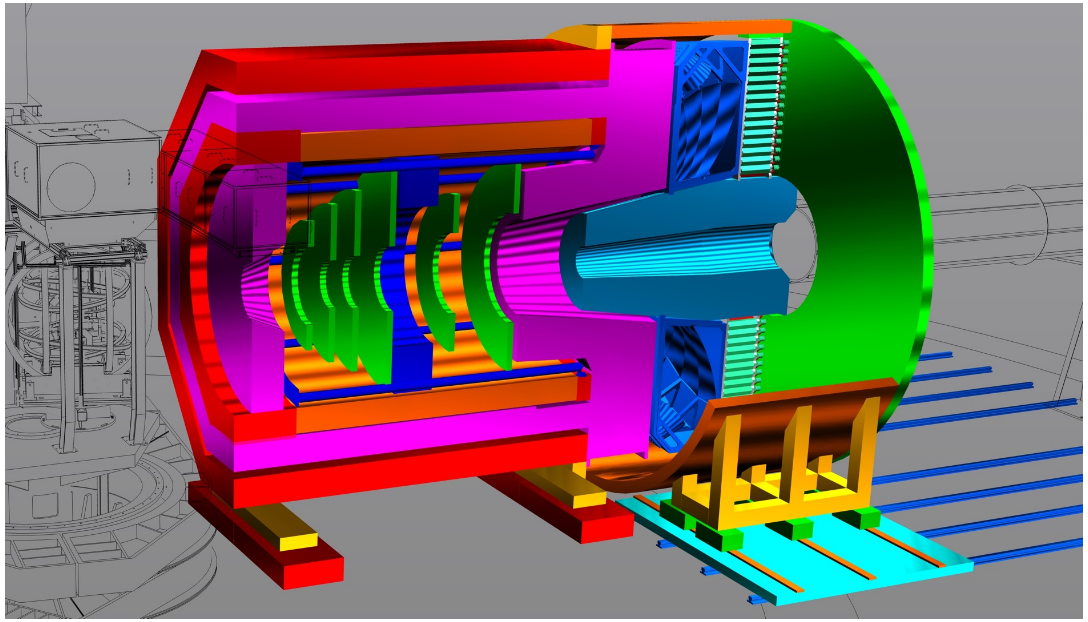
Inclusive Hadron Production and Relation to TMDs

- 3-D hadron structure & 2-Scale observables
- SIDIS, TMDs & Angular modulations
- Collision induced QCD & QED radiations
  – treating them equally
- Prompt inclusive single hadron production
  – test of QCD fragmentation picture
- Summary and Outlook

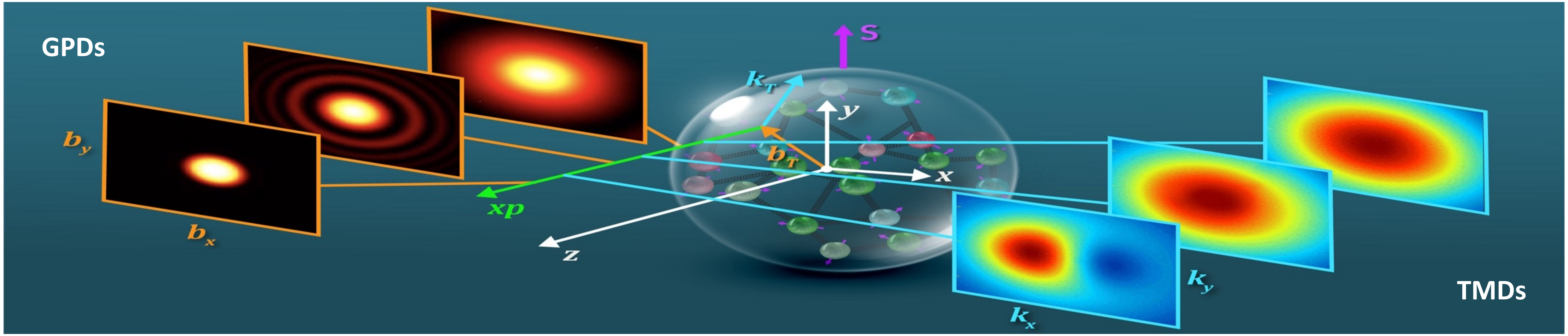


In collaboration with K. Watanabe, T.B. Liu, J.Y. Zhang, …

Jianwei Qiu
Jefferson Lab, Theory Center

3-D Hadron Structure and 2-Scale Observables

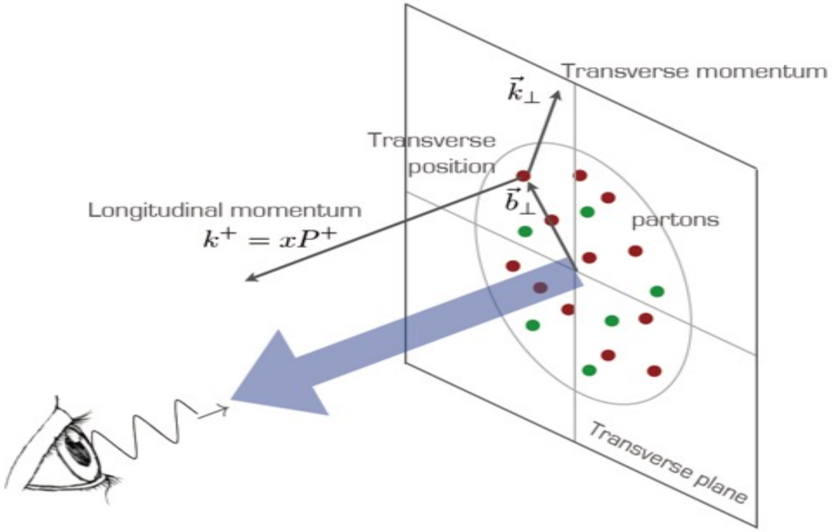
❑ 3-D hadron structure:



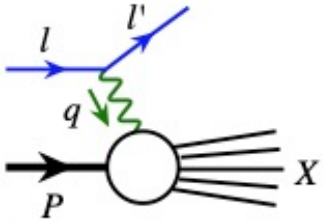
❑ Need new observables with 2 distinctive scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

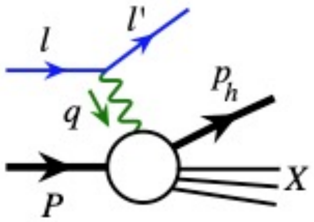
- **Hard scale:** Q_1 to localize the probe to see the particle nature of quarks/gluons
- **"Soft" scale:** Q_2 to be more sensitive to the emergent regime of hadron structure ~ 1/fm



Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering (SIDIS)

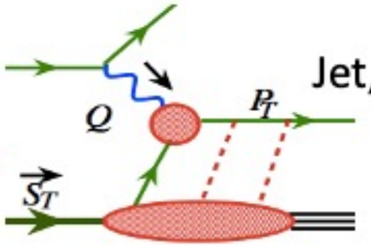


Scale: Q^2 - **PDFs**



$Q^2 \gg P_{h_T}^2$

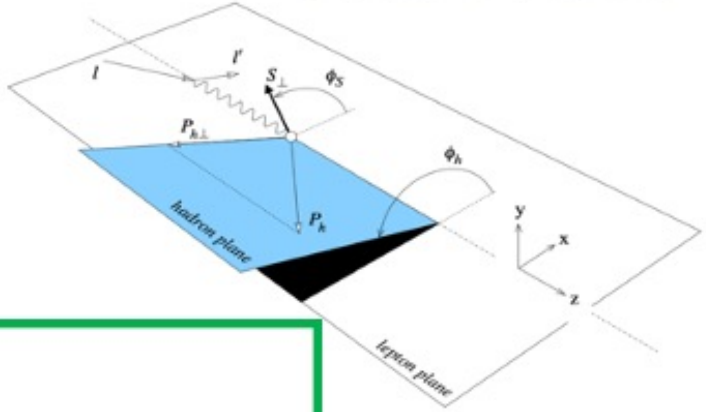
In photon-hadron frame!



$f(x, k_T, Q)$ - **TMDs**

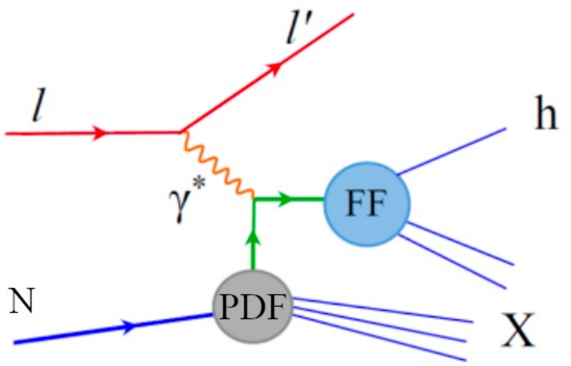
$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =\ & \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
& + S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
& + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
& + |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
& + |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

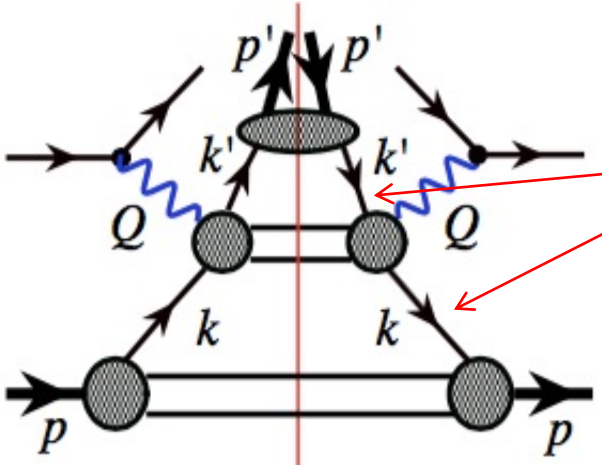
18 SIDIS Structure Functions



QCD Factorization – Linking Structure Functions to TMDs

❑ SIDIS when PT ≪ Q in the photon-hadron frame :





Pinch in both k^2 and k'^2

Active partons are "long-lived" comparing to the time of hard collisions

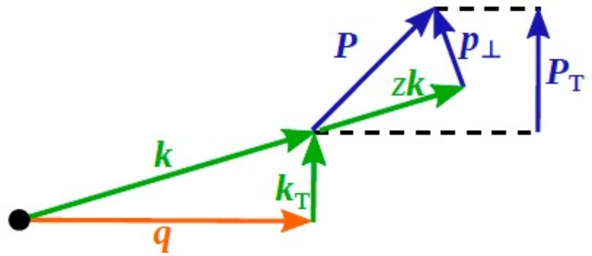
❑ QCD TMD factorization:

$$F_{UU,T} = \mathcal{C}[f_1 D_1]$$

$$\sigma \propto \text{TMD PDF} \otimes \text{hard part} \otimes \text{TMD FF}$$

Convolution of two TMDs:

$$\mathcal{C}[wfD] = x \sum_q e_q^2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{P}_\perp\, \delta^{(2)}(z\boldsymbol{k}_T + \boldsymbol{P}_\perp - \boldsymbol{P}_T) w(k_T, P_\perp)\, f^q(x, k_T, Q^2)\, D^{q\to h}(z, P_\perp, Q^2)$$

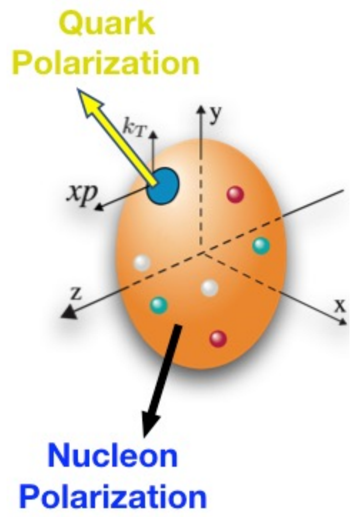


$$\boldsymbol{P}_T \approx z\boldsymbol{k}_T + \boldsymbol{P}_\perp$$

Transverse Momentum Dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

Nucleon Polarization	Quark Polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ Boer-Mulders
L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^\perp(x,k_T^2)$ Long-Transversity
T	$f_1^\perp(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Trans-Helicity	$h_1(x,k_T^2)$ Transversity; $h_{1T}^\perp(x,k_T^2)$ Pretzelosity

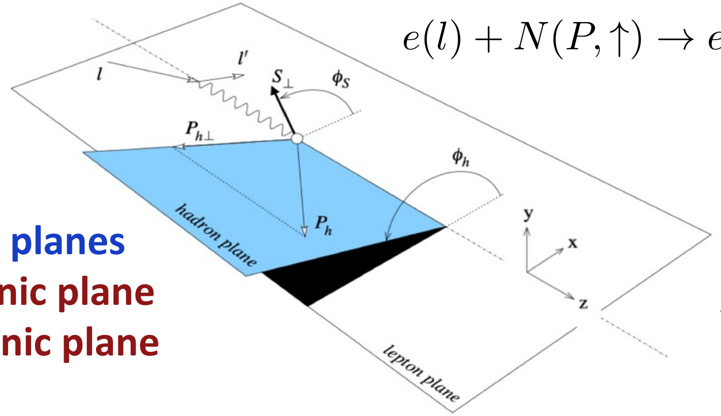


Analogous tables for:

- Gluons $f_1 \to f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

❑ Polarized SIDIS:

$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$



Two planes
Leptonic plane
Hadronic plane

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

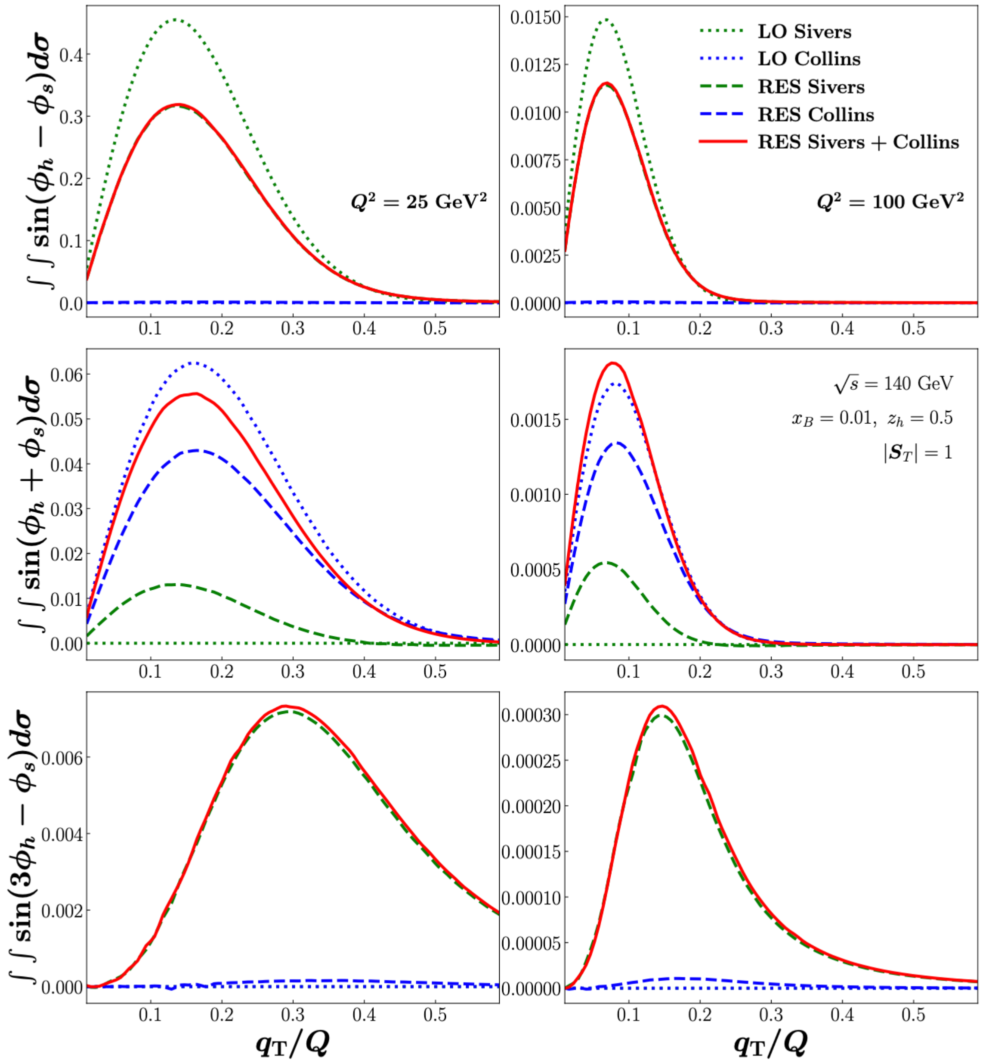
$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

Angular modulation provides the best way to separate TMDs

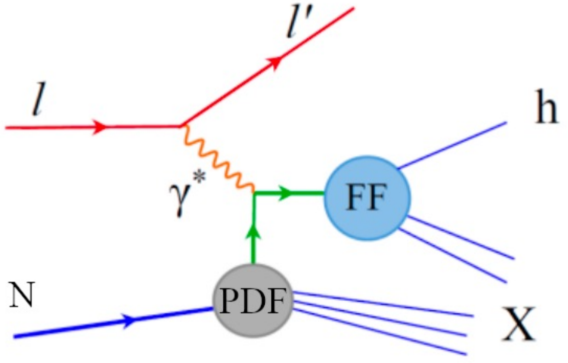
QCD Factorization is an Approximation

- **Impact of collision induced QED radiation:**

Change the angular modulations!



- **Leading power approximation in PT/Q:**



The observed hadron should be:

- A lead hadron within the hadronization jet of the fragmenting parton – “Prompt”
- The invariant mass of the hadronization jet should be much smaller than the energy of the jet

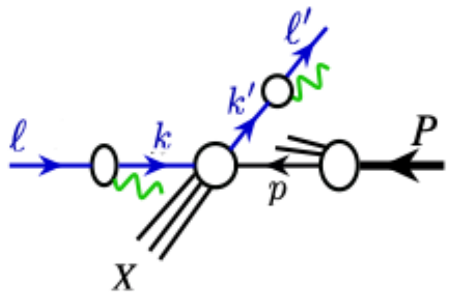
Unlikely to be satisfied by JLab kinematics

Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033; JHEP 11 (2021) 157

Treat QCD and QED equally in terms of Factorization

❑ Inclusive Deep Inelastic Scattering (DIS):

Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033;
JHEP 11 (2021) 157



$$E'\frac{d\sigma_{\ell P\to\ell' X}}{d^3\ell'} \approx \frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\, D_{e/j}(\zeta,\mu^2)\, f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\, f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi\ell, xP, \ell'/\zeta, \mu^2) + (1/\ell_T')^{\alpha}$$

LFFs — LDFs — PDFs — IRS hard coefs

$$i.j = e,\gamma,\bar{e},\ldots,q,g,\ldots$$
$$a = q,g,\bar{q},e,\gamma,\bar{e},\ldots$$

- No DIS "Structure Functions"!
- QED & QCD contribution are factorized at the same scale: μ

$$(x_B, Q^2) \leftrightarrow (y_\ell, \ell_T')$$

$$\widehat{H}_{ia\to jX}(\xi\ell, xP, \ell'/\zeta, \mu^2) \approx \widehat{H}^{(m,n)}_{ia\to jX}(\xi\ell, xP, \ell'/\zeta, \mu^2)$$
$$\approx \mathcal{O}(\alpha^m \alpha_s^n)$$

All collision induced QED radiations are included in

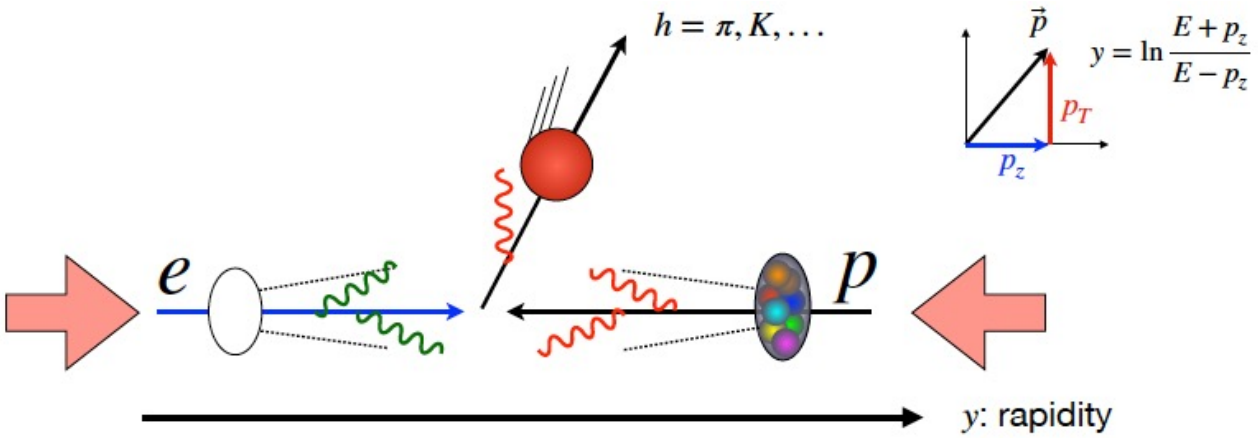
- Universal LDFs and LFFs, or
- Process-dependent, but, calculable infrared Safe short-distance hard parts
- Process-dependent, but, power suppressed ("High twists") contributions !

See also talk by J.Y. Zhang tomorrow

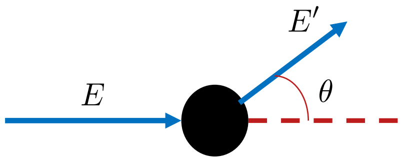
Prompt inclusive single hadron production – Test of fragmentation picture

❑ Single hadron (Jet) production in lepton-hadron scattering:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation



JLab kinematics – fixed target:



$$p_T = E' \sin(\theta) \gtrsim 1 \text{ GeV?}$$

Factorization:

$$E\frac{d\sigma_{\ell P\to pX}}{d^3p} \approx \frac{1}{2s}\sum_{i,a,b}\int_{z_{\min}}^{1}\frac{dz}{z^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{h/b}(z,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to bX}(\xi\ell, xP, p/z, \mu^2) + (1/p_T)^{\alpha}$$

Single hard scale
Collinear factorization

Nayak, Qiu, Sterman, PRD72, 114012 (2005)

The new unknown is $f_{i/e}(\xi,\mu^2)$
lepton distribution functions (LDFs)

Hadron fragmentation functions (FFs):

Known, but, limited knowledge, in particular, at large z !

Evolution of lepton distribution functions (LDFs)

❑ Modified DGLAP equation for LDFs:

Qiu, Watanabe
In preparation

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

- **Factorization scale:**

  $$\mu^2 \sim m_c^2$$

- **Input LDFs at μ^2:**
  - **Perturbatively generated by solving QED evolution from lepton mass threshold**
  - **With perturbatively calculated fixed-order MSbar LDFs**
  - **Test the size of non-perturbative hadronic contribution**
  - …

Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $P_{ij}^{(0,0)} = 0\,,\quad N_F\,,\quad N_l$

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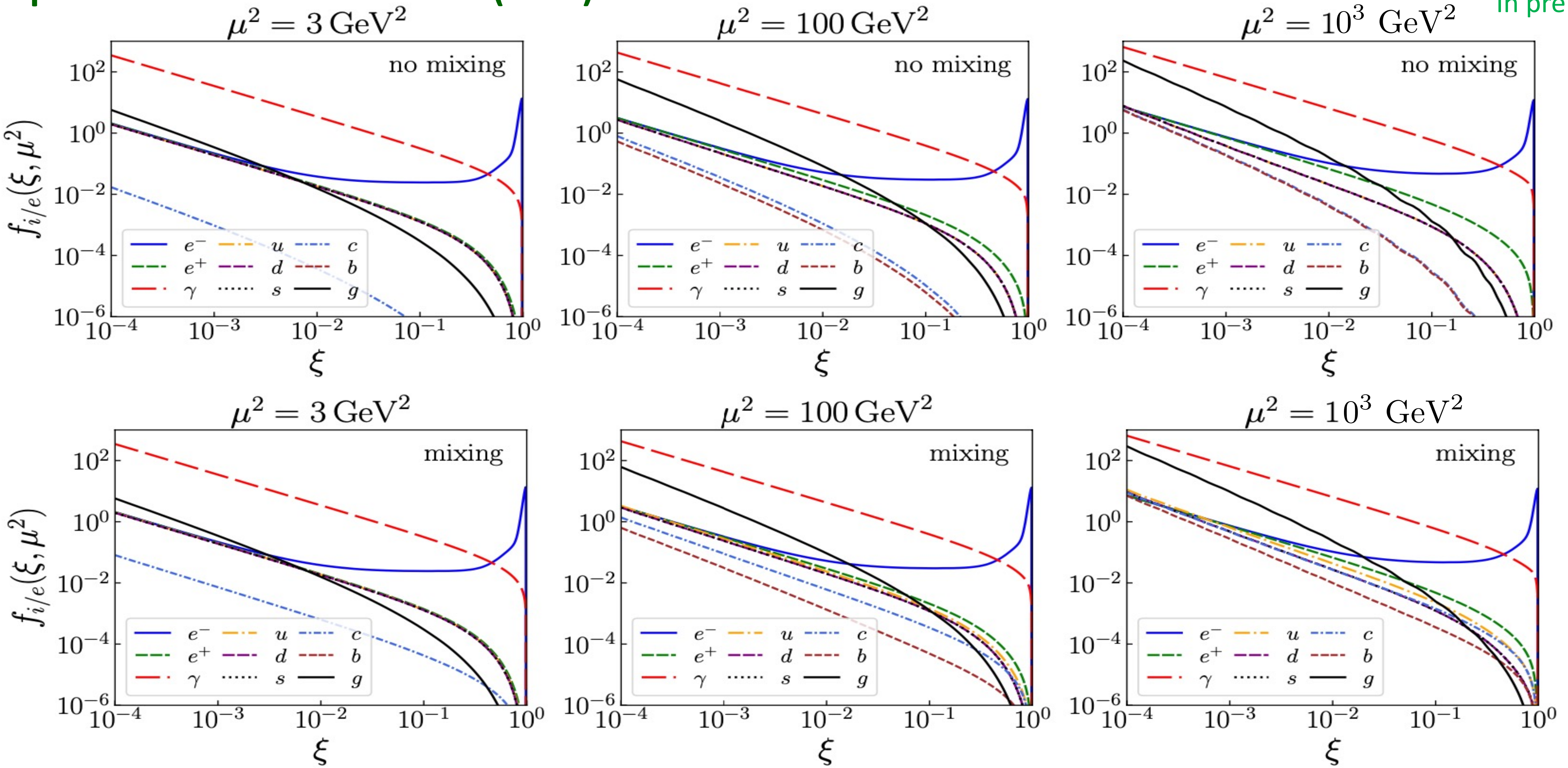
Evolution of lepton distribution functions (LDFs)

❑ **Lepton distribution functions (LDFs):**

Qiu, Watanabe
In preparation



With LDFs, we calculated single hadron production, including J/ψ production at the EIC

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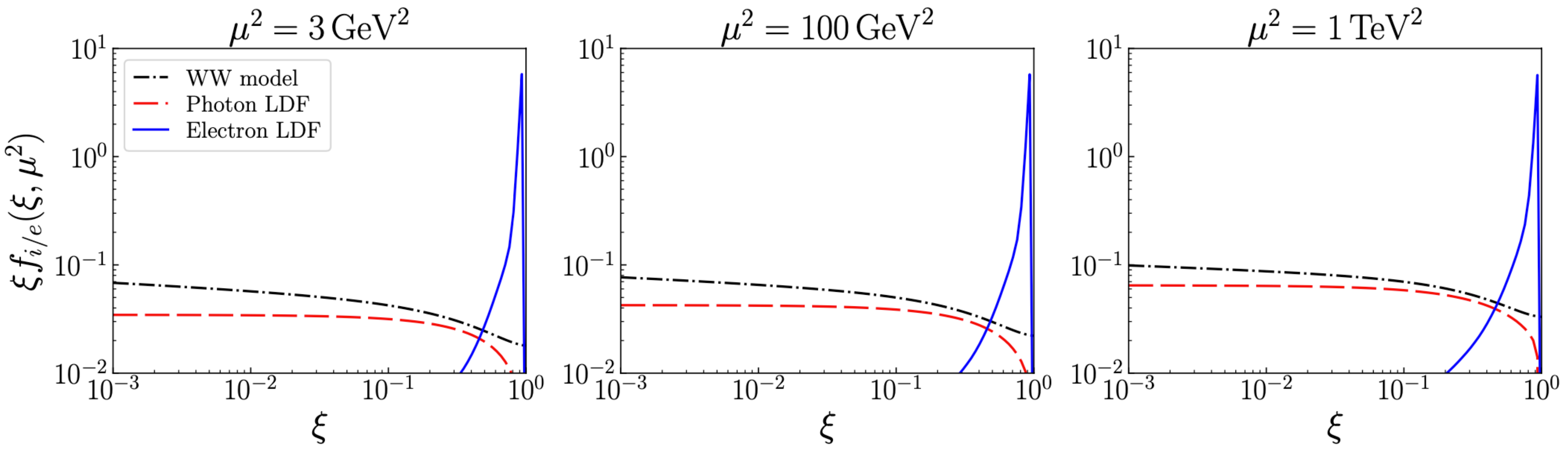
Evolution of lepton distribution functions (LDFs)

Qiu, Watanabe
In preparation

❑ Photon distribution of the electron:

- **Weizsä̈cker-William photon distribution:**

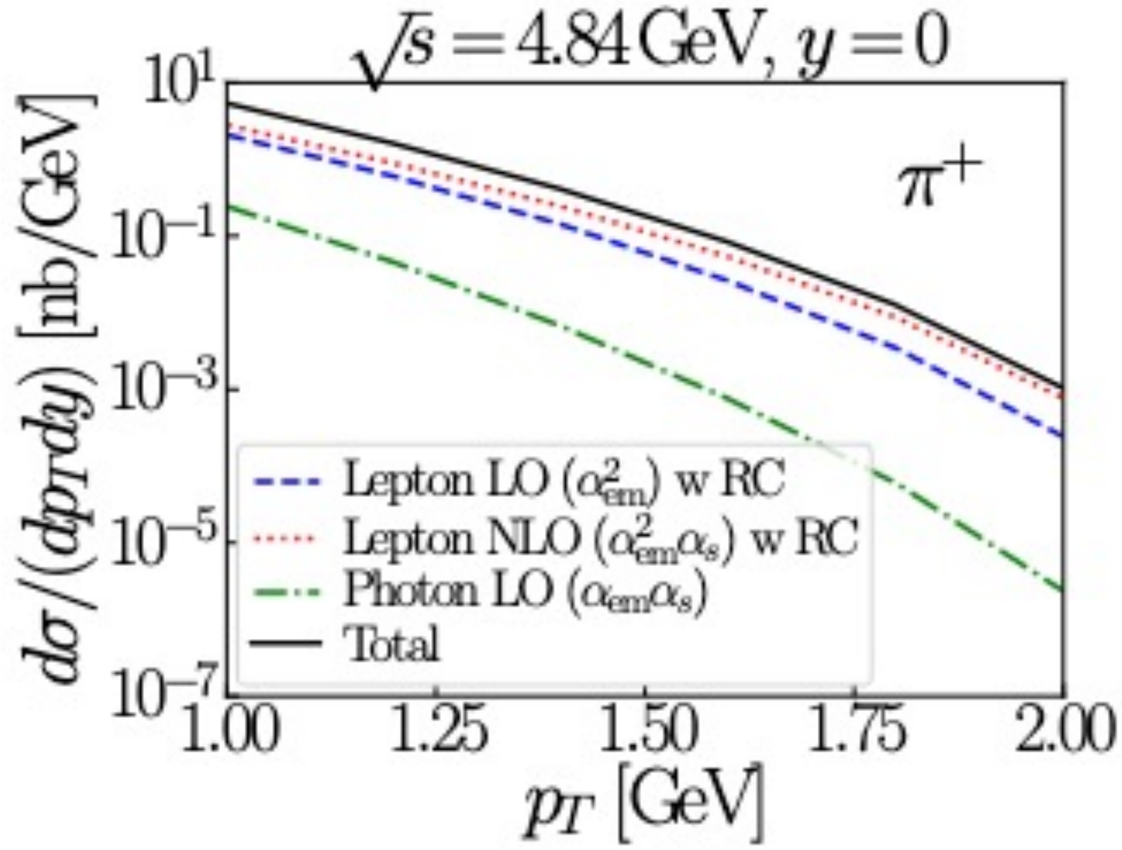
$$f_{\gamma/e}^{\mathrm{WW}}(\xi,\mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi) \left[\ln\left(\frac{\mu^2}{\xi^2 m_e^2}\right) - 1 \right]$$

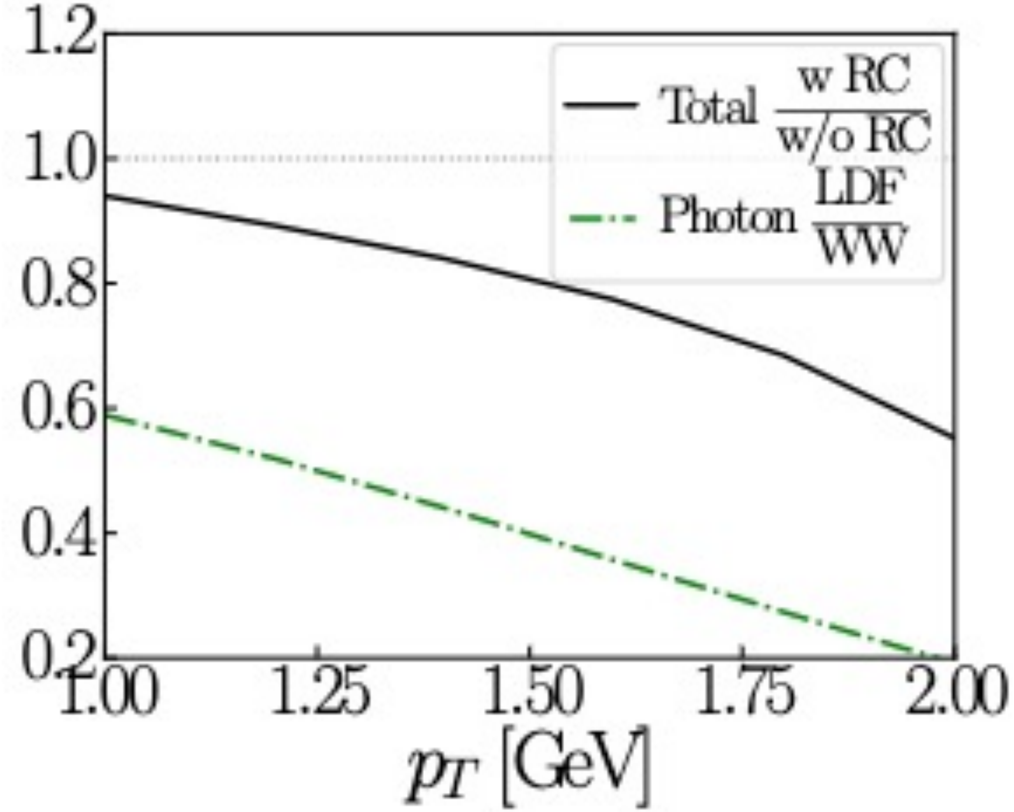


- **LDFs are not purely perturbative in QED or perturbative – need global analysis!**
  - **Precision measurements for BSM physics at the EIC needs reliable lepton distributions**
  - **Joint global analysis of lepton and hadron distribution functions should be carried out.**
  - **Impact on searching BSM at ILC or CEPC, FCC, …**

Predictions at JLab 12 GeV Energy

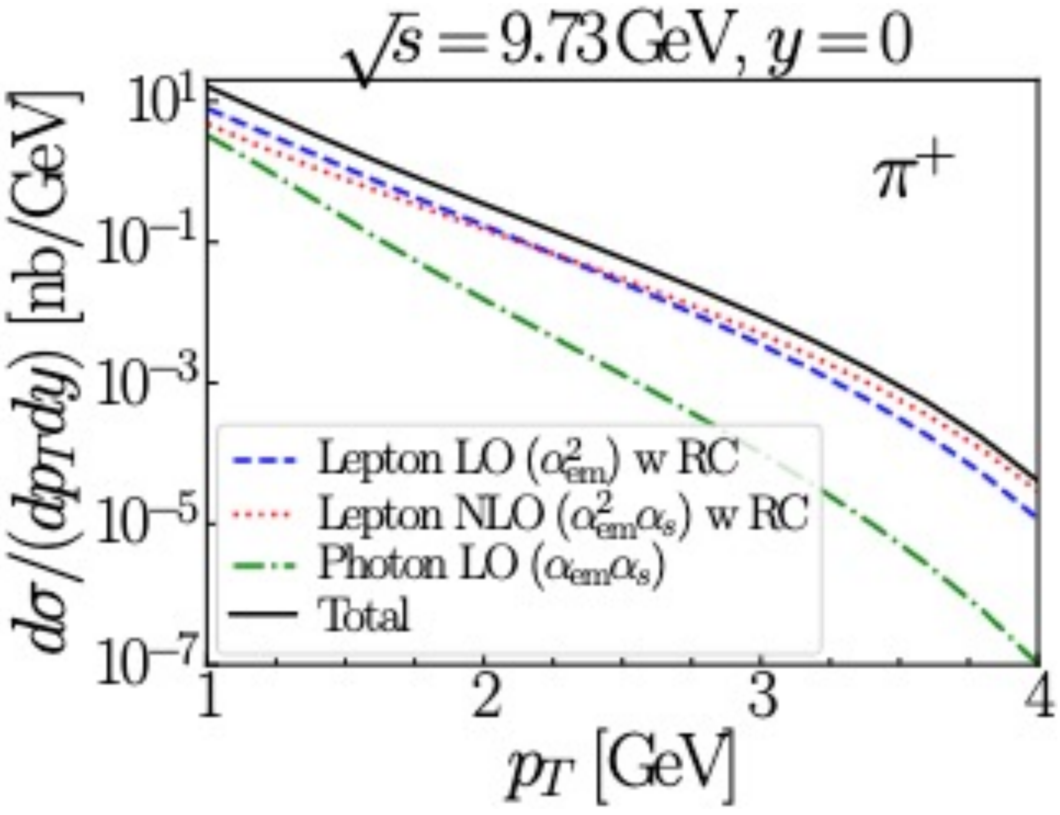
❑ **FFs – JAM20:**

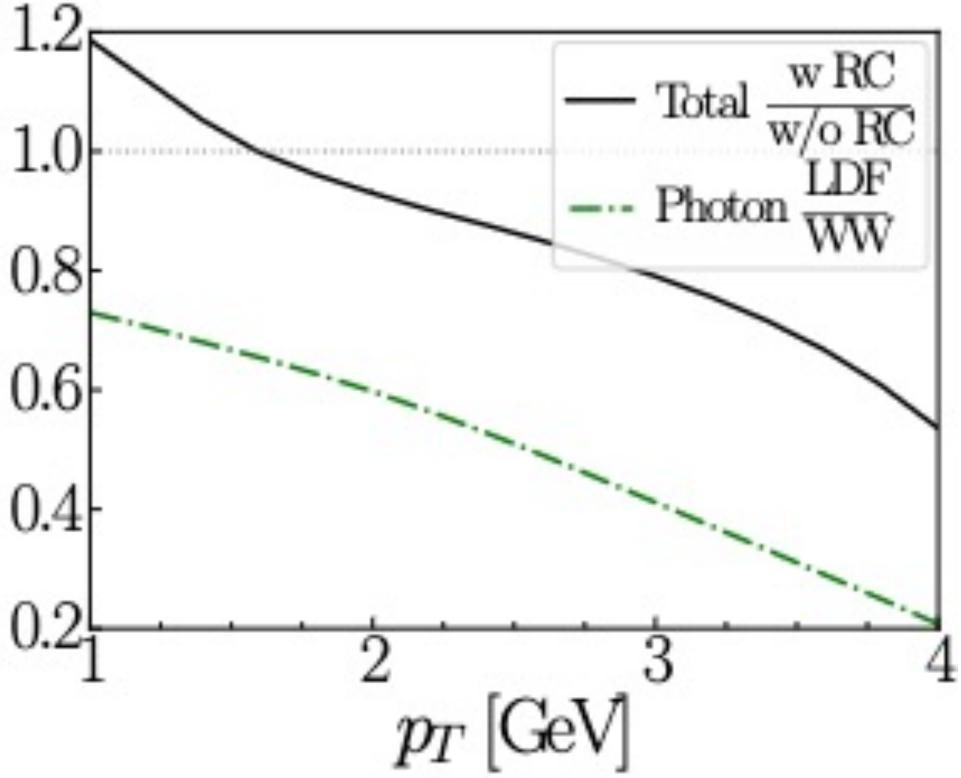




Predictions at SLAC Energy

- **FFs – JAM20:**





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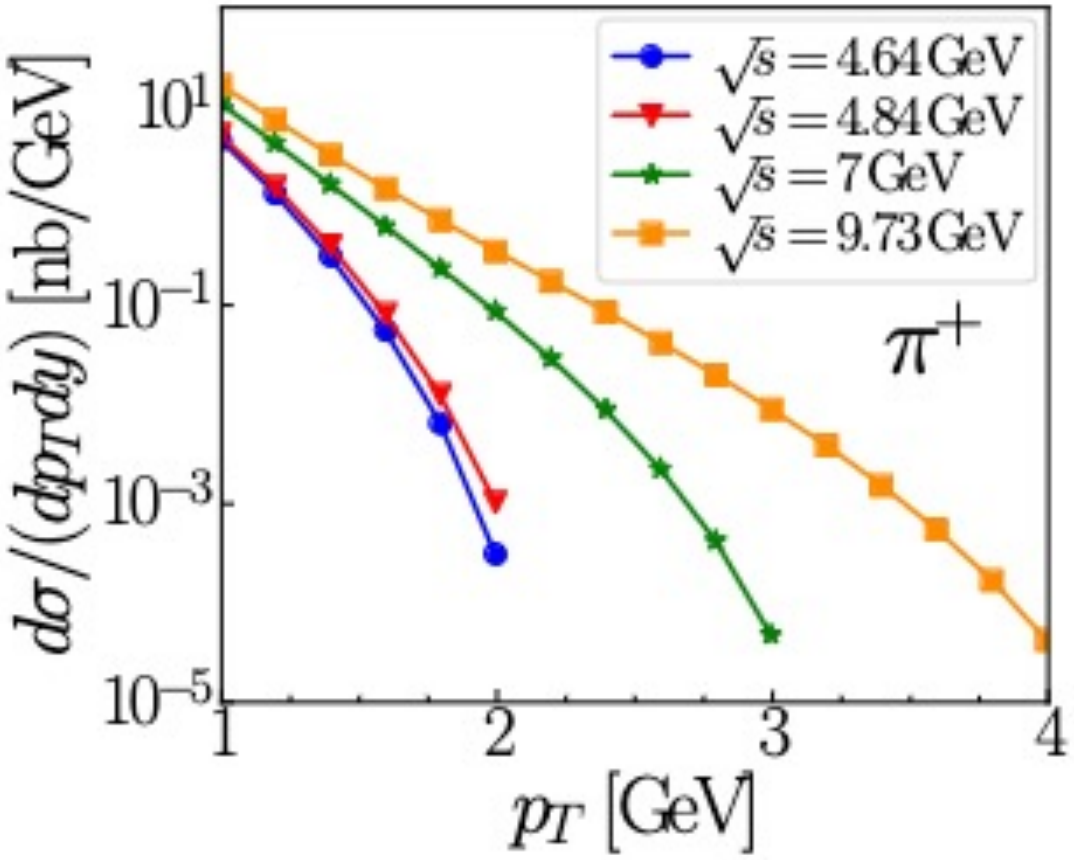
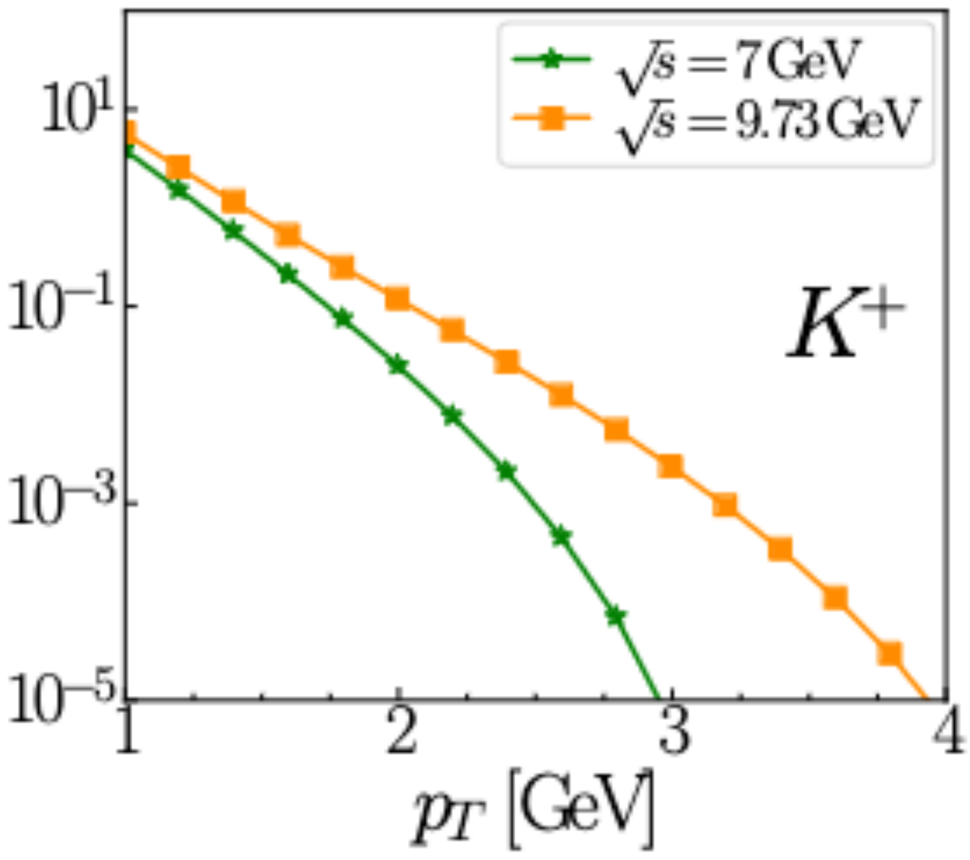
Predictions at Fixed Target Energies

- **FFs – JAM20:**





Explore uncertainties of Fragmentation Functions

❑ Parameterize the JAM20 FFs:

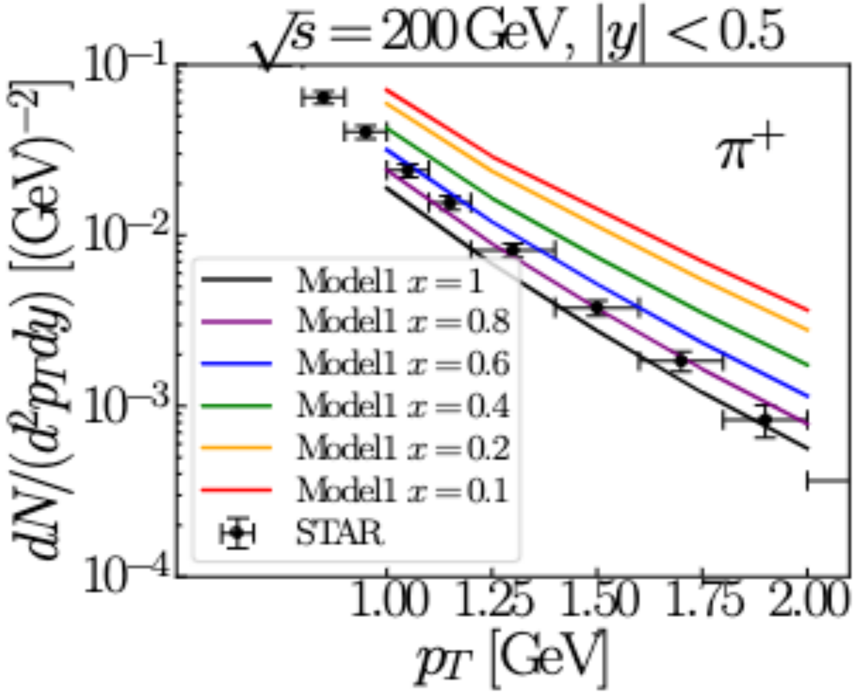
$$(\text{Model1}): zD_i^{\pi^+}(z,x) = N_i \frac{z^{\alpha_i}(1-z)^{\beta_i x}}{B[1+\alpha_i, 1+\beta_i x]},$$

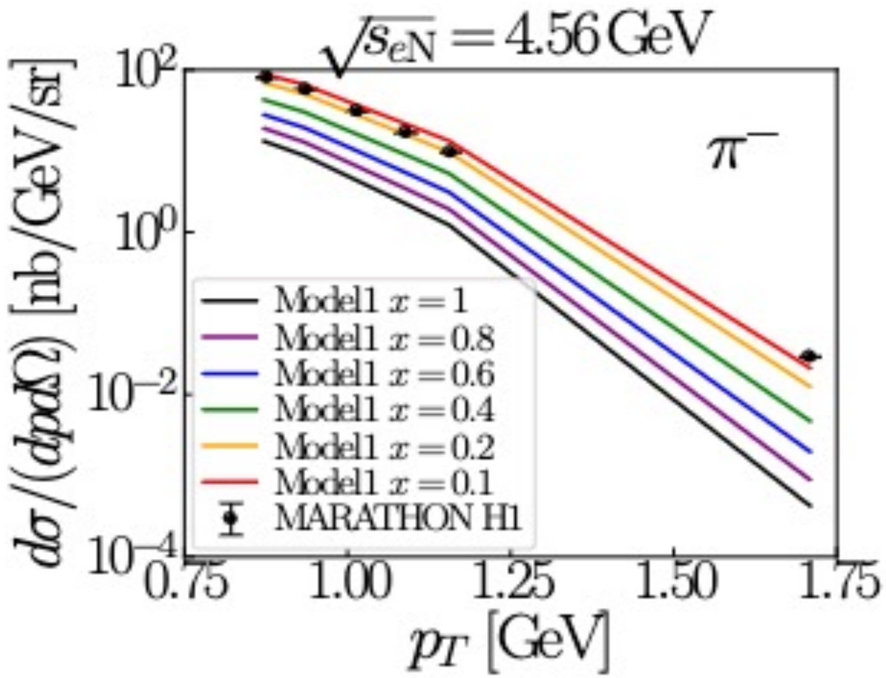
$$(\text{Model2}): zD_i^{\pi^+}(z,x) = N_i \frac{z^{\alpha_i}(1-z)^{\beta_i x}(1+c_i z^{\gamma_i})}{B[1+\alpha_i, 1+\beta_i x] + c_i B[1+\alpha_i+\gamma_i, 1+\beta_i x]}.$$

With a parameter: $x \in [0,1]$

- Smaller x = Larger contribution from large z region
- e+e- has weak constraint on large z

❑ Compare with data:





Summary and Outlook – Thank you!

- Collision induced QED radiation is an integrated part of the lepton-hadron collision
  - Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC

  Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton-hadron scattering processes

- Proposed to use the prompt single hadron inclusive production to test the fragmentation picture in particular at JLab energy – Prerequisite to fitting SIDIS data with current factorization formalism
  - Current fragmentation functions with the NLO perturbative coefficients can fit the RHIC data
  - But, have a difficulty to fit the JLab data, which could have included "non-prompt" pions (those from rho decay, …)
  - Same problem is much less important due the more steep falling spectrum of rho and other VMs.

  Need to consistently subtract the "background" from "non-prompt" source of "leading hadron" when studying SIDIS at JLab energies