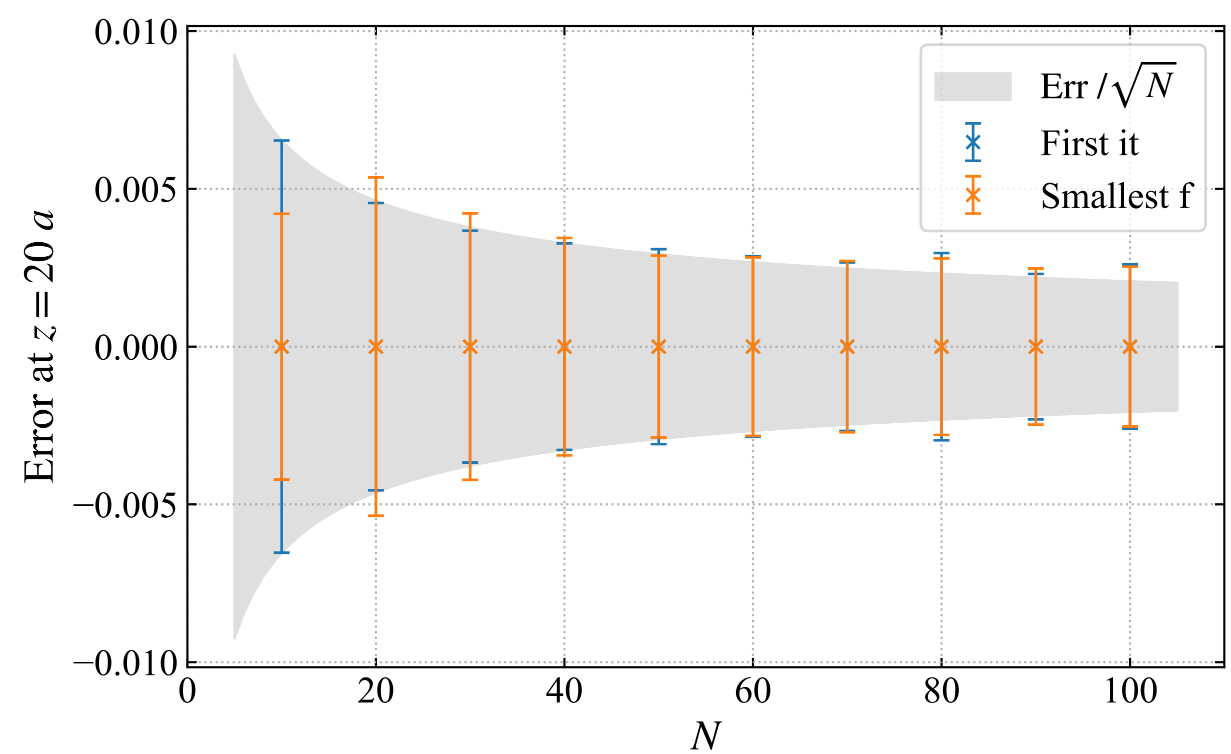
Test of the Gribov copies effect

For each configuration, obtain *n* daughter configurations that satisfy the criteria.

"First it": select the first daughter configuration.

"Smallest f": select the configuration with the lowest functional value.

Measure the pion quasi-PDF matrix element.



J. He et al., work in preparation.

Transverse link and T-odd TMDs

- T-odd light-cone TMD:

P.V. prescription = anti-symmetric boundary condition $A_\perp^\mu(\infty^-) = -A_\perp^\mu(-\infty^-)$

Need a transverse link to define the T-odd TMDs

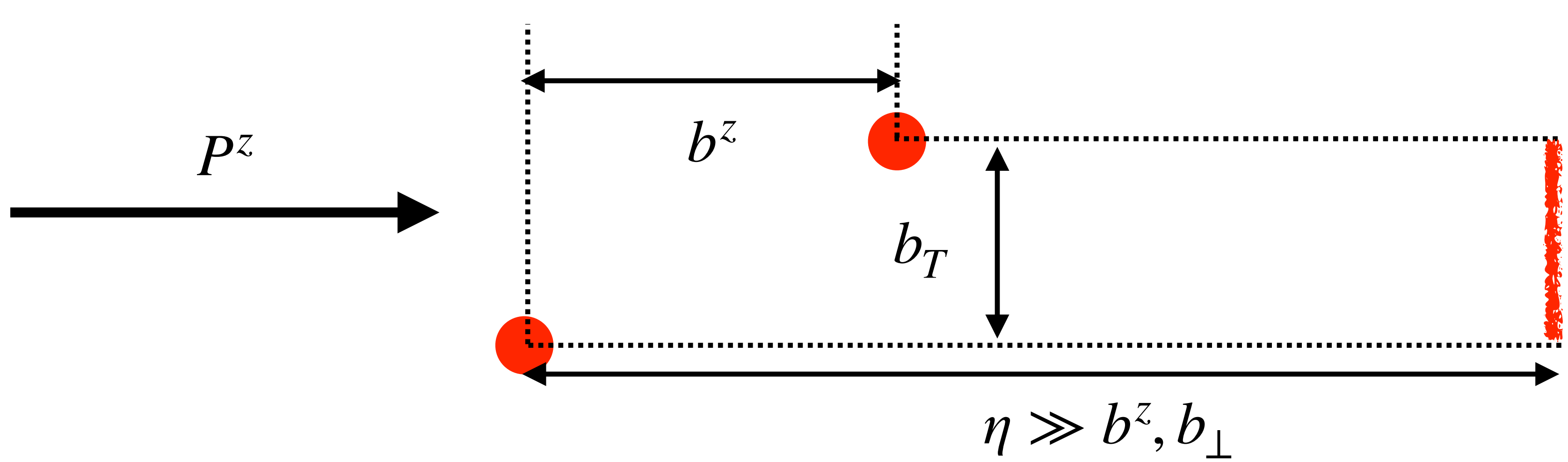
Contribution in a Feynman diagram: $\dfrac{e^{-i\infty^- k^+}}{[k^+]_{\rm pv}} = -i\pi\delta(k^+)$

- Ji and Yuan, PLB **543** (2002)
- Belitsky, Ji and Yuan, NPB **656** (2003)

- Coulomb-gauge quasi-TMD:

$$\tilde{h}(\vec{b}, \vec{P}, \mu, \pm) = \frac{1}{2P^t}\langle P \,|\, \bar{\psi}(\vec{b})\mathscr{W}_\perp(\pm\infty\hat{z}; b_\perp, 0_\perp)\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|\, P\rangle$$

Quasi-TMD with a transverse link



$$\frac{k^z e^{-i\infty^z k^z}}{k_z^2 + k_\perp^2} \xrightarrow{k^z \gg k_\perp} -i\pi\delta(k^z)\ ?$$

Gluon TMDs

- Factorization formula can be derived using SCET as well.
- No linear power divergence.
- Mixing with gauge-variant operators. However, the number of mixings is finite under the constraint of SO(3) symmetry.
- Could be more susceptible to the Gribov noise, but it may still easily beat the statistical precision achieved with Wilson line operators.

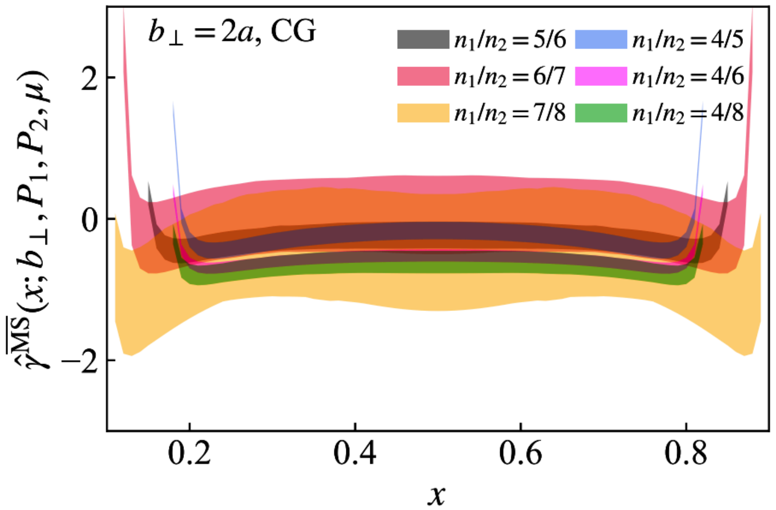
High-order corrections

- Known to be difficult due to non-covariant nature, but it is just one scale in massless integrals.

- No complete 2-loop result so far, even for the quark wave function renormalization.

- Linear renormalon in the matching coefficient.
  - Corresponds to a linear power correction of order $\Lambda_{\rm QCD}/P^z$;
  - However, strength of the renormalon is not easy to estimate. An NNLO calculation can offer a lot of insight.
  - Lattice calculation of the Collins-Soper kernel suggests that the result converges in P^z quite well.

Y. Liu and Y. Su, JHEP **2024** (2024)



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