TMDs early studies … @ sub-leading power
SoLID opportunity

U.S. DEPARTMENT OF ENERGY | Office of Science

Leonard Gamberg
w/ Zhongbo Kang, Ding-Yu Shao, John Terry, Fany Zhao

PennState Berks

Importance of NLP TMDs & Factorization

- Importance of NLP TMD *observables* underscored by observation that while they are suppressed by M/Q wrt *LP* observables:

  - NLP/SLP TMDs can be as sizable as leading-power TMDs in some situations where Q is not that large… not small in the kinematics of fixed-target experiments

- Their understanding is required for a complete description of "*benchmark processes*" SIDIS, DY & e^+e^-…

- Are of interest offer a mechanism to probe physics of quark-gluon-quark correlations, provide novel information about the partonic structure of hadrons, and are largely unexplored.
  - Such correlations may be considered quantum interference effects, related to average transverse forces acting on partons inside (polarized) hadrons as well as other phenomena.

- Experimental information from SIDIS on effects related to subleading TMDs is & has been available DESY/Zeus, Fermi-LAB, HERMES, COMPASS

- **Opportunity for SoLID TMD program large lumi & EIC with its large kinematical coverage will be ideal for making further groundbreaking progress in this area**

  - **NB: Iff factorization can be established beyond "tree level" & leading order**

    -Global analysis of NLP TMDs

TMD fact at NLP w& w/o polarization (incomplete list)

F. Rivindal PLB 1973
Georgi Politzer PRL 1978
Cahn PLB 1978 (response to Georgi Politzer PRL 1978)
A.Kotzinian NPB (1994)
J. Levelt, P.Mulders Phys. Rev. D(1994)
R. Tangerman, P. Mulders hep-ph/9408305 [hep-ph] (1994)
P.Mulders, R. Tangerman, NPB 461(1996)
D. Boer, P. Mulders, Phys.Rev.D 57 (1998)
L. Gamberg, D. Hwang, A Metz, M. Schlegel, PLB 639 (2006), uncanceled rapidity div. @tw3-factorization
Boer Vogelsang DY PRD 2006
Koike Nagashima Vogelsang SIDIS NPB 2006 Large P_T
A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) factorization at NLP consistency checks on matching
A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017)
I. Feige, D.W. Kolodrubetz, I. Moult, I.W. Stewart, J. High Energy Phys. 11 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 12 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 04 (2019)
Moult, I.W. Stewart, G. Vita, arXiv:1905 .07411, 201
A. Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019)
A.Vladimirov Moos, Scimemi, & S.Rodini JHEP 2022
M. Ebert A. Gao I. Stewart JHEP 06 (2022)
S. Rodini, A. Vladimirov JHEP 08 (2022)
L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209 (2022)
I.Balitsky, JHEP 03 (2023) and 2024

Also Spin transverse spin-dependence Qui Sterman collinear higher twist 1991 NLB
X. Ji, J.W. Qiu, W. Vogelsang, and F. Yuan, Phys.Rev.Lett. 97 (2006), Phys.Lett.B 638 (2006),Phys.Rev.D 73 (2006)

Challenges of *SLP/NLP* TMDs

NLP TMD observables challenging in comparison to the current state-of-the-art of leading power observables
Treatments in the literature are mostly limited to a tree-level formalism until recently

<u>First studies beyond tree level</u> : ***Bacchetta et al. JHEP 2008, Chen et al. PLB 2017*

<u>More recently results beyond *LO*</u>
A.P. Chen, J.P. Ma, PLB (2017)
Bacchetta et al. PLB 2019
MIT group, Gao, Ebert, Stewart JHEP 2022
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209
Vladimirov, Rodini, Scimemi, Moos, JHEP 2021, 2022, arXiv 2023
Balitsky 2023 rapidity only TMD evolution
See also Ch. 10 TMD handbook, e-Print:2304.03302 [hep-ph]

- ***In arXiv: e-Print:221.13209*** **present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP** ***using CSS formalism*** **which addresses disagreements in the literature**

TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

10 - Subleading TMDs

L. Gamberg, A. Metz, I. Stewart

10 Subleading TMDs . . . **308**
10.1 Introduction . . . 308
10.2 Observables for Subleading TMDs . . . 309
10.3 Subleading TMD Distribution Functions . . . 310
10.3.1 Quark-gluon-quark correlators . . . 310
10.3.2 Subleading quark-quark correlators and equations of motion . . . 314
10.4 Factorization for SIDIS with Subleading Power TMDs . . . 318
10.4.1 Status of SIDIS factorization at next-to-leading power . . . 318
10.4.2 SIDIS structure functions in terms of next-to-leading power TMDs . . . 320
10.5 Experimental Results for Subleading-Power TMD Observables . . . 324
10.6 Estimating Subleading TMDs and Related Observables . . . 326

From a historical perspective it is very interesting that the subleading-power $\cos\phi_h$ azimuthal modulation of the unpolarized SIDIS cross section was important for the development of the TMD field, since one of the earliest discussions of transverse parton momenta in DIS is related to this observable [290, 291, 1237]; see also Sec. 5.1 for more details. Generally, although suppressed by Λ/Q with respect to leading-power observables, subleading TMD observables are typically not small, especially in the kinematics of fixed-target experiments. In fact, the first-ever observed SSA in SIDIS was a sizeable power-suppressed longitudinal target SSA for pion production from the HERMES Collaboration [480]. Those measurements, which triggered many theoretical studies and preceded the first measurements of the (leading-power) Sivers and Collins SSAs, were critical for the growth of TMD-related research.

[290] R. N. Cahn, *Azimuthal Dependence in Leptoproduction: A Simple Parton Model Calculation*, *Phys. Lett. B* **78** (1978) 269.

[1237] F. Ravndal, *On the azimuthal dependence of semiinclusive, deep inelastic electroproduction cross-sections*, *Phys. Lett. B* **43** (1973) 301.

[480] HERMES collaboration, A. Airapetian et al., *Observation of a single spin azimuthal asymmetry in semiinclusive pion electro production*, *Phys. Rev. Lett.* **84** (2000) 4047

Outline

- **Heuristic disc. factorization-key approach to probe partonic structure of hadrons QCD**
- Predictability based on universality & evolution equation of factorized cross sections in terms of QCFs (e.g. TMDs GPDs) and hard cross sections
- Bench-mark processes to probe partonic 3-D momentum-spatial structure of hadrons

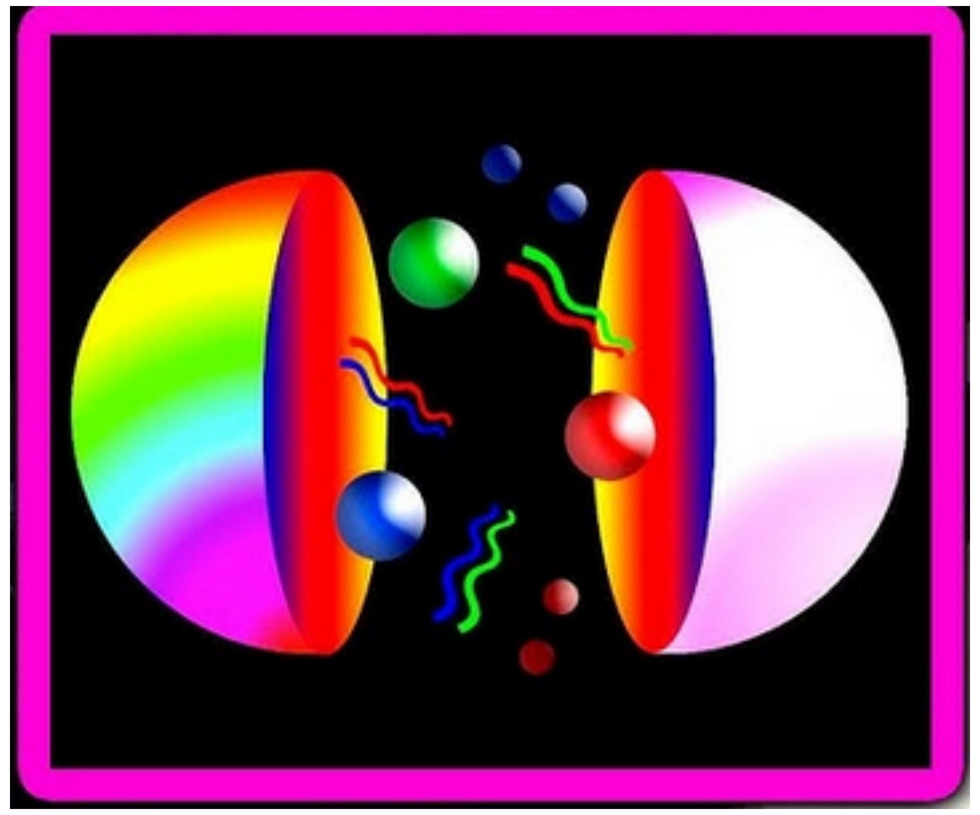
- **The beginning of TMD Physics ? "The observable"** $\langle \cos\phi \rangle$
- **Georgi & Cahn, PRL 1978, PLB 1978 (Ravndal, PLB 1972) & Feynman PR 1978**
  Critique of the perturbative QCD calculation of azimuthal dependence in leptoproduction emphasize importance intrinsic $\boldsymbol{k_T}$ the early days/birth of TMD physics

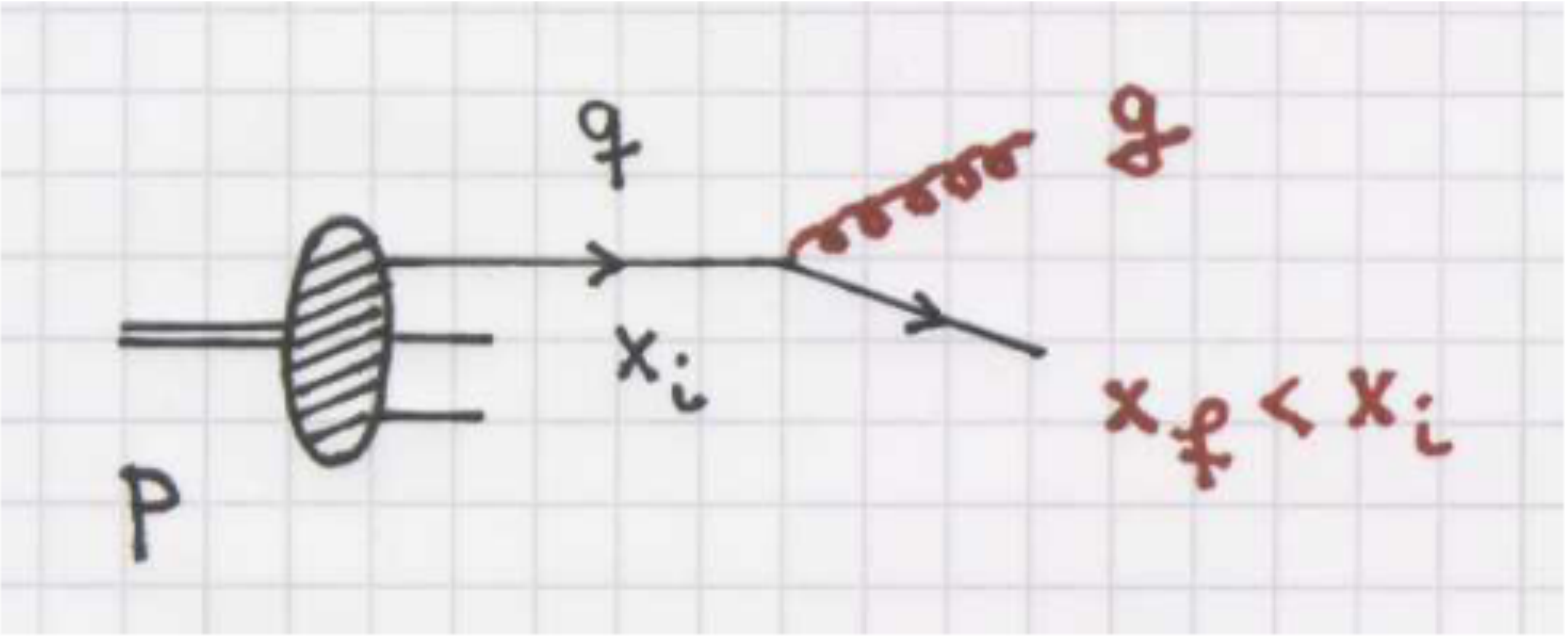
"Led to /Leads to…"

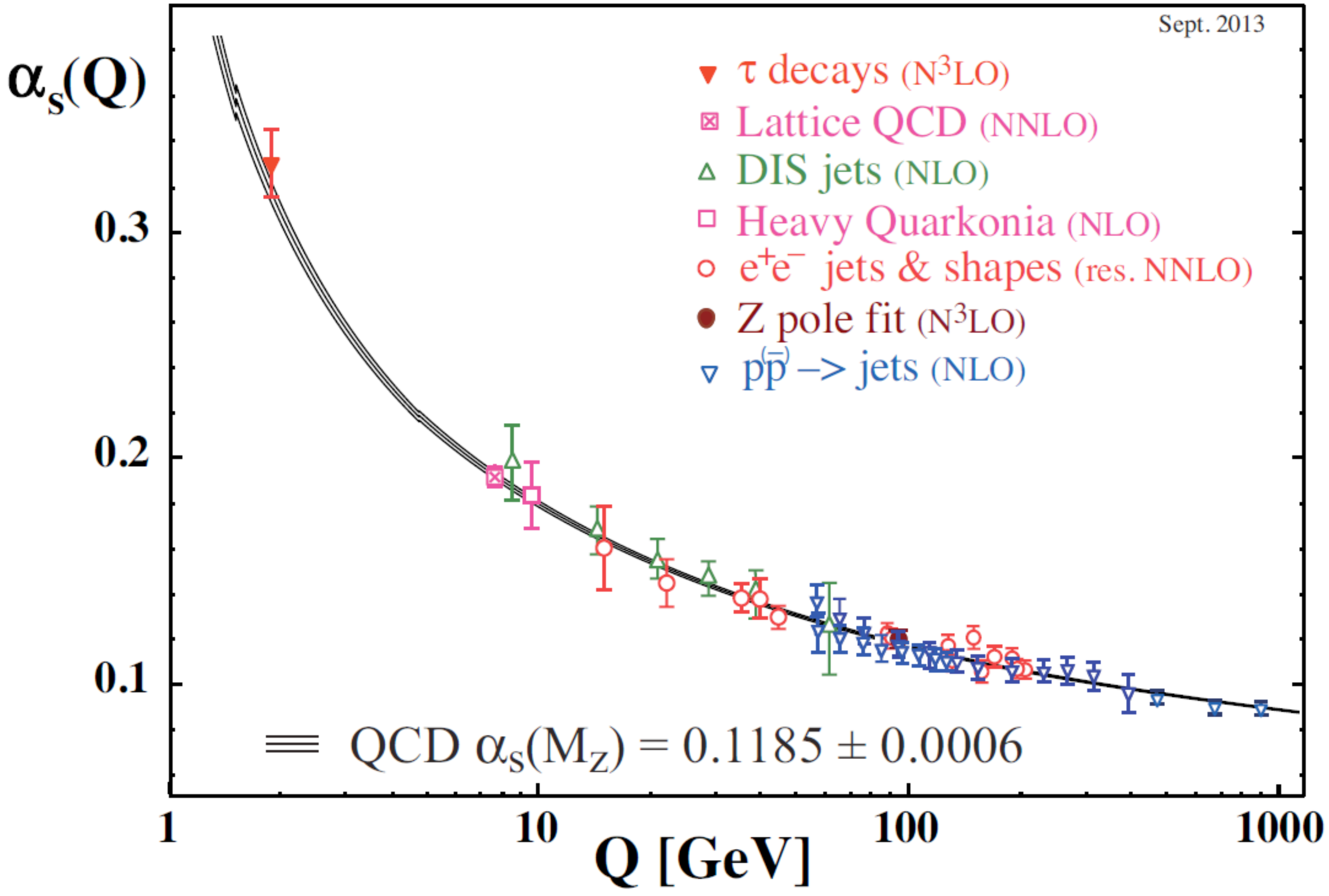
1. The challenge of mapping "low" to "high" transverse momentum spectrum q_T or P_{hT}
2. **Factorization BUT(!) @ NLP order α_s … issues … necessary (but not sufficient) consistency checks**
3. "Ongoing work"

Intro Comments Factorization

- QCD predicts hadrons are *dynamical system* of quarks & gluons governed by predictions of the running "QCD" coupling displaying **asymptotic freedom** of interactions at *short distance*, and **confinement** at *long distance* **scales**

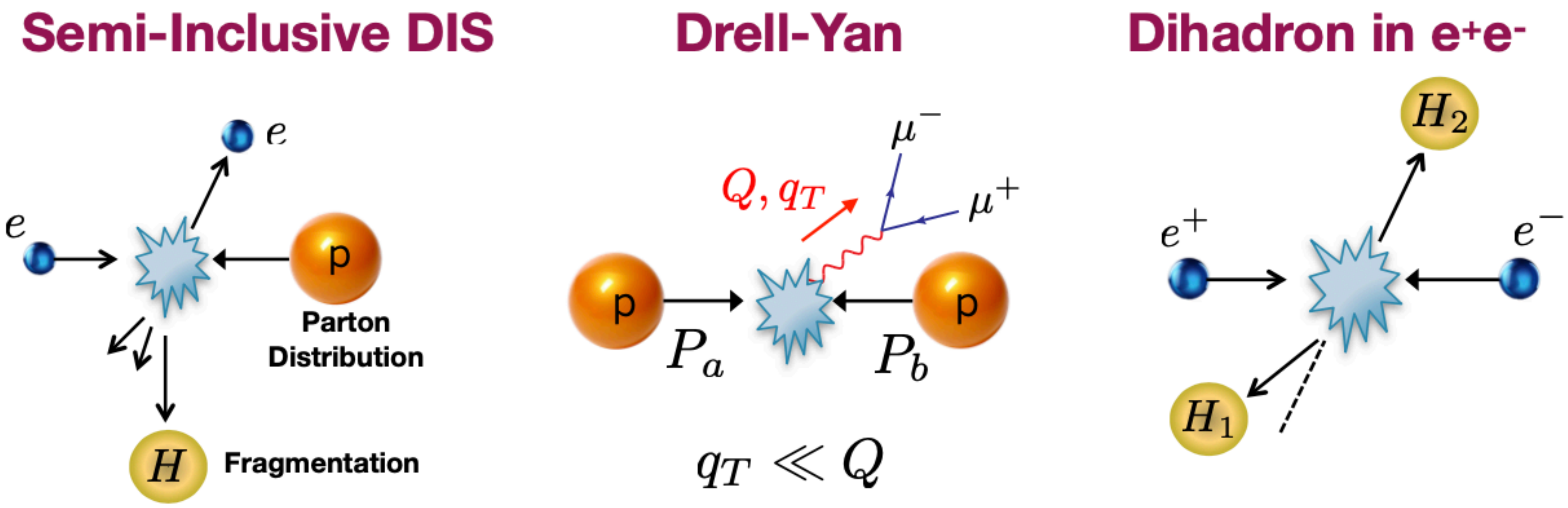




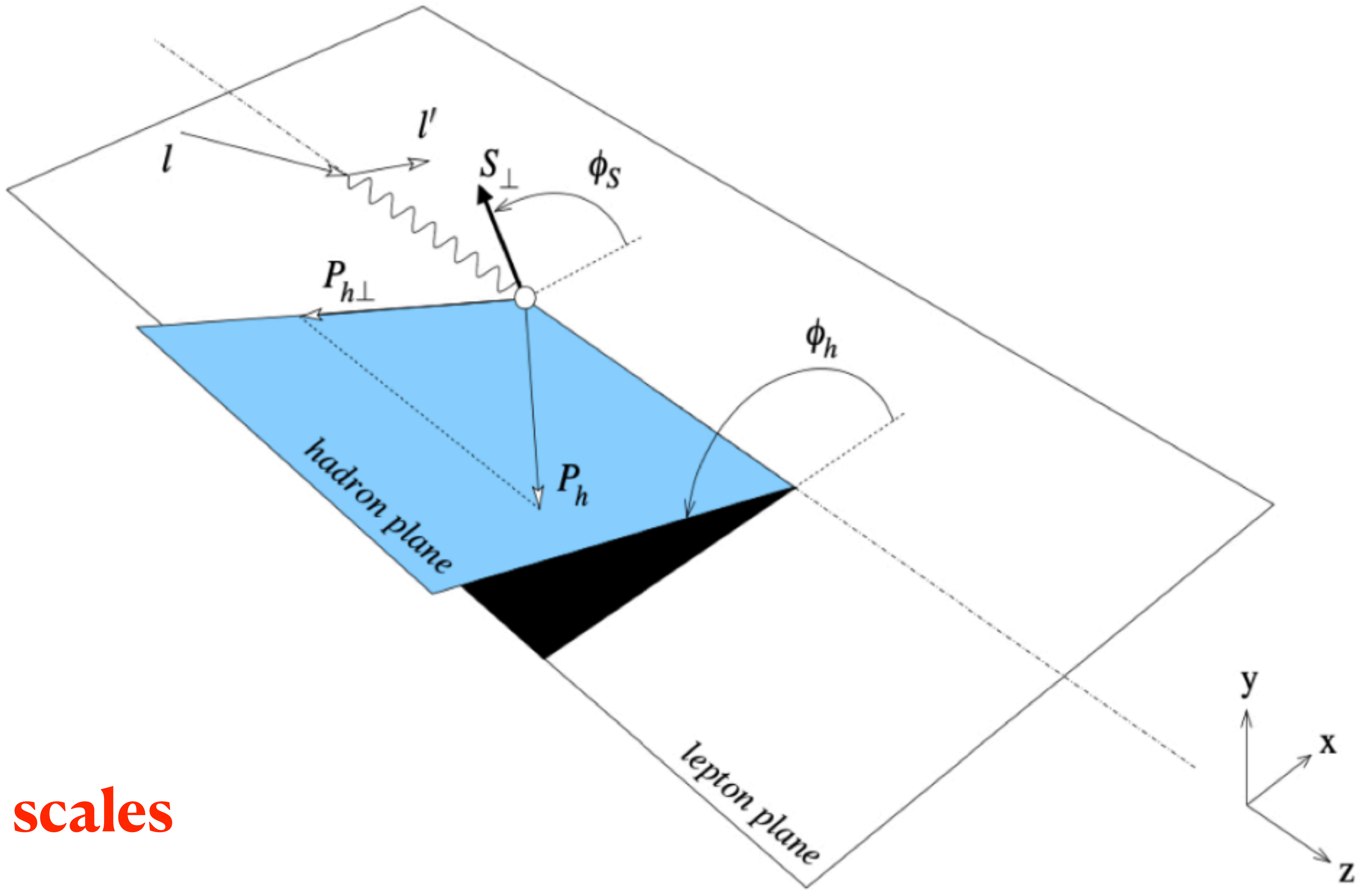


Intro Comments Factorization

- The delicate interplay of **confinement** coexisting **asymptotic freedom** allows link quarks & gluons @ short time and distance scales to hadrons measured in high energy deep inelastic scattering experiments.
- **Asymptotic freedom**, makes it possible to use the formalism of **QCD factorization** to quantify the partonic structure & dynamics of hadrons in terms of quantum field theoretic (universal) **parton correlation functions called "benchmark processes"**



Factorization and scales

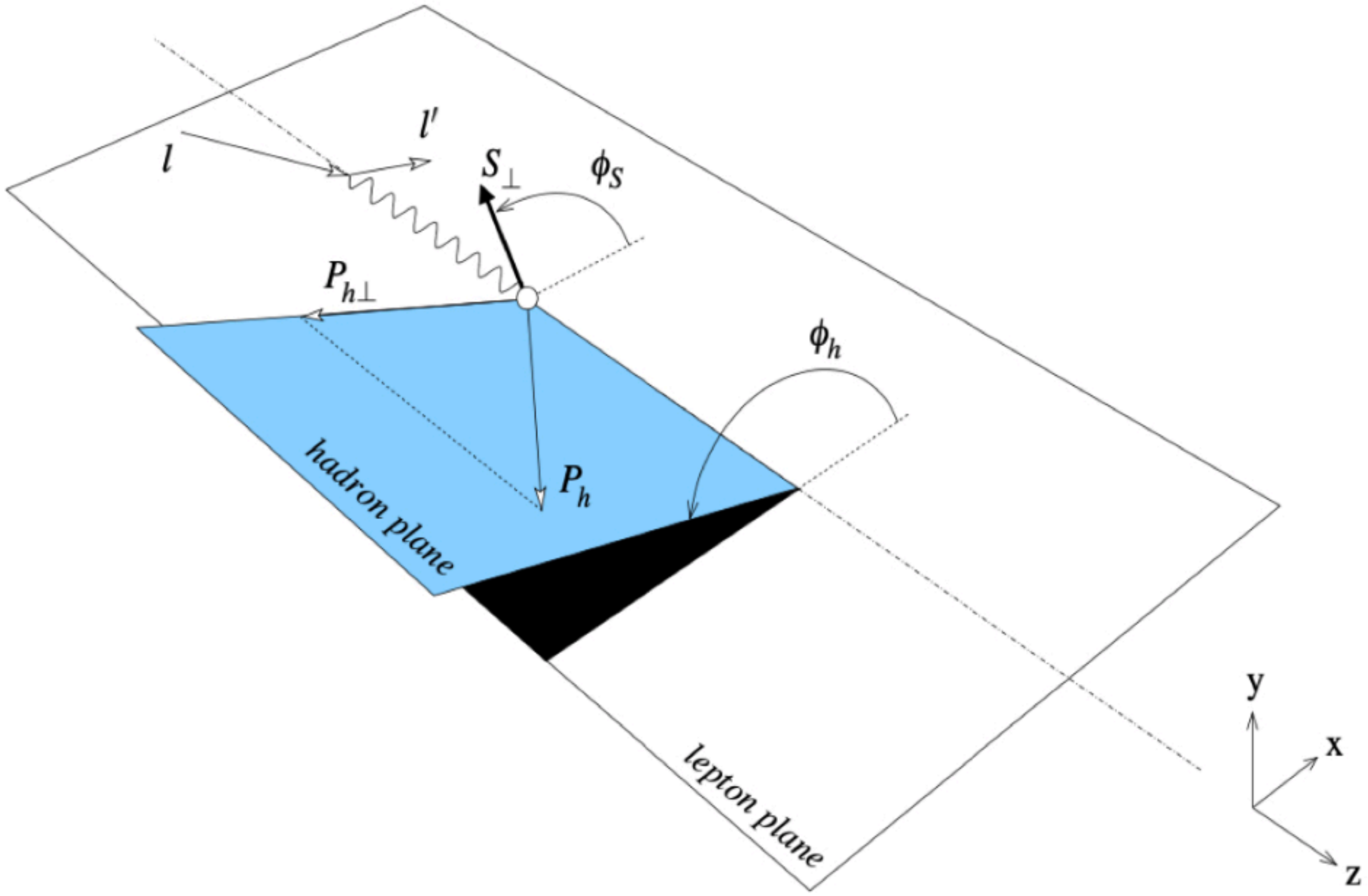


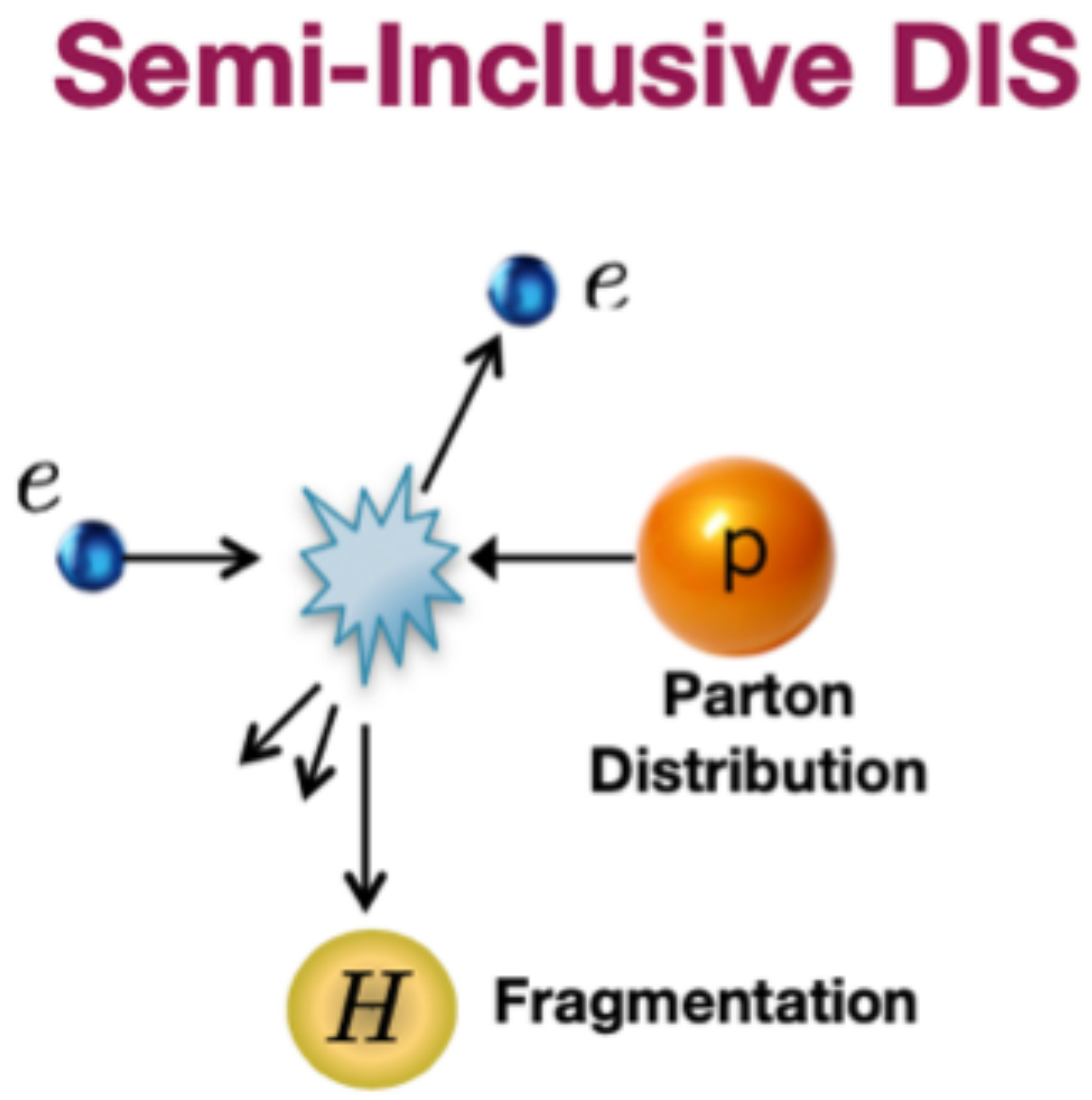
"TMD" physics problem characterized in terms of the 3 scales namely:

- the scale of nonperturbative QCD dynamics, which we represent by the nucleon mass $M \sim \Lambda_{QCD}$
- the transverse momentum $P_{h\perp}$ of the produced hadron,
- the hard scale of photon/probe Q, which we require to be large compared with M

Intro Comments

- There are two basic descriptions for the production of particle with specified transverse momentum q_T or $P_{h\perp}$ $(or\ P_{hT}\ )$

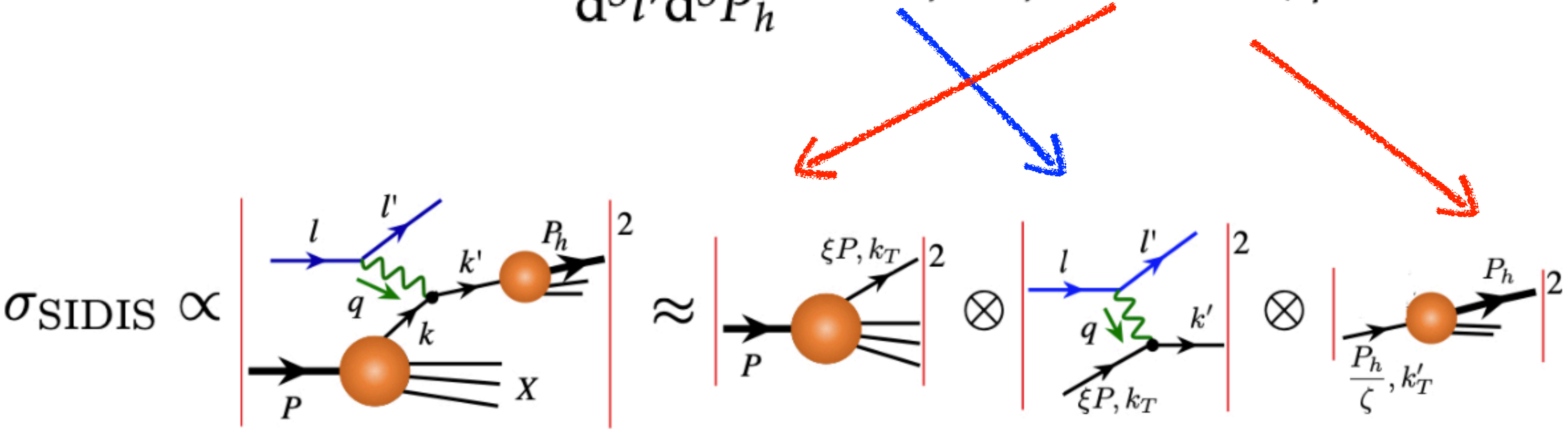


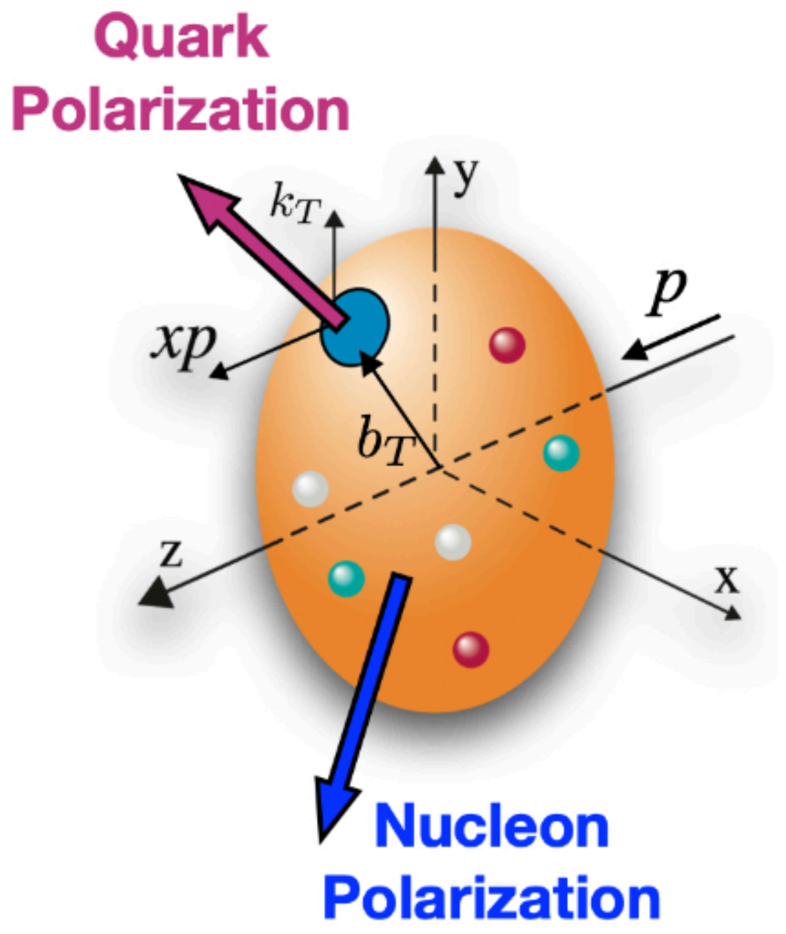


TMD Framework

- One framework is applicable when $\Lambda_{QCD} \sim P_{h\perp} \ll Q$ (hard scale)
- QCD theory predicts that $P_{h\perp} \sim \mathbf{k}_T$ or $\mathbf{p}_T$ (intrinsic transverse momentum partons in hadrons), the non-perturbative structure is given by transverse momentum dependent (TMD) parton distribution functions (PDFs) and or fragmentation functions (FFs),
  while the perturbative hard scattering cross sections probe the short distance dynamics of partons

$$E'E_h \frac{d\sigma_{ep\to e'hX}}{d^3 l' d^3 P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \,\widetilde{\otimes}\, D_{h/q'}.$$



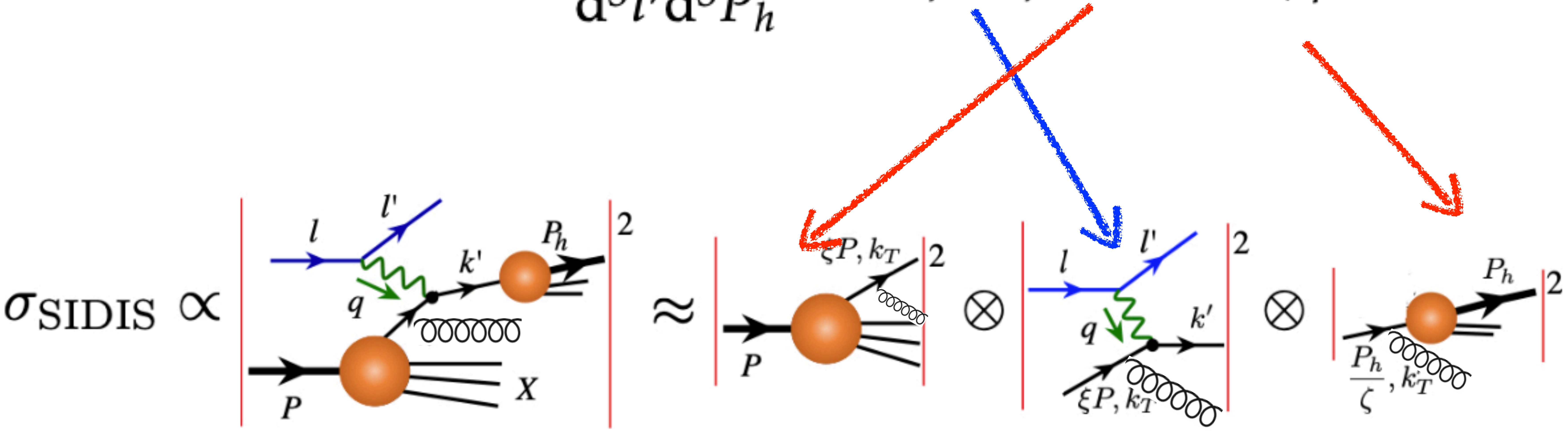


ıre 1.1: Illustration of the mo-
ıtum and spin variables probed
ΓMD parton distributions.

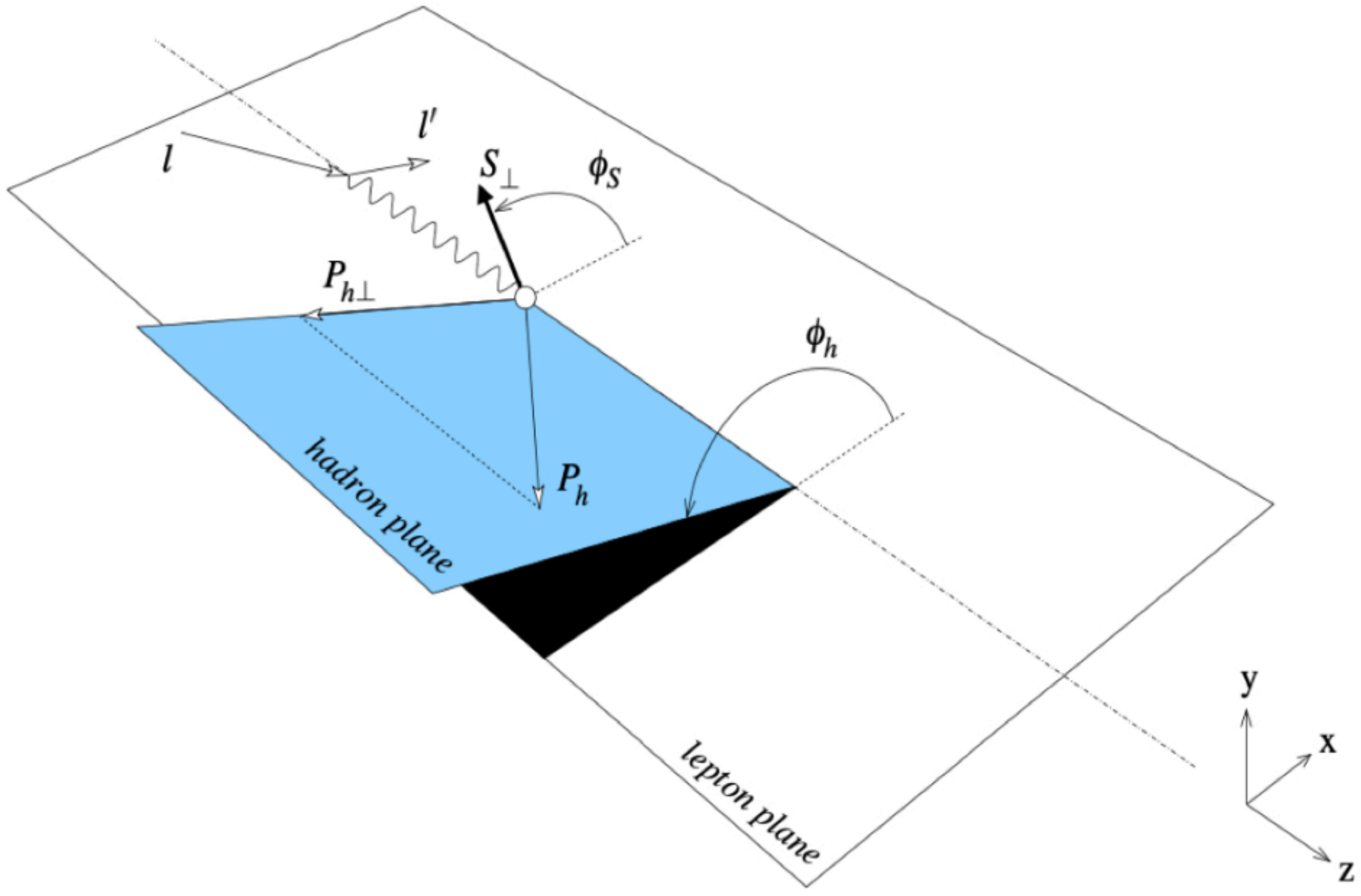
Collinear Framework

- Another framework is applicable when $P_{h\perp} \sim Q \gg \Lambda_{QCD}$
- QCD theory predicts that $P_{h\perp} \gg \mathbf{k}_T$ or $\mathbf{p}_T$ (generates transverse momentum in the final state as perturbative radiation where the non-perturbative structure is given by collinear (integrated) parton distribution functions (PDFs) and or fragmentation functions (FFs)

$$E'E_h \frac{d\sigma_{ep\to e'hX}}{d^3l'd^3P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \tilde{\otimes} D_{h/q'}.$$



Factorization and angular distributions



A number of nontrivial issues for factorization arise when one observes the transverse momentum $P_{h\perp}$ and the angular distribution of the produced particle with respect to a suitable reference direction
Will see in context of "LP vs. NLP" factorization

Goes back to $\sim$ 1978

TMDs @ "twist-3 " NLP-the beginning?

Historical-context

- **Georgi Politzer, PRL 1978**
  Performed QCD analysis of hard gluon radiation in SIDIS to predict absolute value of final state hadron's $\boldsymbol{P_T}$, and the angular distribution relative to lepton scattering plane $\langle \cos \phi \rangle$
- **~12-15% …clean test of QCD since such effects would not arise as a result of limited transverse momentum associated with confined quarks**
- **"Measurement of $\langle \cos \phi \rangle$ provide very clean test of the perturbative predictions of QCD"**

- **Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972)**
  Critique of the QCD calculation of azimuthal dependence in leptoproduction; emphasize importance intrinsic $\boldsymbol{k_T}$ …
- **"We conclude that the azimuthal dependence in vector exchange interactions is inevitable since the partons have transverse momentum as a consequence of being confined and such dependence certainly does not require a special mechanism like gluon bremstrahlung"**
- **"…Results (of Cahn78) cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics " (i.e. of G&P78)**

The observable $\langle \cos\phi \rangle$

No assumption of mechanism

$$\frac{d\sigma}{dx_H\, dy\, dz_H\, d^2P_T} := \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

$$\int d\sigma^{(1)} \cos\phi = \int d^2P_T \cos\phi \frac{d\sigma}{dx_H\, dy\, dz_H\, d^2P_T}$$

SIDIS Kinematics dictionary

$$Q^2 = -q^2, \quad \mathbf{P}_T = \mathbf{P}_{2T}, \quad \phi,$$

$$x_H = \frac{Q^2}{2P_1\cdot q}, \quad y = \frac{P_1\cdot q}{P_1\cdot k_1}, \quad z_H = \frac{P_1\cdot P_2}{P_1\cdot q},$$

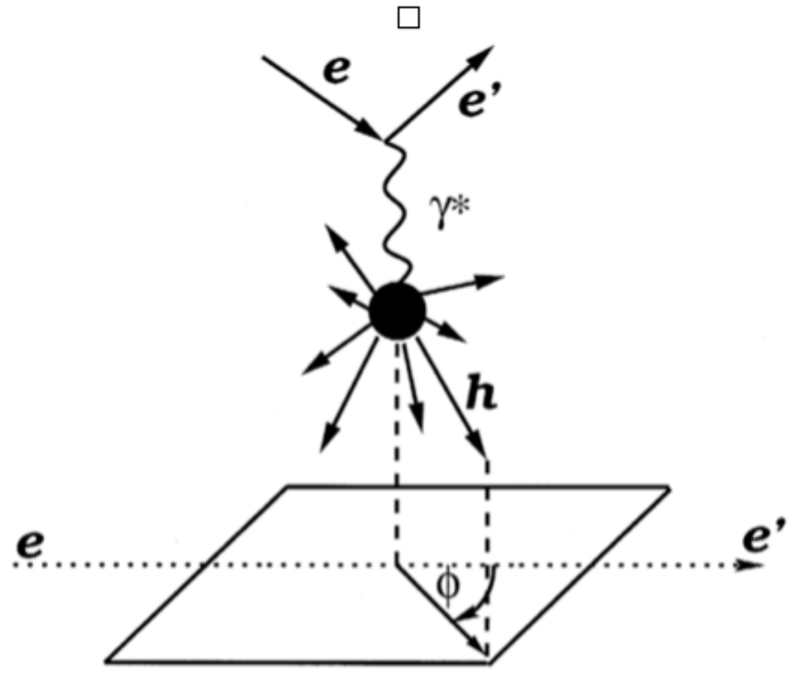
and the parton variables

$$x = \frac{x_H}{\xi} = \frac{Q^2}{2p_1\cdot q}, \quad z = \frac{z_H}{\xi'} = \frac{p_1\cdot p_2}{p_1\cdot q}.$$

Clean tests of QCD ?

PHYSICAL REVIEW LETTERS

VOLUME 40 | 2 JANUARY 1978 | NUMBER 1



Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

H. David Politzer
California Institute of Technology, Pasadena, California 91125
(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

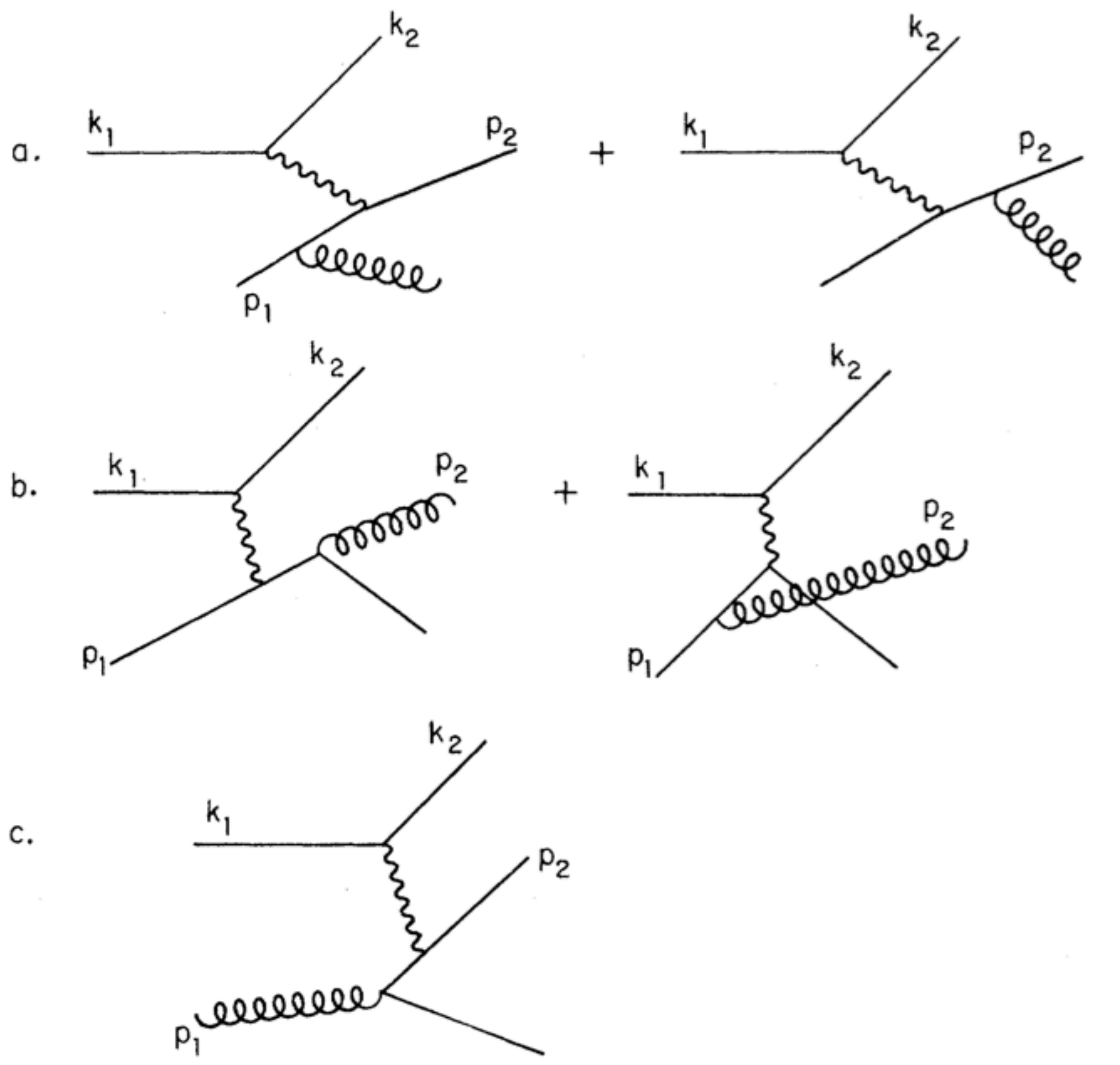


FIG. 1. Diagrams contributing to semi-inclusive μ-parton scattering to first order in α_s. k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Pert. QCD

$$\langle \cos\varphi \rangle_{ep} = -\frac{\alpha_s}{2}\kappa\sqrt{1-z}\,\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

$$\alpha_s = g^2/4\pi$$

Cahn intrinsic k_T

Volume 78B, number 2,3 PHYSICS LETTERS 25 September 1978

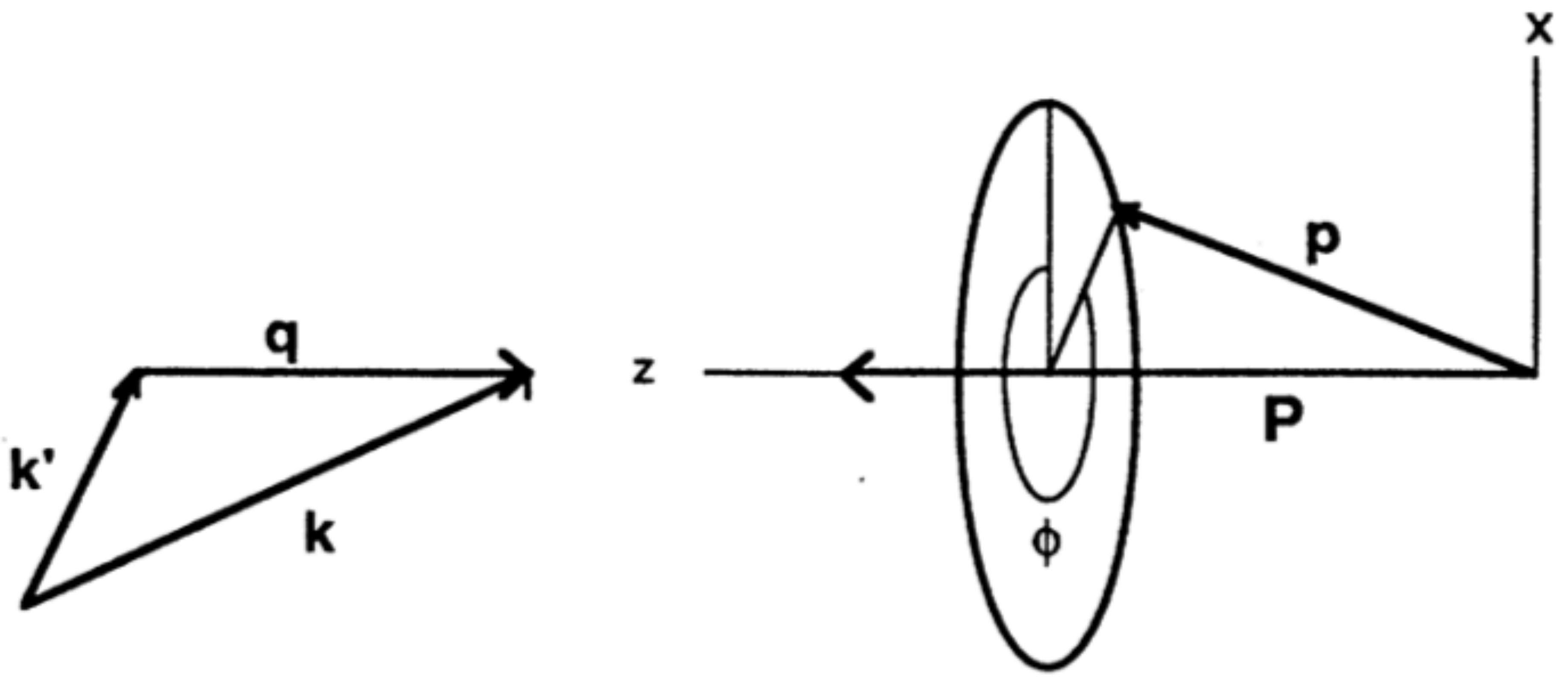
AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION☆

Robert N. CAHN
Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

parton model argument allowing for transverse momentum in Mandelstam variables...

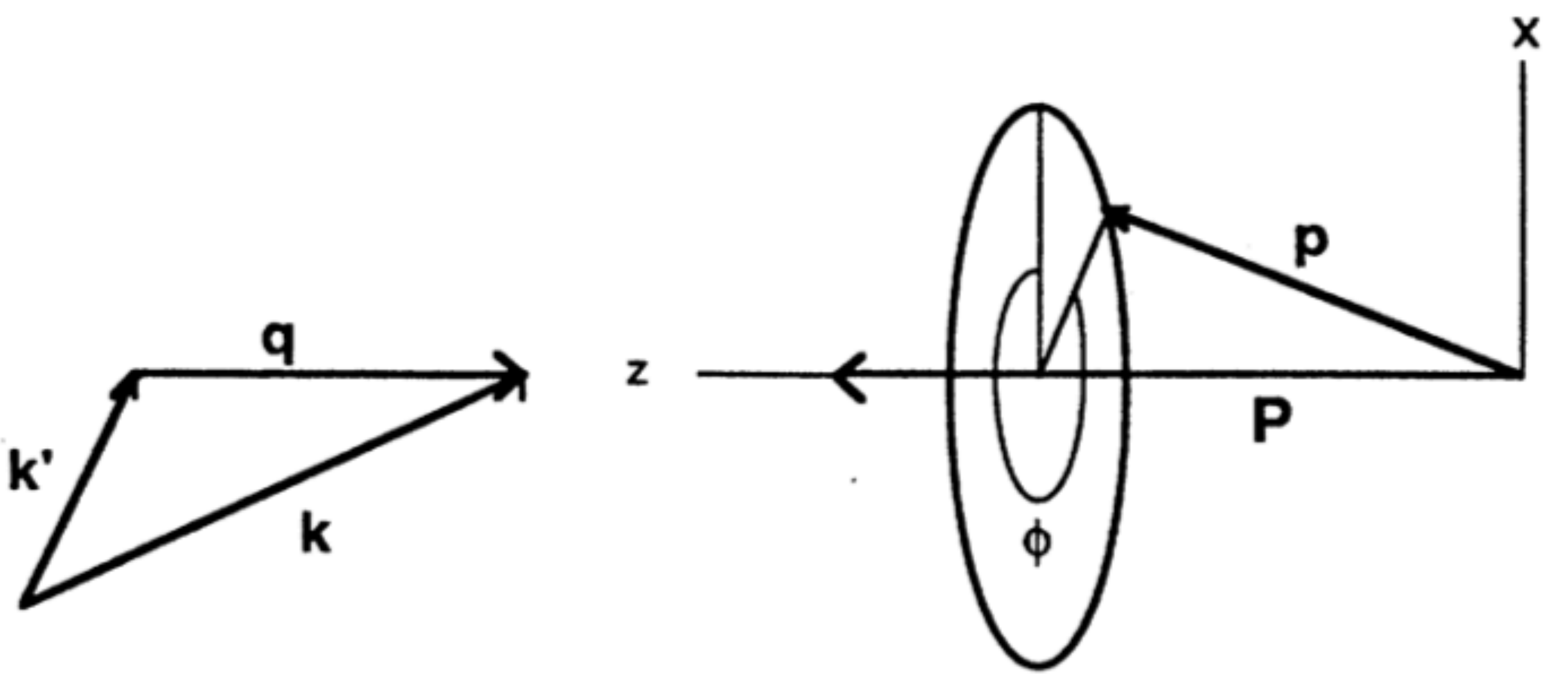
Semi-inclusive leptoproduction, $\ell + p \to \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, νp and $\bar{\nu}$p scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.



NLP !

$$\langle \cos\phi \rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Cahn intrinsic k_T



Simple parton model argument allowing for transverse momentum Mandelstam variables...

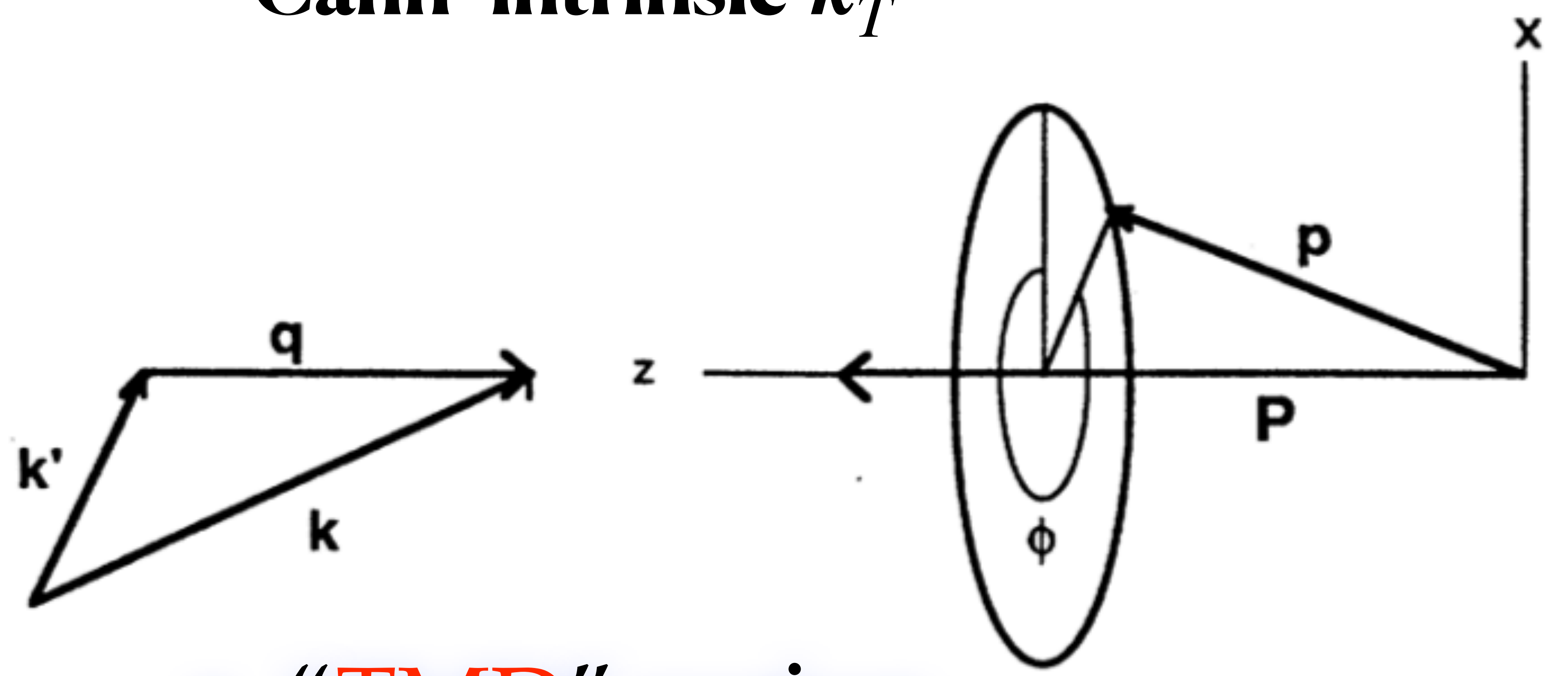
$$\sigma_{ep} \propto \hat{s}^2 + \hat{u}^2 \propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right]^2 + (1-y)^2\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2$$

NLP !

$$\langle \cos\phi \rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Two mechanisms ? Collinear Factorization

Cahn intrinsic k_T

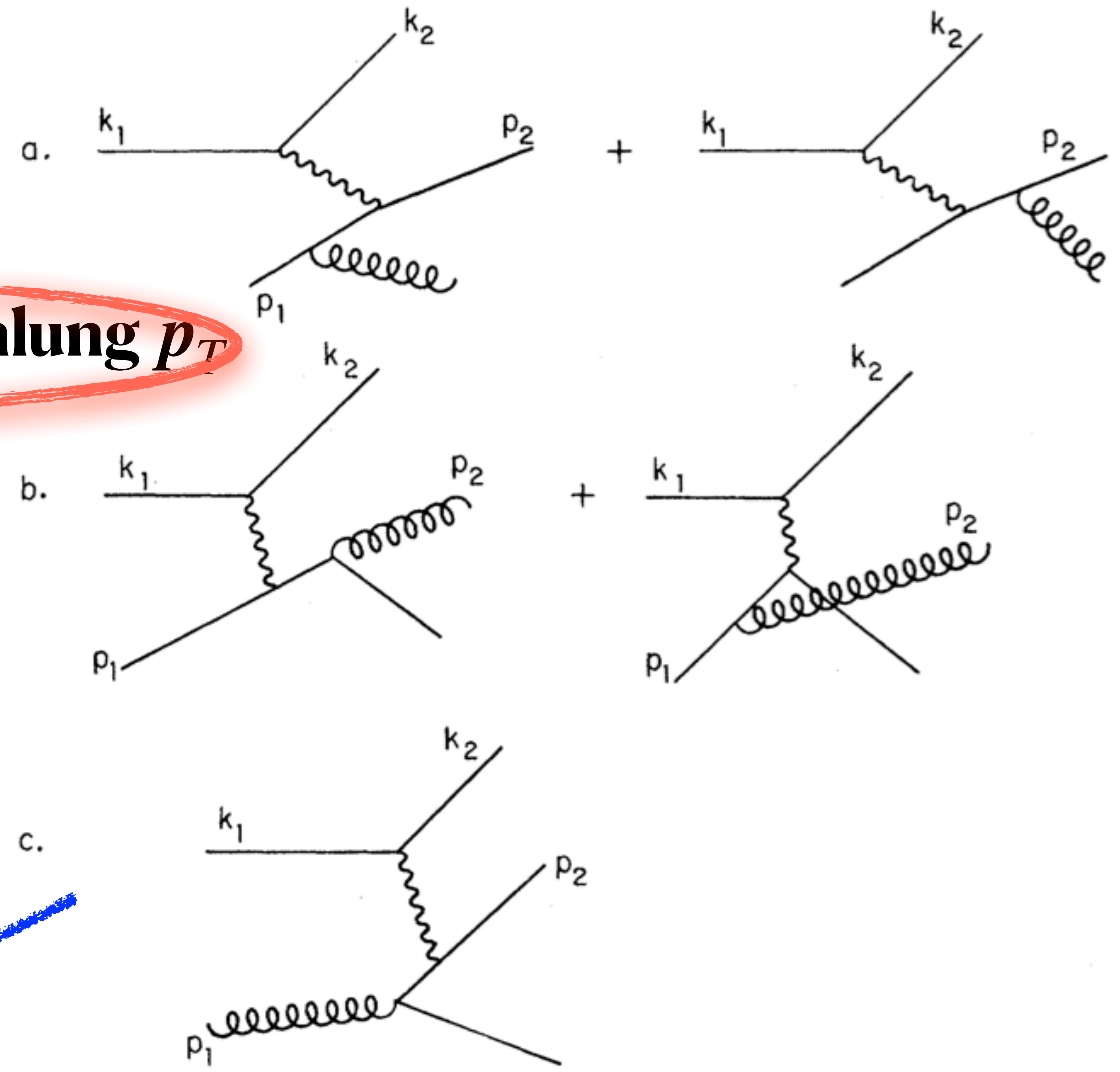


- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$

Georgi & Politzer

hard gluon bremsstrahlung p_T



$$\frac{d^5\sigma}{dx_{bj}\, dQ^2\, dz_f\, dq_T^2\, d\phi} = \frac{\alpha_e^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \int_{x_{\min}}^{1} \frac{dx}{x} \int_{z_f}^{1} \frac{dz}{z} [f \otimes D \otimes \hat{\sigma}_k \times \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right)\right)$$

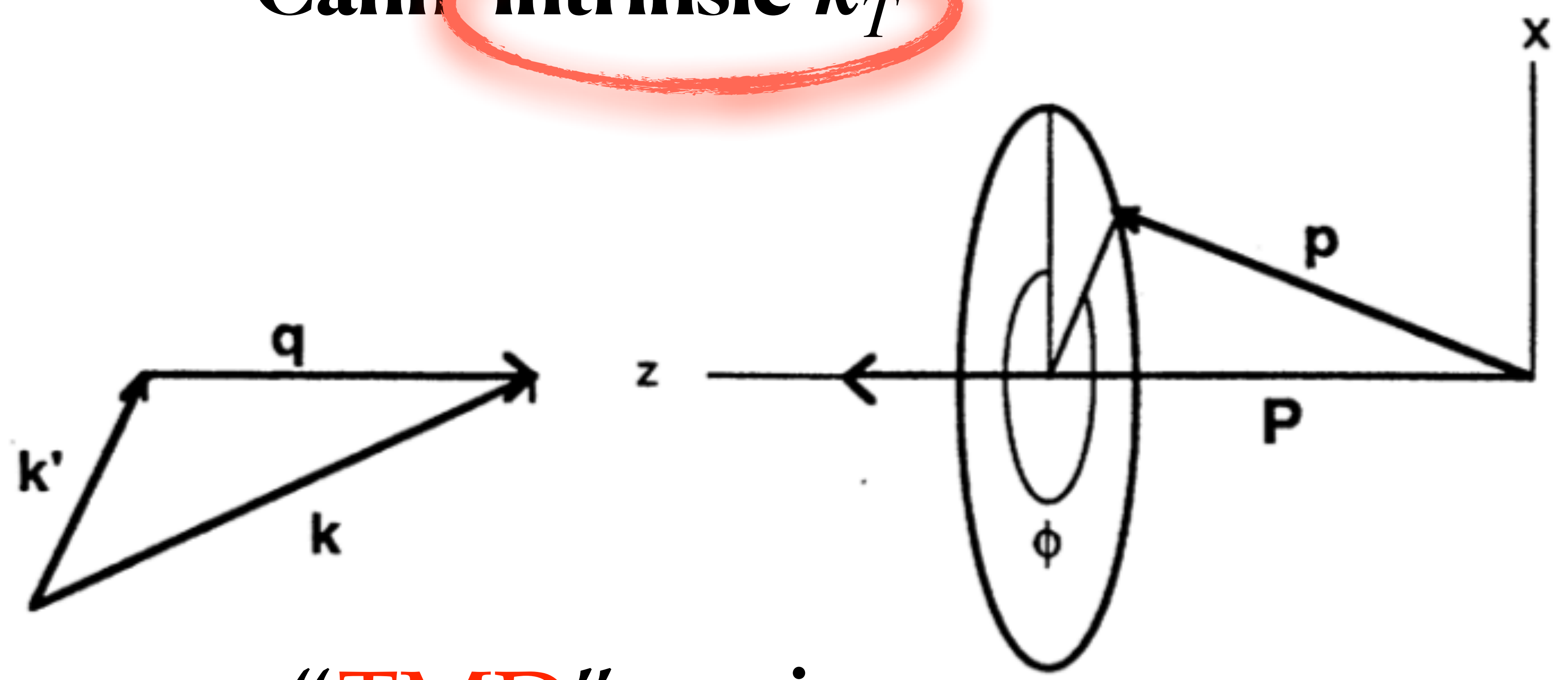
- "Collinear " region

$\Lambda_{qcd} \ll q_T \sim Q$

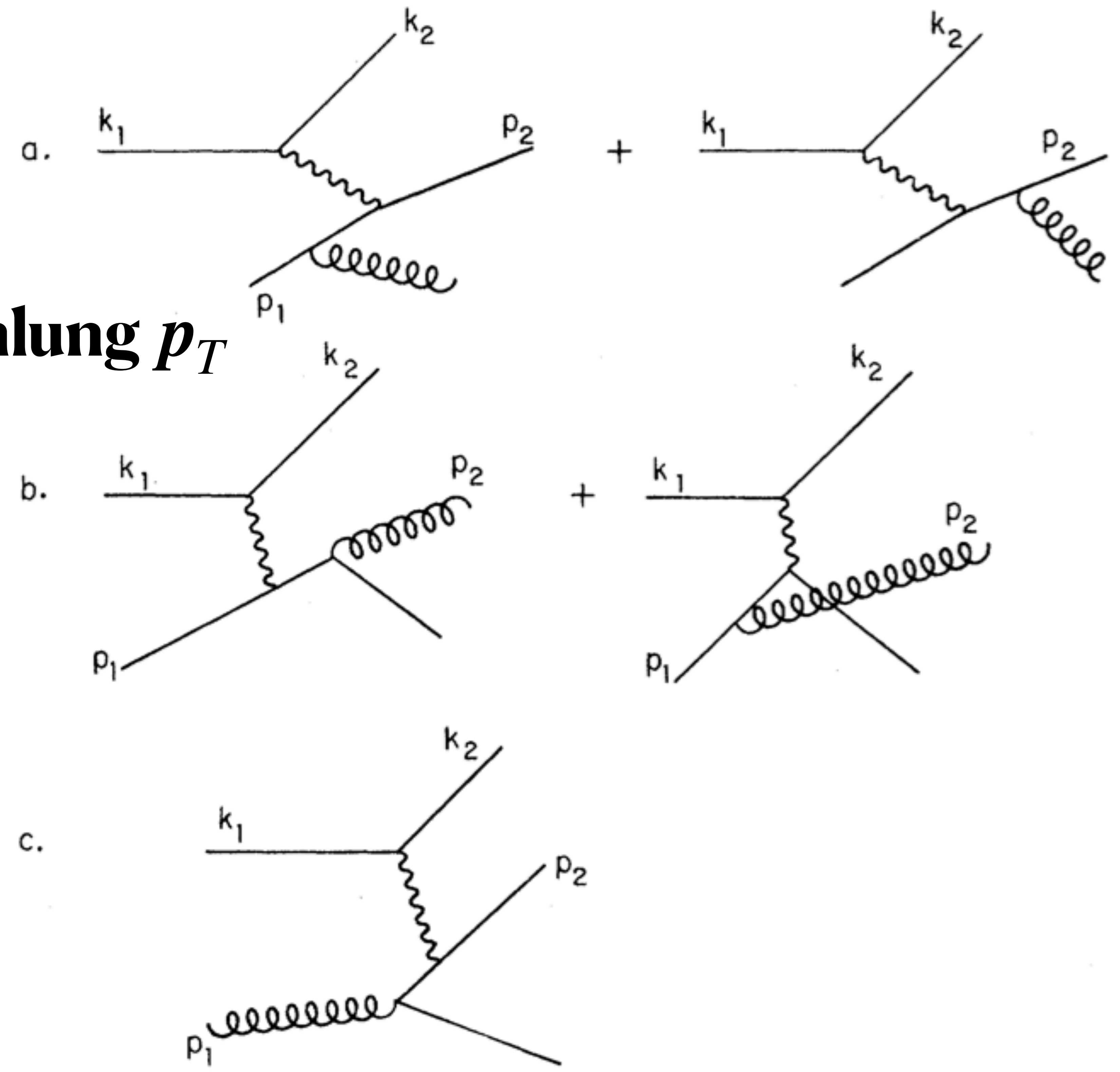
See e.g. Mendez NPB 1978, Koike, Vogelsang, Nagashima NPB 2006

Two mechanisms ? TMD Factorization

Cahn intrinsic k_T



Georgi & Politzer
hard gluon bremsstrahlung p_T



- "TMD" region

$$(p_T \sim k_T) \sim q_T \ll Q$$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

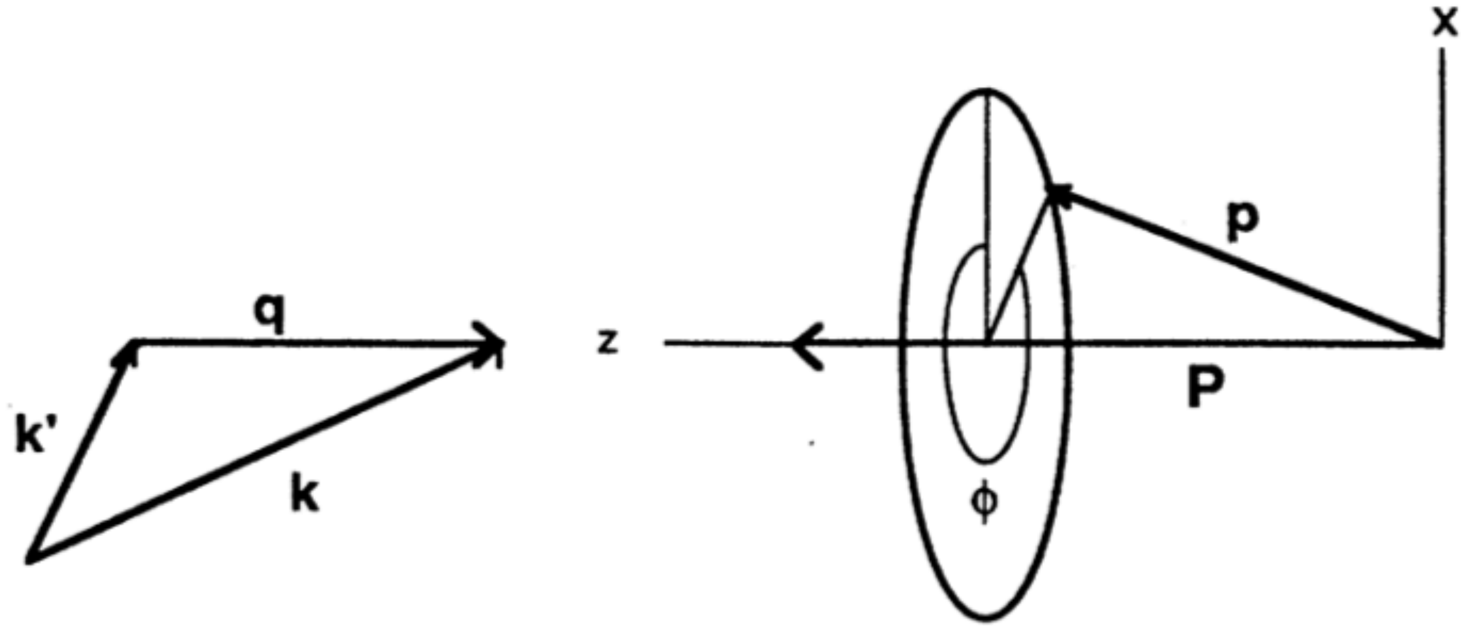
$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\left\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \quad \ldots \right\}$$

e.g. $F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_1 D_1\right]$

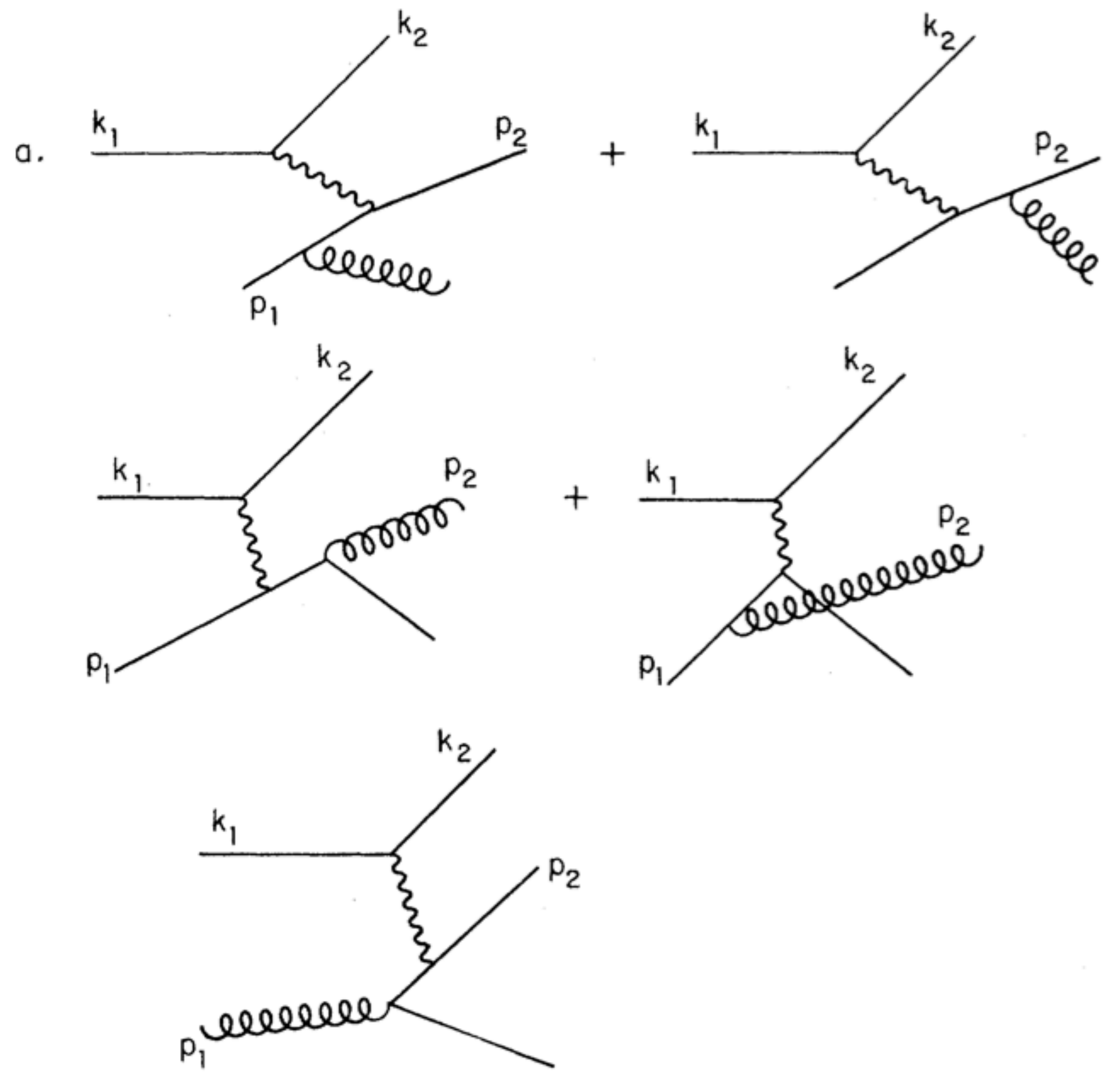
P.Mulders, R. Tangerman, NPB (1996), Bacchetta et al. JHEP 2007

Two mechanisms ? Matching ...

Cahn intrinsic k_T

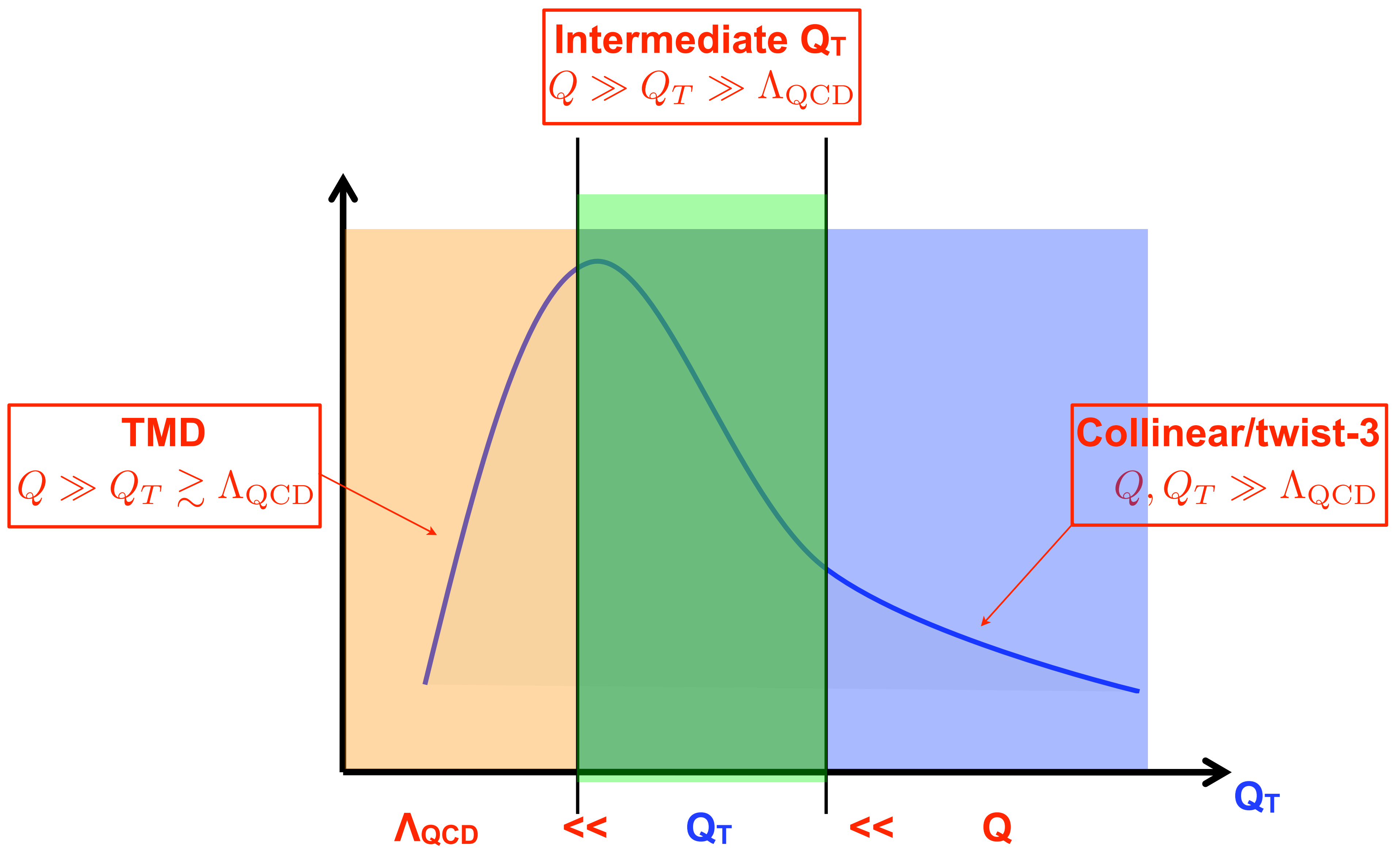


Georgi & Politzer
hard gluon bremsstrahlung p_T



- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$



- "Collinear " region

$\Lambda_{qcd} \ll q_T \sim Q$

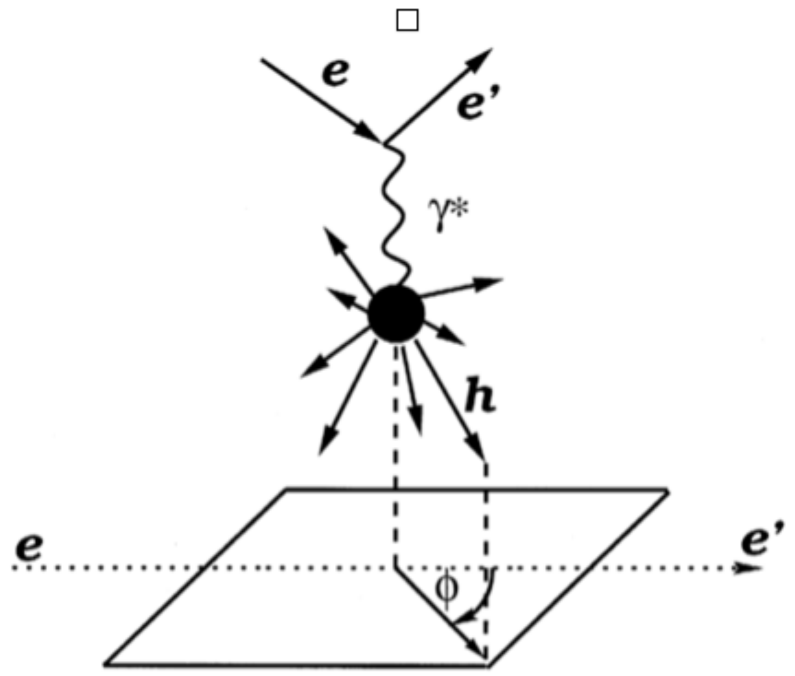
A comprehensive study of matching the hi & low Q_T in the overlap region in SIDIS was carried out by JHEP (2008) Bacchetta et al. where attention was given to azimuthal and polarization dependence

DATA

$(p_T \sim k_T) \sim q_T \ll Q$

"TMD region??"

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$



$\Lambda_{qcd} \ll q_T \sim Q$

"Collinear region??"

EMC collaboration Phys. Lett. B 130 (1983) 118, & Z. Phys. C 34 (1987) 277

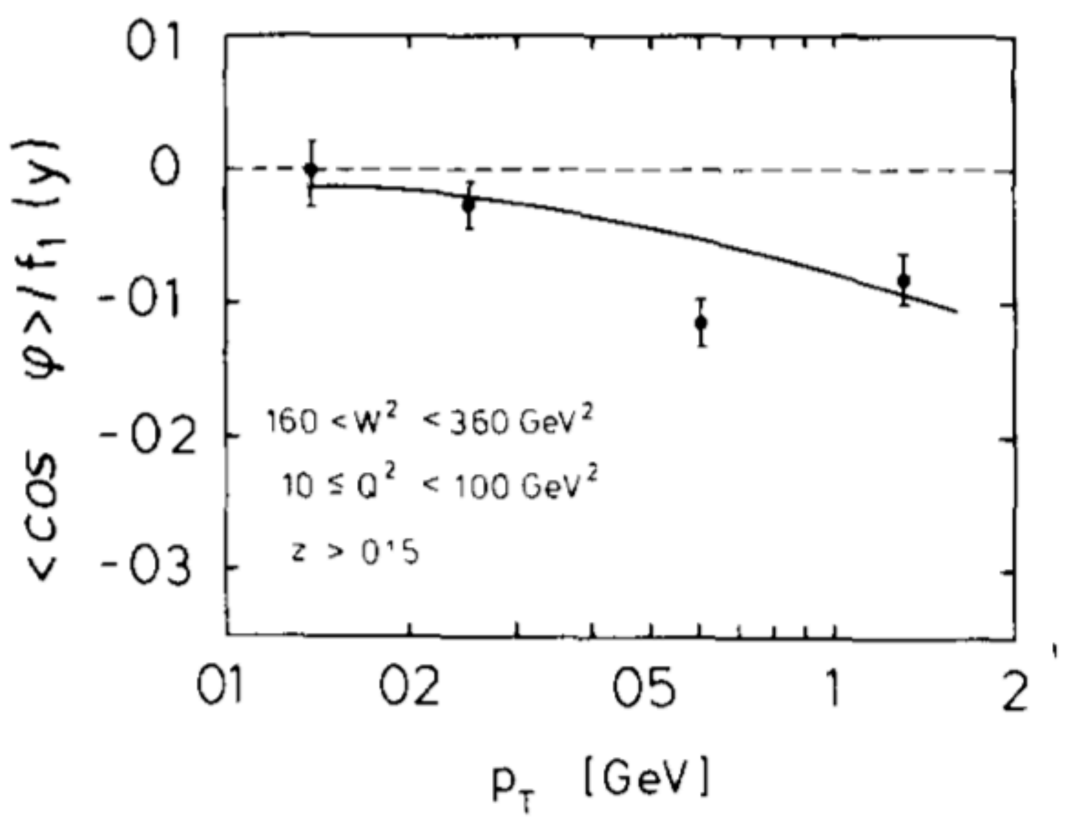
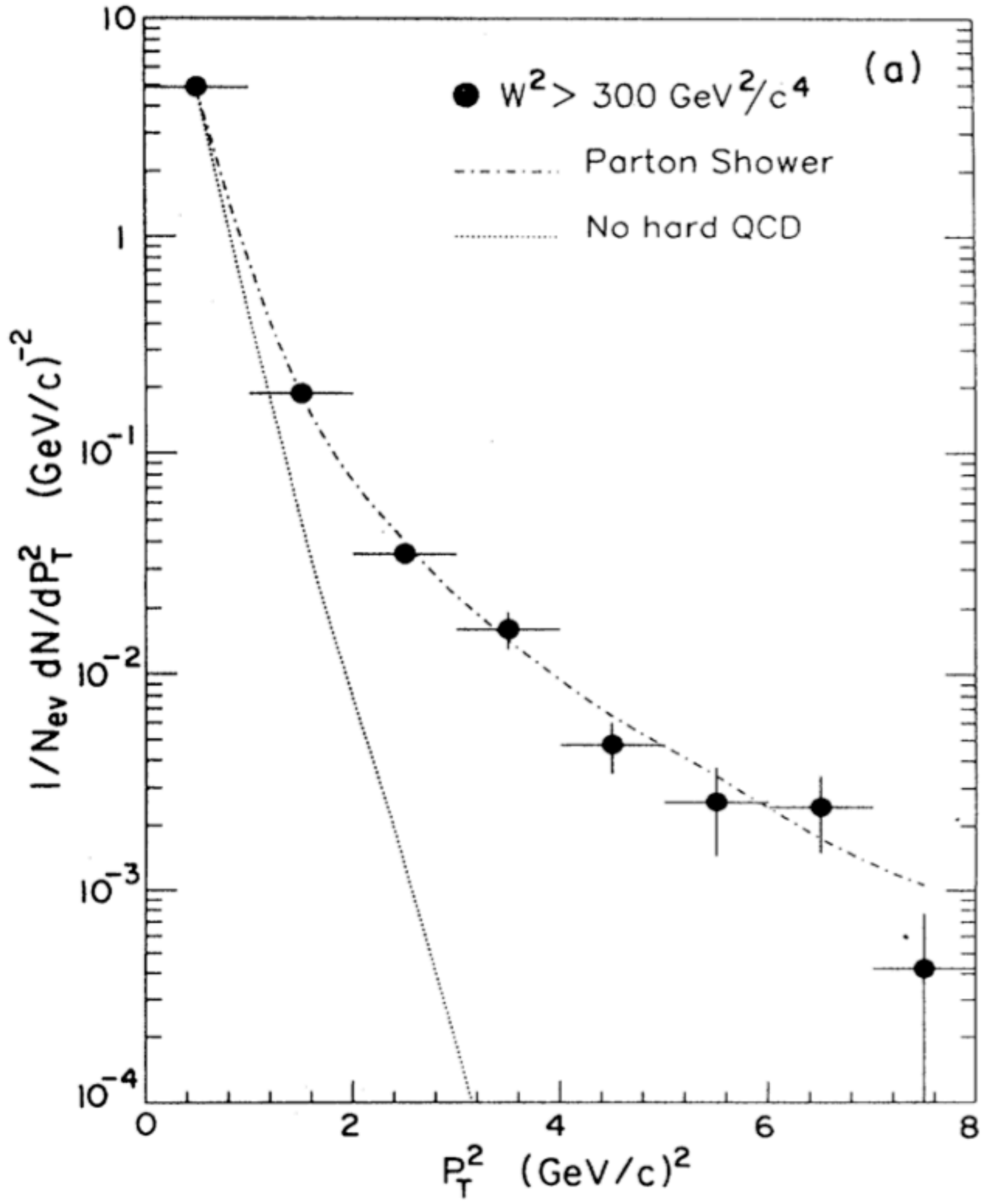


Fig 4 p_T dependence ($p_T > 50$ MeV) of cos φ moment for $160 \leqslant W^2 < 360$ GeV^2, $Q^2 > 10$ GeV^2 and $z > 0.15$ compared with model calculations described in ref [8] (statistical errors on model curve from Monte Carlo ±0 03 not shown)

In conclusion a finite ⟨cos φ⟩ has been observed in deep inelastic muon scattering. The sign of the effect is negative and shows little Q^2 or W^2 dependence. There is a significant increase of the asymmetry as a function of z and p_T. The general trend of the data is reproduced by a model containing a large effective intrinsic momentum. A contribution from leading order QCD cannot be excluded but is at present not required by the data.

Non-pert.

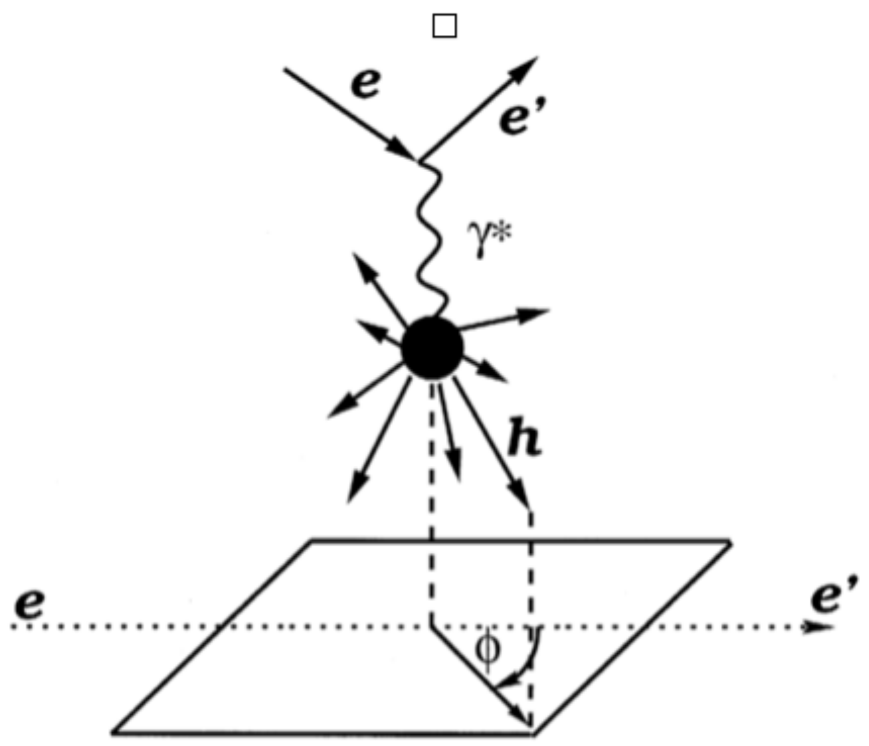
E665 Phys. Rev. D 48 (1993) 5057



Pert.?

DATA

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

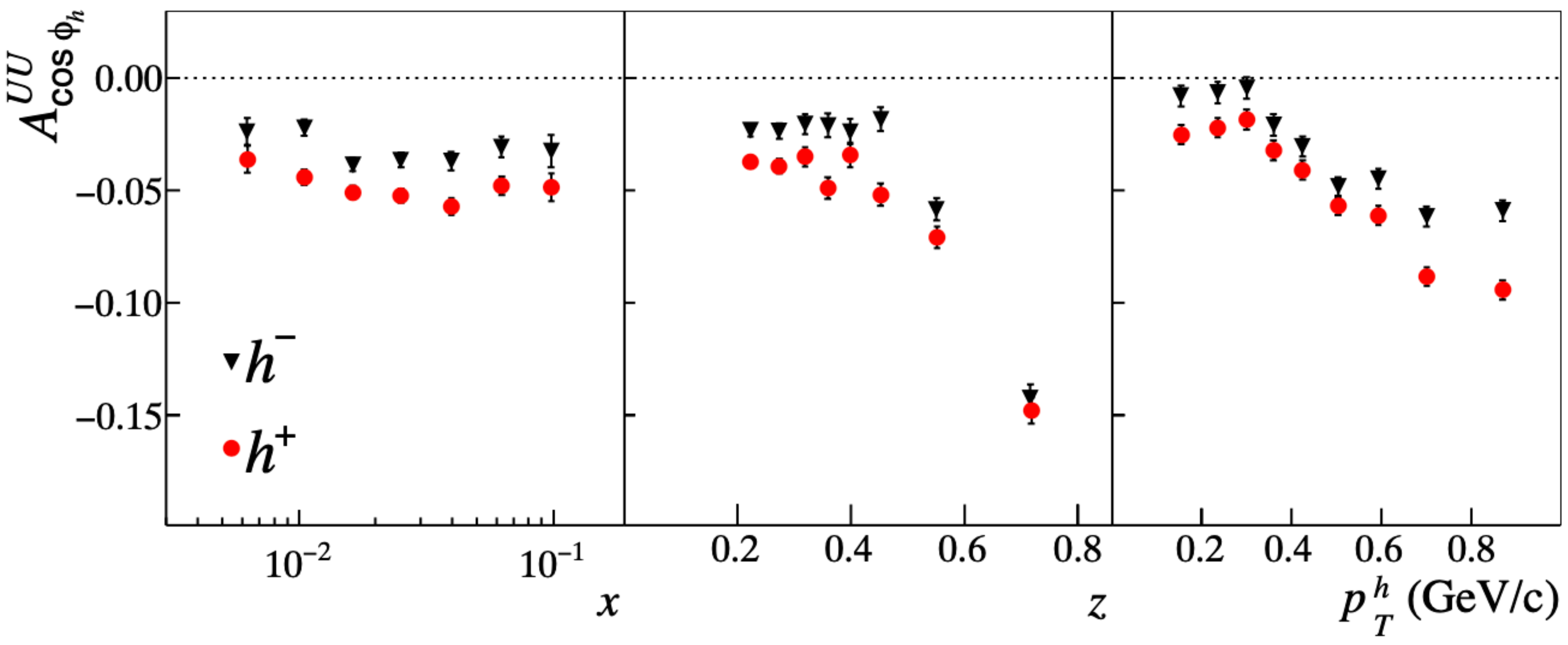


$(p_T \sim k_T) \sim q_T \ll Q$

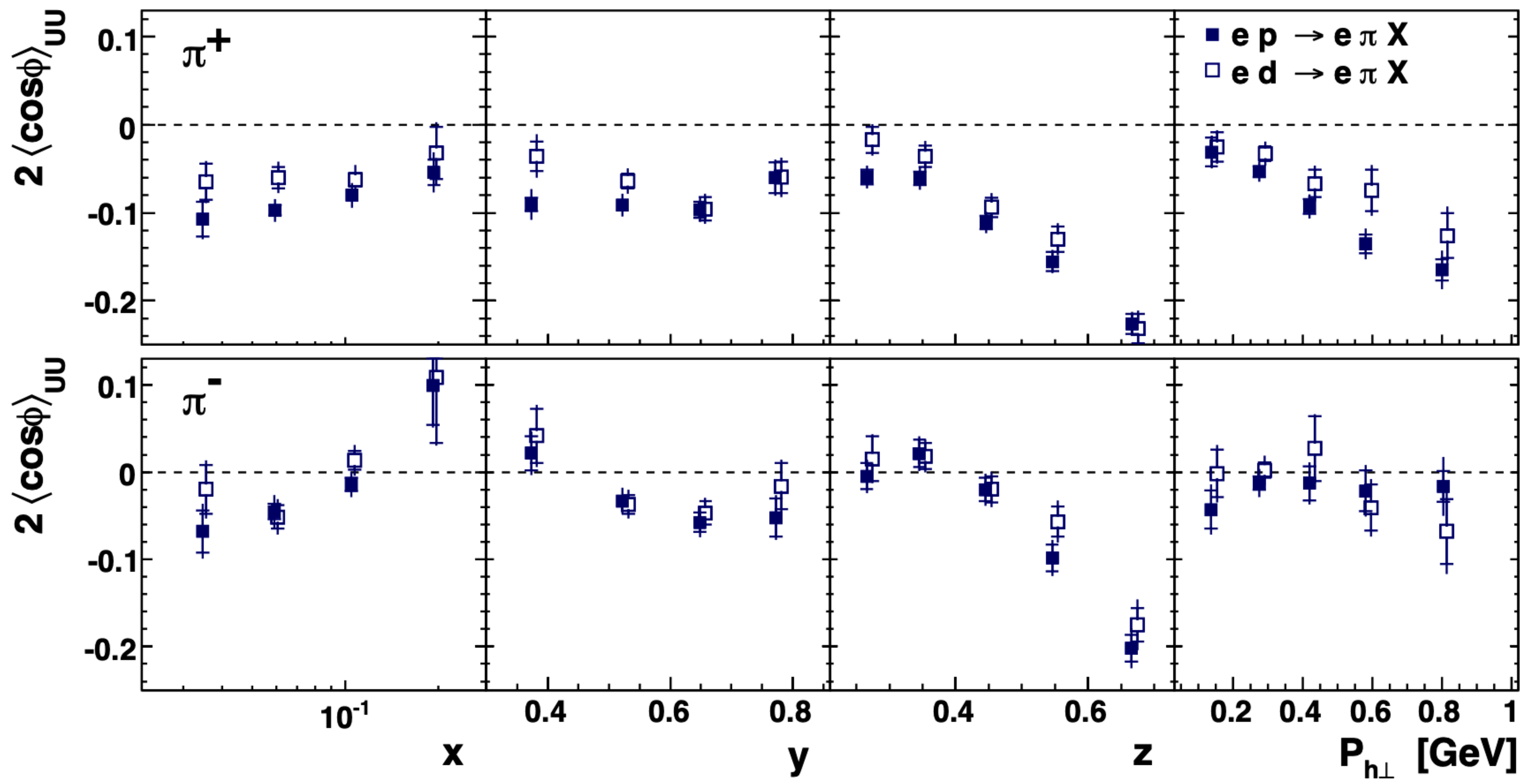
"TMD region"

More recent experiments

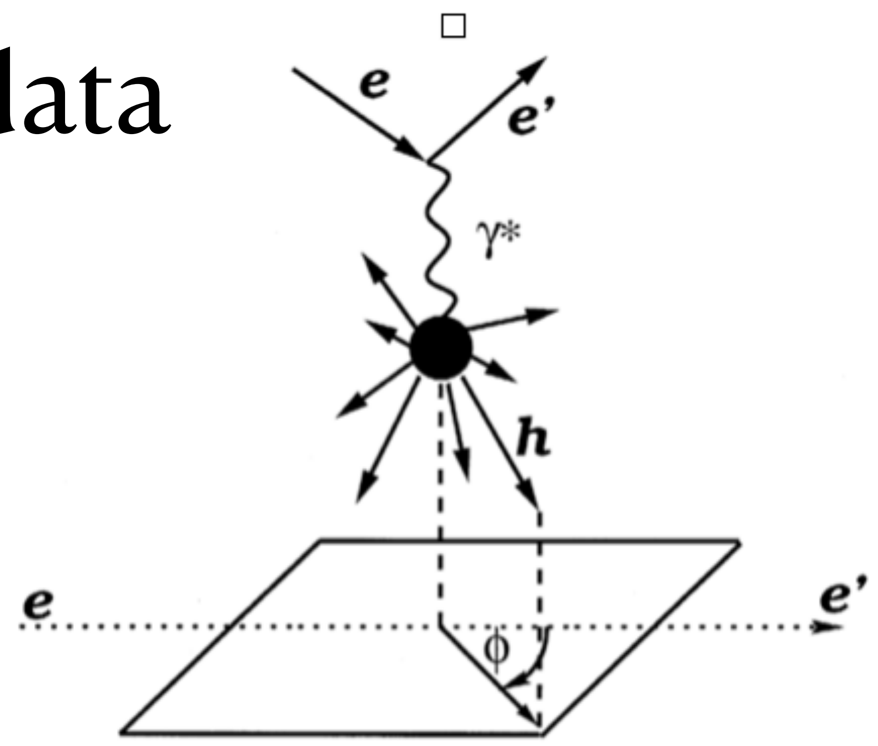
COMPASS, Nucl. Phys. B 886 (2014) 1046



HERMES, Phys. Rev. D 87 (2013) 012010



More recent 2016 2017 data

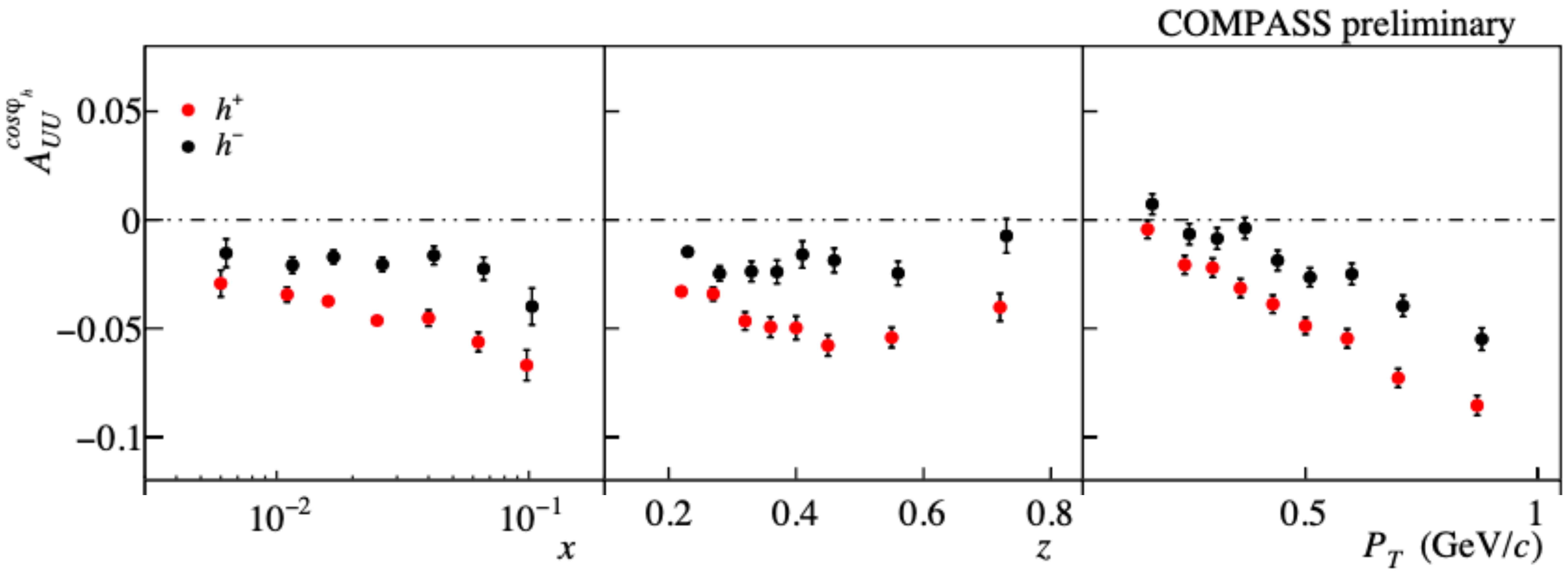


DATA

$(p_T \sim k_T) \sim q_T \ll Q$

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

"TMD region"



2016 2017 data

TMD observables in unpolarised Semi-Inclusive DIS at COMPASS, see talk of Riccardo Longo

Theory/Pheno Studies

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)}\, \cos\phi + \int d\sigma^{(1)}\, \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

Chay, S.D. Ellis, Stirling, Phys. Lett. B (1991)

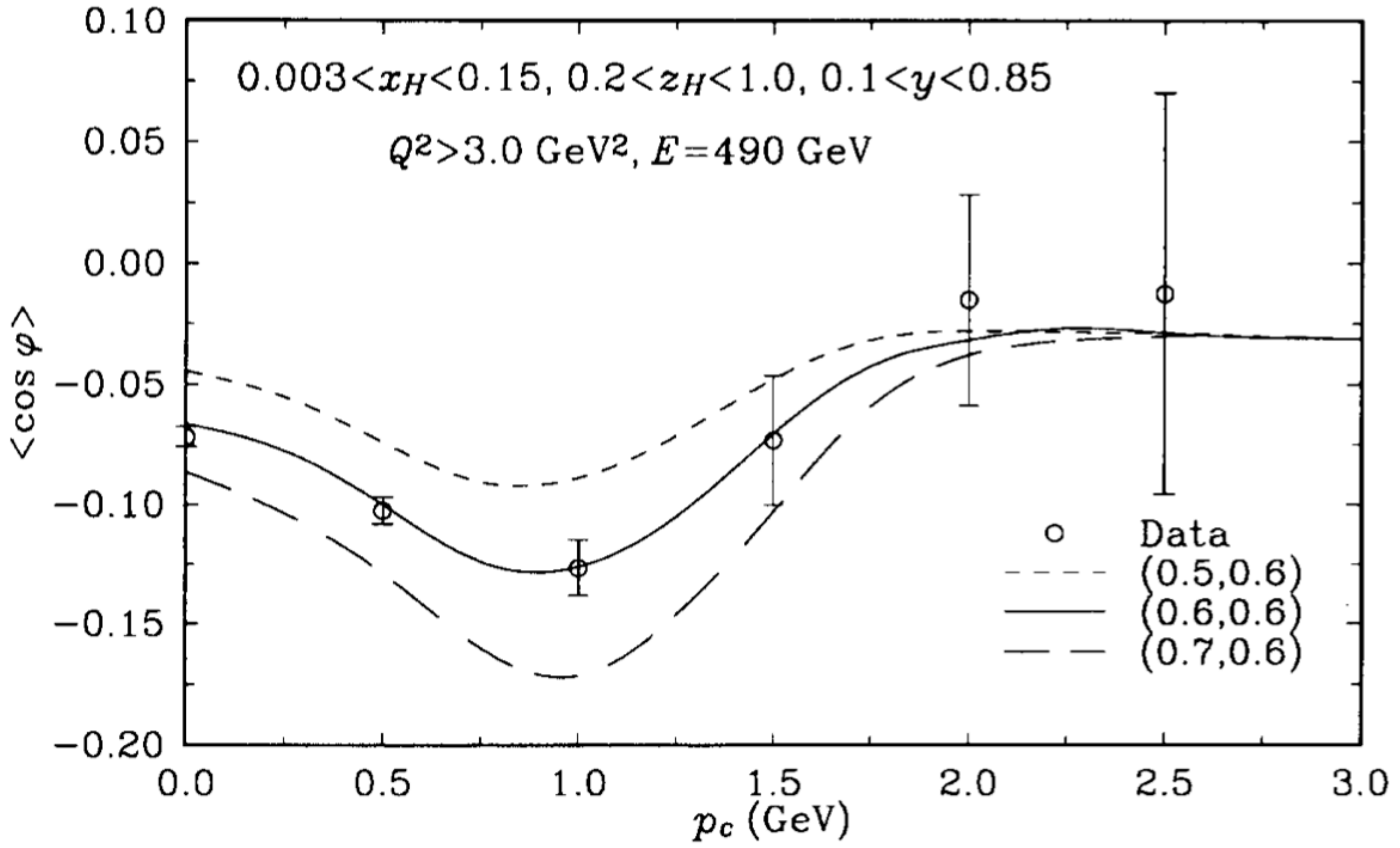
Oganessyan, Avakian, Bianchi, EPJC (1998)

$$\int d\sigma^{(0)} = 2\pi \frac{\alpha^2}{Q^2} \sum_j Q_j^2 F_j(x_H) D_j(z_H) \exp\left(-\frac{p_c^2}{b^2 + z_H^2 a^2}\right) \times \left\{ \frac{1+(1-y)^2}{y} + 4\frac{1-y}{yQ^2}\left[\frac{a^2 b^2}{b^2 + z_H^2 a^2} + \left(\frac{z_H a^2}{b^2 + z_H^2 a^2}\right)^2 (p_c^2 + b^2 + z_H^2 a^2)\right]\right\}$$

$$\int d\sigma^{(1)} \cos\phi = \int d^2 P_T \cos\phi \frac{d\sigma}{dx_H\, dy\, dz_H\, d^2 P_T} = \frac{8}{3}\frac{\alpha_s \alpha^2}{Q^2}\frac{(2-y)\sqrt{1-y}}{y}\int_{x_H}^{1}\frac{dx}{x}\int_{z_H}^{1}\frac{dz}{z}\sum_j Q_j^2 (A_j + B_j + C_j)$$

$$A_j = -\sqrt{\frac{xz}{(1-x)(1-z)}}\,[xz + (1-x)(1-z)]\, F_j\left(\frac{x_H}{x}, Q^2\right) D_j\left(\frac{z_H}{z}, Q^2\right)$$

Simple addition ... "double counting"



$\langle\cos\phi\rangle$ as a function of transverse momentum cutoff

- non-perturbative Cahn-like dominate at low p_c
- negligible at large values p_c because " intrinsic transverse momentum" in distribution &. FF too small to produce effect $P_T > p_c$ (data E665 Fermi-lab).

Theory/Pheno Studies

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

Chay, S.D. Ellis, Stirling, Phys. Lett. B (1991)

Oganessyan, Avakian, Bianchi, EPJC (1998)

$$\int d\sigma^{(0)} = 2\pi \frac{\alpha^2}{Q^2} \sum_j Q_j^2 F_j(x_H) D_j(z_H) \exp\left(-\frac{p_c^2}{b^2 + z_H^2 a^2}\right) \times \left\{ \frac{1+(1-y)^2}{y} + 4\frac{1-y}{yQ^2}\left[\frac{a^2 b^2}{b^2+z_H^2 a^2} + \left(\frac{z_H a^2}{b^2 + z_H^2 a^2}\right)^2 (p_c^2 + b^2 + z_H^2 a^2)\right]\right\}$$

$$\int d\sigma^{(1)} \cos\phi = \int d^2 P_T \cos\phi \frac{d\sigma}{dx_H\, dy\, dz_H\, d^2 P_T} = \frac{8}{3}\frac{\alpha_s \alpha^2}{Q^2}\frac{(2-y)\sqrt{1-y}}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\sum_j Q_j^2 (A_j + B_j + C_j)$$

$$A_j = -\sqrt{\frac{xz}{(1-x)(1-z)}}\,[xz + (1-x)(1-z)]\, F_j\left(\frac{x_H}{x}, Q^2\right) D_j\left(\frac{z_H}{z}, Q^2\right)$$

$\langle \cos\phi \rangle$ as a function of transverse momentum cutoff

- non-perturbative Cahn-like small at large Q

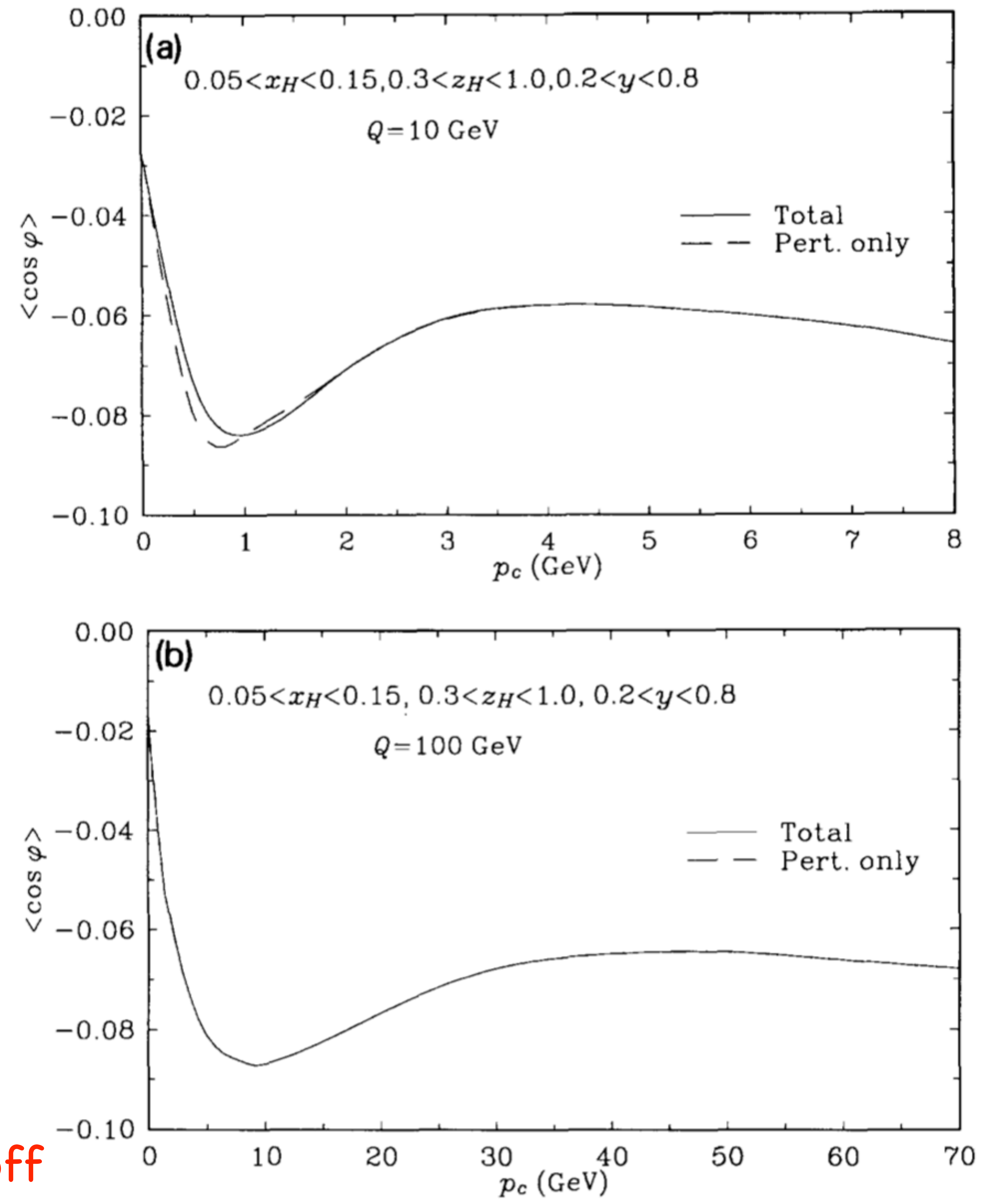


Fig. 2. $\langle \cos\varphi \rangle$ for (a) Q=10 GeV and (b) Q=100 GeV.

Simple addition ... "double counting"

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)}\ \cos\phi + \int d\sigma^{(1)}\ \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

$$\int d\sigma^{(0)} = 2\pi \frac{\alpha^2}{Q^2} \sum_j Q_j^2 F_j(x_H) D_j(z_H) \exp\left(-\frac{p_c^2}{b^2 + z_H^2 a^2}\right) \times \left\{ \frac{1+(1-y)^2}{y} + 4\frac{1-y}{yQ^2}\left[\frac{a^2 b^2}{b^2 + z_H^2 a^2} + \left(\frac{z_H a^2}{b^2 + z_H^2 a^2}\right)^2 (p_c^2 + b^2 + z_H^2 a^2)\right]\right\}$$

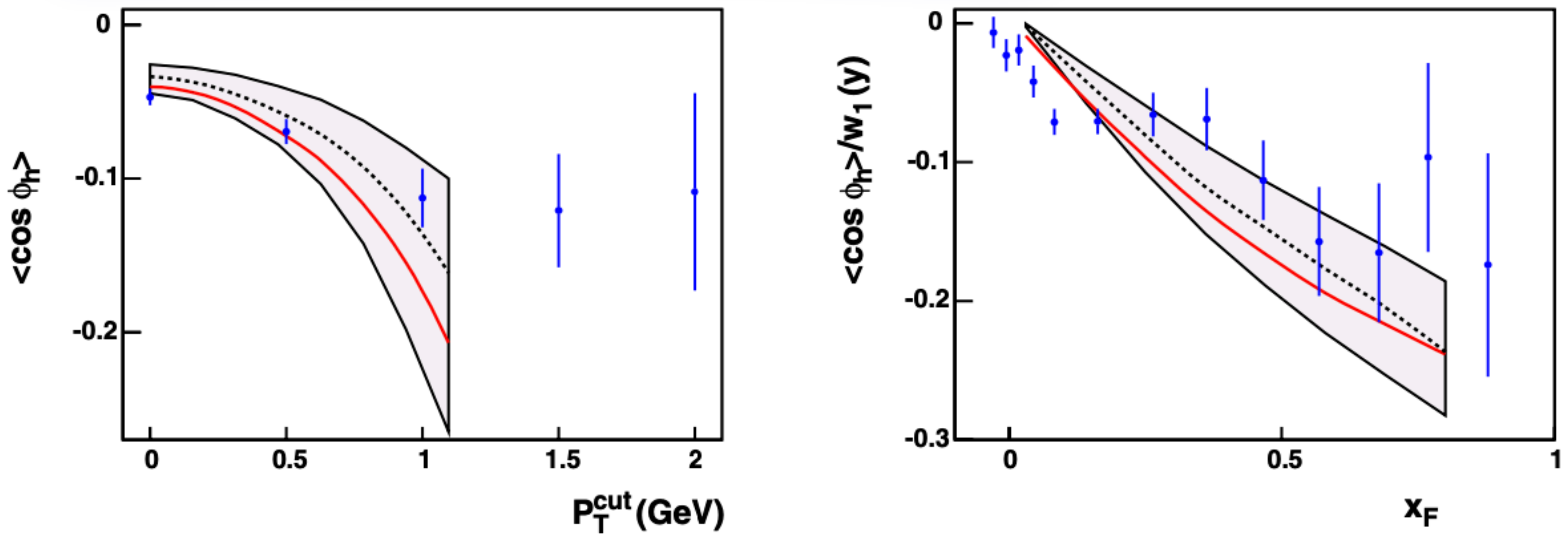
$$\int d\sigma^{(1)}\ \cos\phi = \int d^2 P_T\ \cos\phi \frac{d\sigma}{dx_H\, dy\, dz_H\, d^2 P_T} = \frac{8}{3}\frac{\alpha_s \alpha^2}{Q^2}\frac{(2-y)\sqrt{1-y}}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\sum_j Q_j^2 (A_j + B_j + C_j)$$

“W” term Theory/Pheno studies

Anselmino, Boglione, D’Alesio, Kotzinian, Murgia, Prokudin

PRD **71,** 074006 (2005)

One of the “first” TMD analysis
Role of Cahn effect in SIDIS from TMD framework
Modeling tree level result comparing w/ E665 data



$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\right].$$

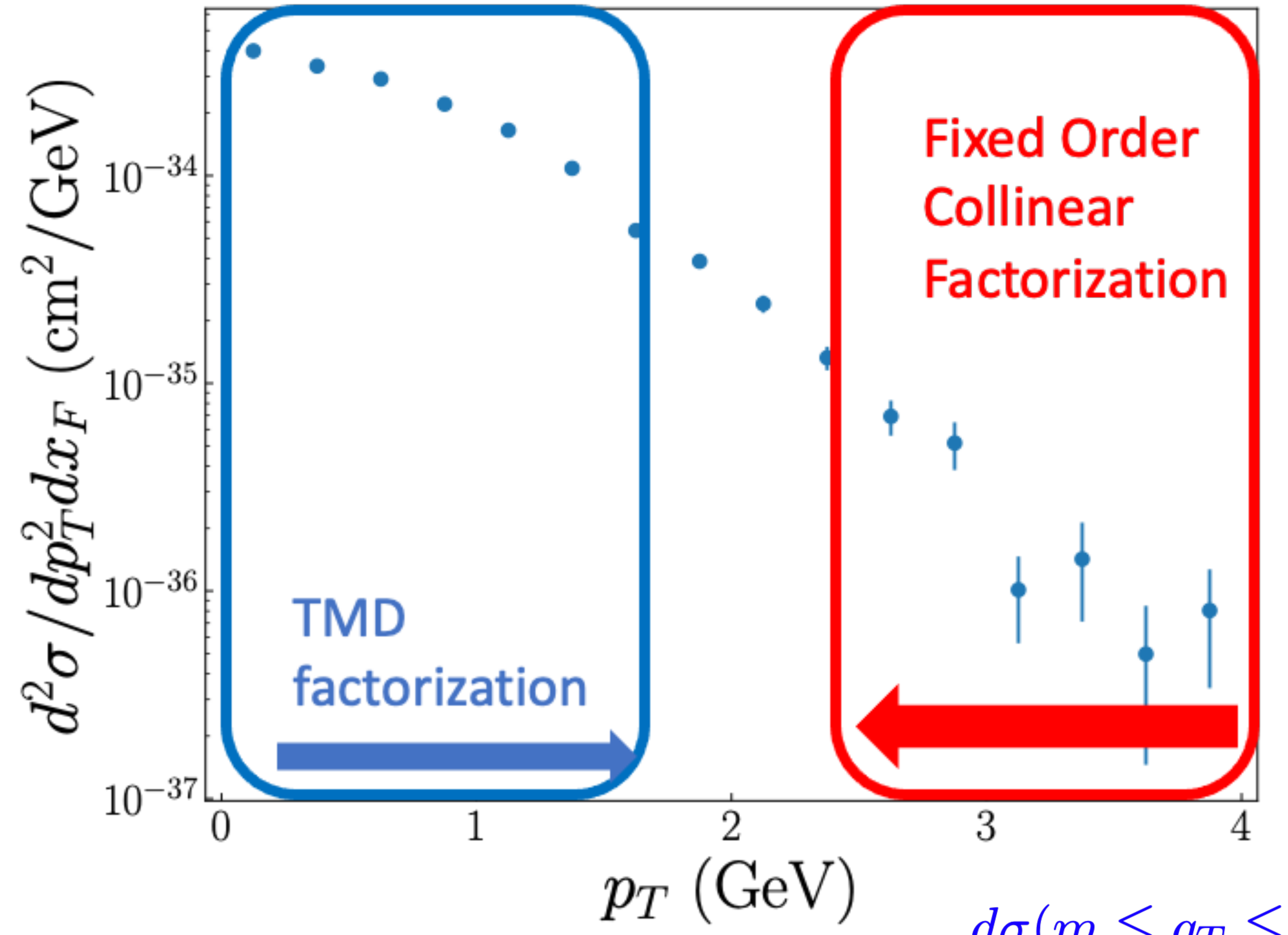
Wandzura Wilzeck approx in TMD Bacchetta et al. JHEP 2007

$$\frac{d^5\sigma^{\ell p\to\ell hX}}{dx_B dQ^2 dz_h d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4} f_q(x_B) D_q^h(z_h)\left[1+(1-y)^2 - 4\frac{(2-y)\sqrt{1-y}\langle k_\perp^2\rangle z_h P_T}{\langle P_T^2\rangle Q}\cos\phi_h\right]\frac{1}{\pi\langle P_T^2\rangle}e^{-P_T^2/\langle P_T^2\rangle},$$

Regions and matching

NPB Collins & Soper(1982), & Sterman 1985

Requires systematic factorization approach



E615 πW Drell-Yan
Phys. Rev. D **39** (1989).

Collins 2011 Foundations of pQCD Cambridge
Collins Gamberg Prokudin Rogers Sato Phys.Rev.D 94 (2016)

- Goal to use p_T (q_T) data over full range &
- simultaneous fit of pdfs & TMDs

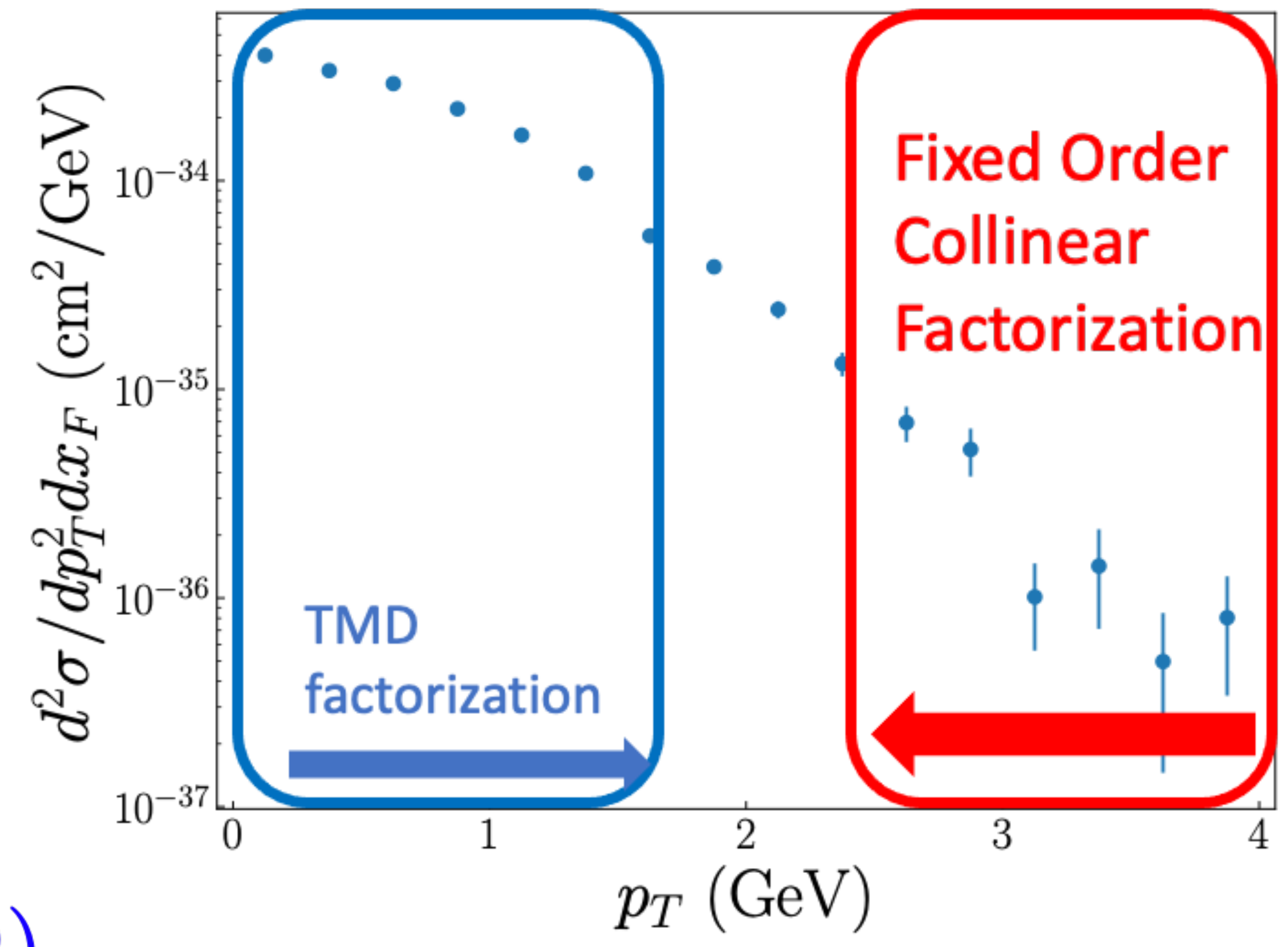
- **<u>Cross section in terms of different "regions"</u>**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- ASY subtracts d.c. & in principle
- $ASY \to W,\ p_T \to \infty$ and $ASY \to FO, p_T \to 0$

$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dydq^2dp_T^2} = \left.\frac{d\sigma^W(q_T,Q)}{dydq^2dp_T^2}\right|_{m\lesssim q_T \ll Q} + \left.\frac{d\sigma^{FO}(q_T,Q)}{dydq^2dp_T^2}\right|_{m\ll q_T \lesssim Q} - \left.\frac{d\sigma^{ASY}(q_T,Q)}{dydq^2dp_T^2}\right|_{m\lesssim q_T \ll Q}$$

$$\equiv W(p_T,Q) + FO(p_T,Q) - AY(p_T,Q) + O\left(\frac{m}{Q}\right)^c$$

nb $\quad q_T \to 0, \quad Y \equiv F0 \to 0\,(!!)$

"Mis"-Matches Factorization @ sub-leading power



$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dy\,dq^2\,dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

- **Bacchetta, Boer, Diehl, Mulders JHEP (2008)**
  Mis-match/inconsistency breakdown of factorization at NLP?

"… the requirement to match the high-q_T result (4.25) for $F_{UU}^{\cos\phi_h}$ at intermediate q_T can be used as a consistency check for any framework that extends Collins-Soper factorization to the twist-three sector."

- **Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, PLB (2019)**

“Mis”-matches Factorization @ sub-leading power

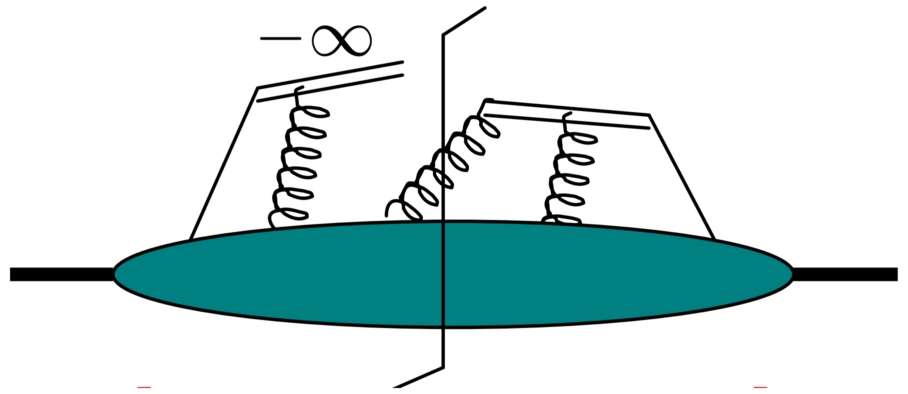
$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

To cure mismatch, Bacchetta et al. speculated that soft factor subtraction from LP TMD same as NLP TMDs: PLB (2019)

What’s the soft factor ???

Advertisement TMD Handbook 2023 e-Print:2304.03302 [hep-ph]

Collins QCD book 2011, Aybat Rogers 2011 PRD, Echevarria et al. 2012 JHEP



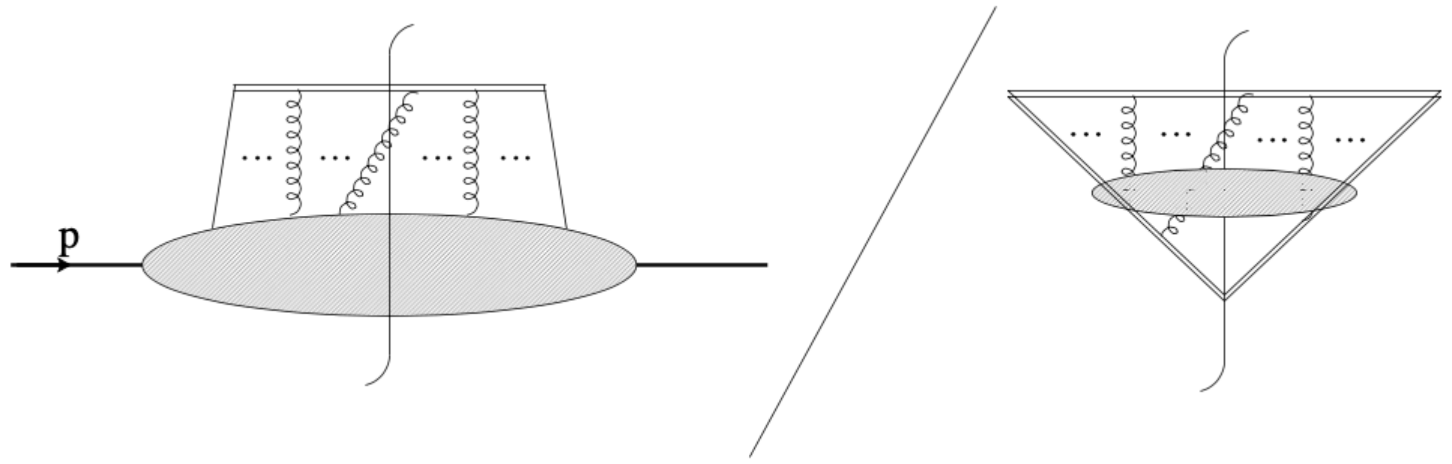
$$\tilde{f}^{\text{sub}}_{j/H}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}^{\text{unsub}}_{j/H}(x, b_T; \mu, y_P - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

$$\Updownarrow$$

$$\tilde{f}^{\text{unsub}}_{j/H}(x, b_T; \mu, y_P - y_B) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}(0)\gamma^+ \mathcal{U}_{[0,b]}\psi(b)|P\rangle|_{b^+=0}$$

JCC Soft factor further “repartitioned”

1) cancel LC divergences in “unsubtracted” TMDs
2) separate “right & left” movers i.e. full factorization
3) remove double counting of momentum regions



Conjecture of Bacchetta et al 2019 based on Matches and mis-matches 2008

$???$ $\quad W \to AY \leftarrow FO \quad$ $???$

$S(l_T)$

$$\mathcal{C}[wfD] = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T) \quad \times w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2, \ldots(l_T))$$

$$F_{UU}^{\cos\phi} = \mathscr{C}\left[\frac{\hat{h}\cdot p_\perp}{Q} f_1(x, p_\perp)\frac{\tilde{D}^\perp(z, k_\perp)}{z} S(l_\perp) + \frac{\hat{h}\cdot k_\perp}{Q} x f^\perp(x, p_\perp) D_1(z, k_\perp) S(l_\perp)\right]$$

Method: Bacchetta et al. 2008

let one of $p_\perp, k_\perp\, l_\perp \to q_\perp$ *large* other small
No contribution from Soft factor term to cancel
"mis-match" term from expanding the W term at large
Offending mismatch term remains

Method: Bacchetta et al. 2019

Conjecture based on replacement based on LP analysis in $F_{UU,T}$

$$\frac{1}{2}L(\eta^{-1}) \longrightarrow \frac{1}{2}L\left(\frac{Q^2}{k_\perp^2}\right) + C_F\,,$$
$$\frac{1}{2}L(\eta_h^{-1}) \longrightarrow \frac{1}{2}L\left(\frac{z^2Q^2}{P_\perp^2}\right) + C_F$$

Unpolarized Structure function

$$F_{UU,T} = \frac{\alpha_s}{2\pi^2} \frac{1}{z^2 q_T^2} \sum_a x e_a^2$$
$$\times \Bigg\{ \Bigg[\frac{L(\eta^{-1})}{2} f_1^a(x) - C_F f_1^a(x) + \left(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\right)(x) \Bigg] D_1^a(z)$$
$$+ f_1^a(x) \Bigg[\frac{L(\eta_h^{-1})}{2} D_1^a(z) - C_F D_1^a(z) + \left(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\right)(z) \Bigg]$$
$$+ 2C_F f_1^a(x) D_1^a(z) \Bigg\}$$
$$= \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \Bigg[f_1^a(x) D_1^a(z) L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) \left(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\right)(z)$$
$$+ \left(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\right)(x) D_1^a(z) \Bigg],$$

Noted that this replacement accomplish cancellation but don't need to do this replacement—very little justification

$$\frac{1}{2} L(\eta^{-1}) \longrightarrow \frac{1}{2} L\left(\frac{Q^2}{k_\perp^2}\right) + C_F\,,$$
$$\frac{1}{2} L(\eta_h^{-1}) \longrightarrow \frac{1}{2} L\left(\frac{z^2 Q^2}{P_\perp^2}\right) + C_F$$

$\cos\phi$ Structure function

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\frac{\alpha_s}{2\pi^2}\frac{1}{2z^2q_T^2}\sum_a x\,e_a^2$$
$$\times\Big\{\Big[\frac{L(\eta^{-1})}{2}f_1^a(x) + (P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g)(x)\Big]D_1^a(z)$$
$$+ f_1^a(x)\Big[\frac{L(\eta_h^{-1})}{2}D_1^a(z) - 2C_F D_1^a(z) + (D_1^a\otimes P'_{qq} + D_1^g\otimes P'_{gq})(z)\Big]\Big\}$$
$$= -\frac{1}{Q\,q_T}\frac{\alpha_s}{2\pi^2z^2}\sum_a x\,e_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,(D_1^a\otimes P'_{qq} + D_1^a\otimes P'_{gq})(z)$$
$$+ (P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g)(x)\,D_1^a(z) - 2C_F\,f_1^a(x)\,D_1^a(z)\Big], \qquad (8.55)$$

which is *not identical* to the high-q_T result (4.25) because of the extra term $2C_F\,f_1^a(x)\,D_1^a(z)$

Noted that this replacement accomplish cancellation—conjecture

$$\frac{1}{2}L(\eta^{-1}) \longrightarrow \frac{1}{2}L\Big(\frac{Q^2}{k_\perp^2}\Big) + C_F\,,$$
$$\frac{1}{2}L(\eta_h^{-1}) \longrightarrow \frac{1}{2}L\Big(\frac{z^2Q^2}{P_\perp^2}\Big) + C_F$$

Here there is no soft contribution to in $F_{UU}^{\cos\phi}$ without some transverse momentum in tensor structure of soft factor

To understand appreciate the subtleties review
Tree level TMD @ LP and NLP factorization

In reviewing will remind about the utility of using
Fierz decamp & “good and bad” LC quark fields “

Then onto Factorization at NLO address soft factor calculation

Factorization at sub-leading power ... revisit Tree level

$$\frac{d\sigma}{dx\, dy\, d\Psi\, dz\, d^2 P_{h\perp}} = \kappa \frac{\alpha_{\rm em}^2}{4Q^4} \frac{y}{z} L_{\mu\nu} W^{\mu\nu}$$

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

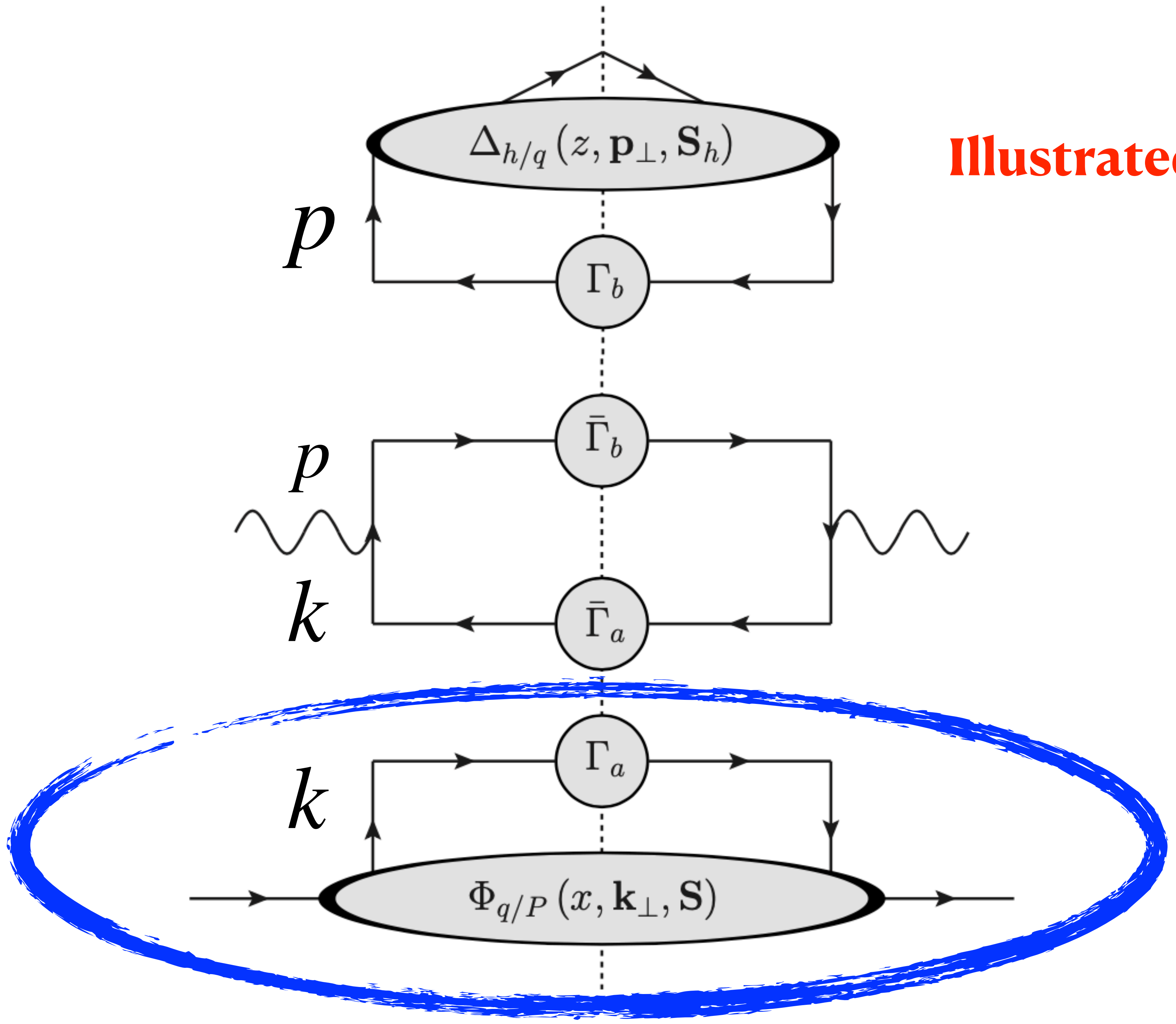
$$W_{\mu\nu} = \frac{1}{(2\pi)^4} \sum_X \int d^4x\, e^{-iqx} \langle P | J_\mu^\dagger(0) | h, X \rangle \langle h, X | J_\nu(x) | P \rangle ,$$

$$J_\mu(x) = J_\mu^{(2)}(x) + J_\mu^{(3)}(x)$$

$$k^\mu \sim Q(1,\lambda^2,\lambda),\ p^\mu \sim Q(\lambda^2,1,\lambda)$$

Working @ NLP, the current contains 3(!) contributions:

- One with 2 partons entering from each correlation function
- Another with 3 partons entering from one correlation function
- **& partonic kinematic power corrections-momentum scaling**



Illustrated at "tree level"

Factorization at sub-leading power ... revisit Tree level

$$\frac{d\sigma}{dx\,dy\,d\Psi\,dz\,d^2P_{h\perp}} = \kappa\frac{\alpha_{em}^2}{4Q^4}\frac{y}{z}L_{\mu\nu}W^{\mu\nu}$$

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

$$W_{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X \int d^4x\, e^{-iqx}\langle P|J_\mu^\dagger(0)|h,X\rangle\langle h,X|\,J_\nu(x)|P\rangle\,,$$

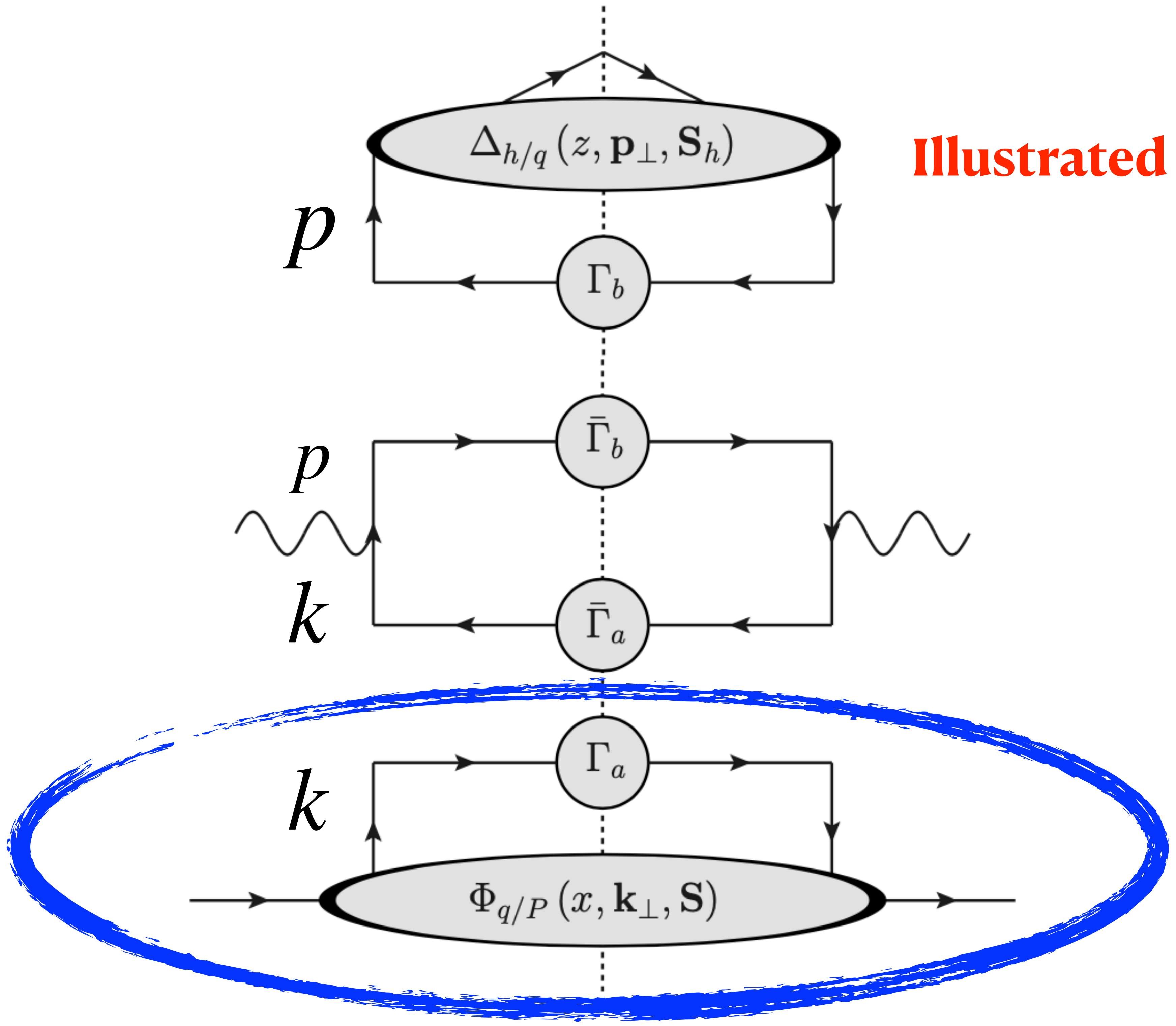
$$J_\mu(x) = J_\mu^{(2)}(x) + J_\mu^{(3)}(x)$$

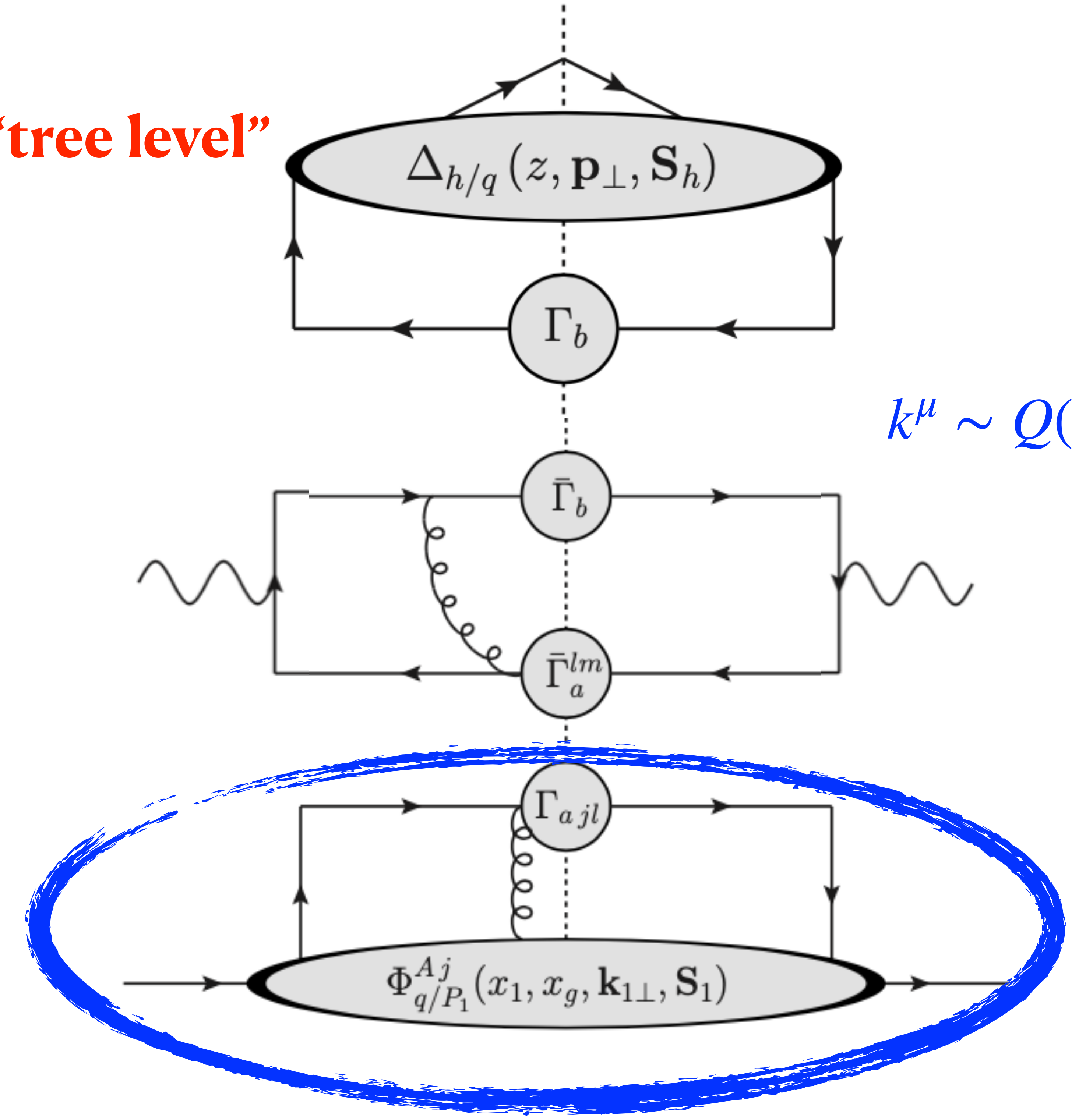
Working @ NLP, the current contains 3(!) contributions:

- One with 2 partons entering from each correlation function
- Another with 3 partons entering from one correlation function
- **& partonic kinematic power corrections-momentum scaling**

$k^\mu \sim Q(1,\lambda^2,\lambda),\ p^\mu \sim Q(\lambda^2,1,\lambda)$

Illustrated at "tree level"





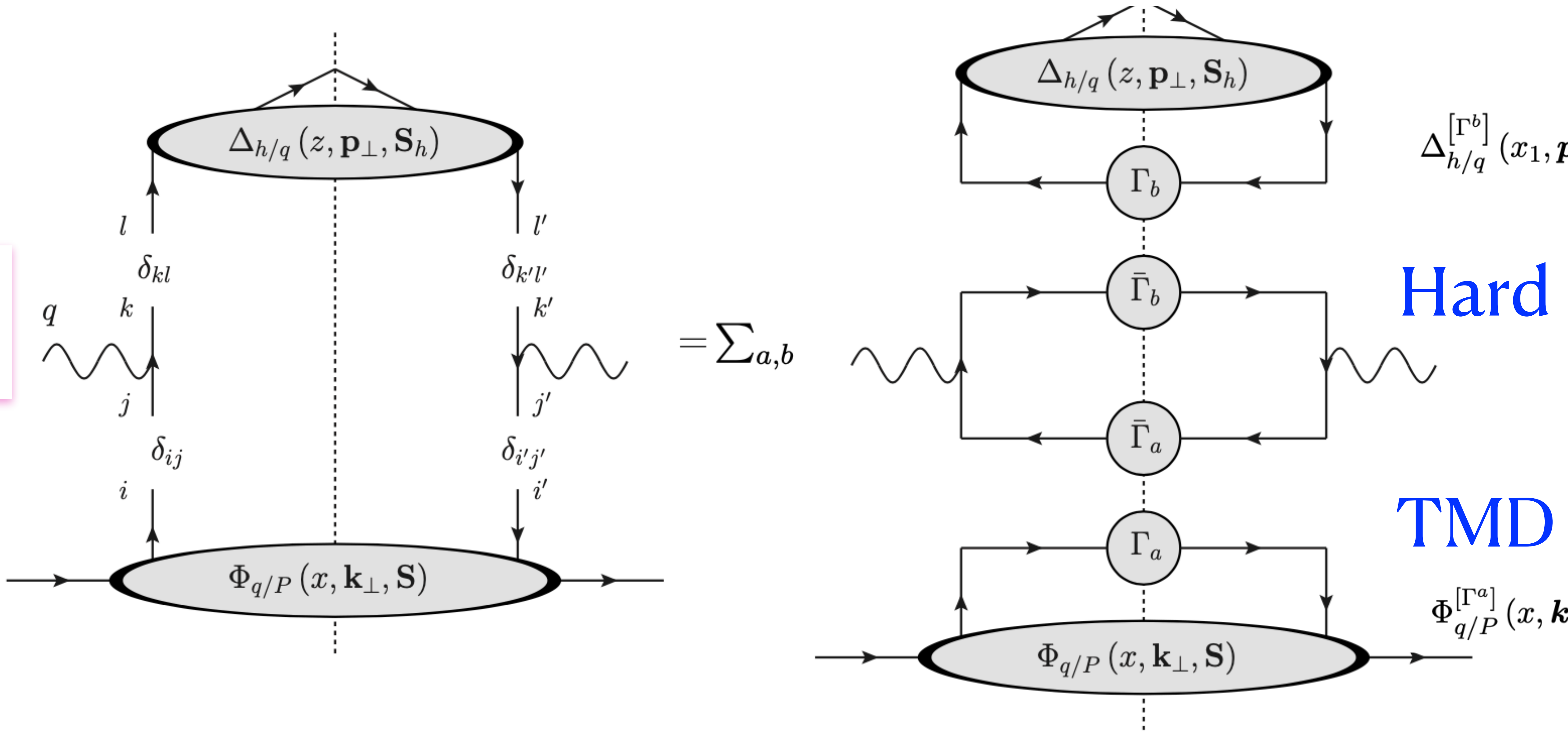
$k^\mu \sim Q(1,\lambda^2,\lambda),\ p^\mu \sim Q(\lambda^2,1,\lambda)$

Factorization at leading and sub-leading power "Tree level" parton mdl. Fierz decomposition

2 parton hadronic tensor can be organized contributions @ given twist by Fierz decomposition of the quark lines

Fierz decomposition of 2 parton correlation function $\delta_{ij}\delta_{j'i'} = \sum_a \Gamma^a_{ii'}\,\bar{\Gamma}^a_{j'j}$ Illustrated in Fig.

$$\delta_{ij}\delta_{j'i'} = \sum_a \Gamma^a_{ii'}\,\bar{\Gamma}^a_{j'j}$$



$$\Delta^{[\Gamma^b]}_{h/q}(x_1, \boldsymbol{p}_\perp, \boldsymbol{S}_h) = \mathrm{Tr}\left[\Delta_{h/q}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h)\,\Gamma^b\right]$$

$$\Phi^{[\Gamma^a]}_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \mathrm{Tr}\left[\Phi_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S})\,\Gamma^a\right]$$

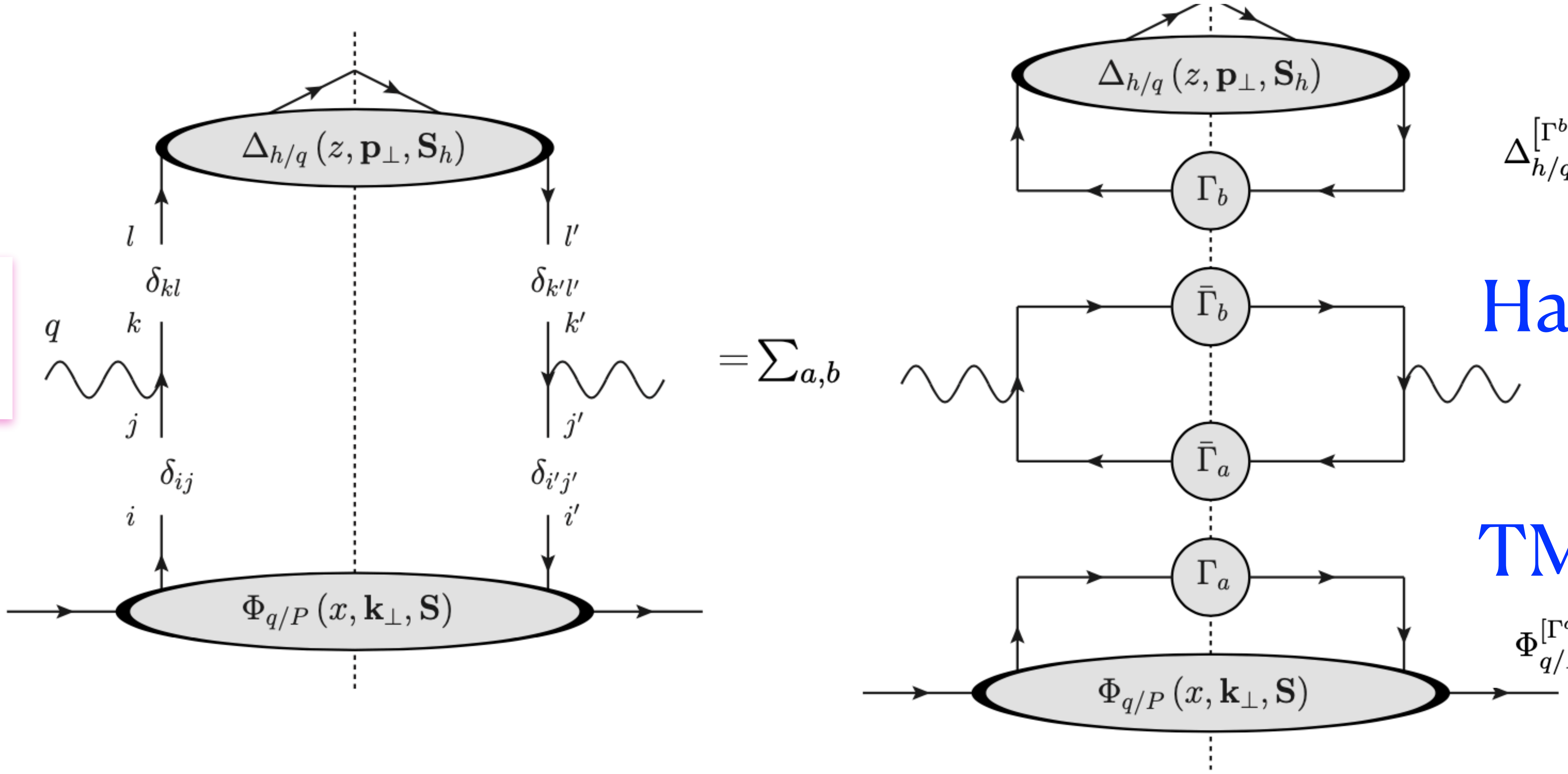
$$\Gamma_a \in \left\{ \underbrace{\frac{\not{n}}{4}, \frac{\not{n}\gamma^5}{4}, \frac{i}{4}\sigma^{i+}\gamma^5}_{\text{LP}}, \underbrace{\frac{1}{2}, \frac{\gamma^5}{2}, \frac{\gamma^i}{2}, \frac{\gamma^i\gamma^5}{2}, \frac{i}{2}\sigma^{ij}\gamma^5, \frac{i}{4}\sigma^{+-}\gamma^5}_{\text{NLP}} + \ldots\ldots \right\}$$

Factorization at sub-leading power Tree level employ Fierz decomposition

2 parton hadronic tensor can be organized contributions @ given twist by Fierz decomposition of the quark lines

Fierz decomposition of 2 parton correlation function $\delta_{ij}\delta_{j'i'} = \sum_a \Gamma^a_{ii'}\,\bar{\Gamma}^a_{j'j}$ Illustrated in Fig.

$$\delta_{ij}\delta_{j'i'} = \sum_a \Gamma^a_{ii'}\,\bar{\Gamma}^a_{j'j}$$



$$\Delta^{[\Gamma^b]}_{h/q}(x_1,\boldsymbol{p}_\perp,\boldsymbol{S}_h) = \mathrm{Tr}\left[\Delta_{h/q}(z,\boldsymbol{p}_\perp,\boldsymbol{S}_h)\,\Gamma^b\right]$$

Hard

TMD

$$\Phi^{[\Gamma^a]}_{q/P}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \mathrm{Tr}\left[\Phi_{q/P}(x,\boldsymbol{k}_\perp,\boldsymbol{S})\,\Gamma^a\right]$$

Factorized !!

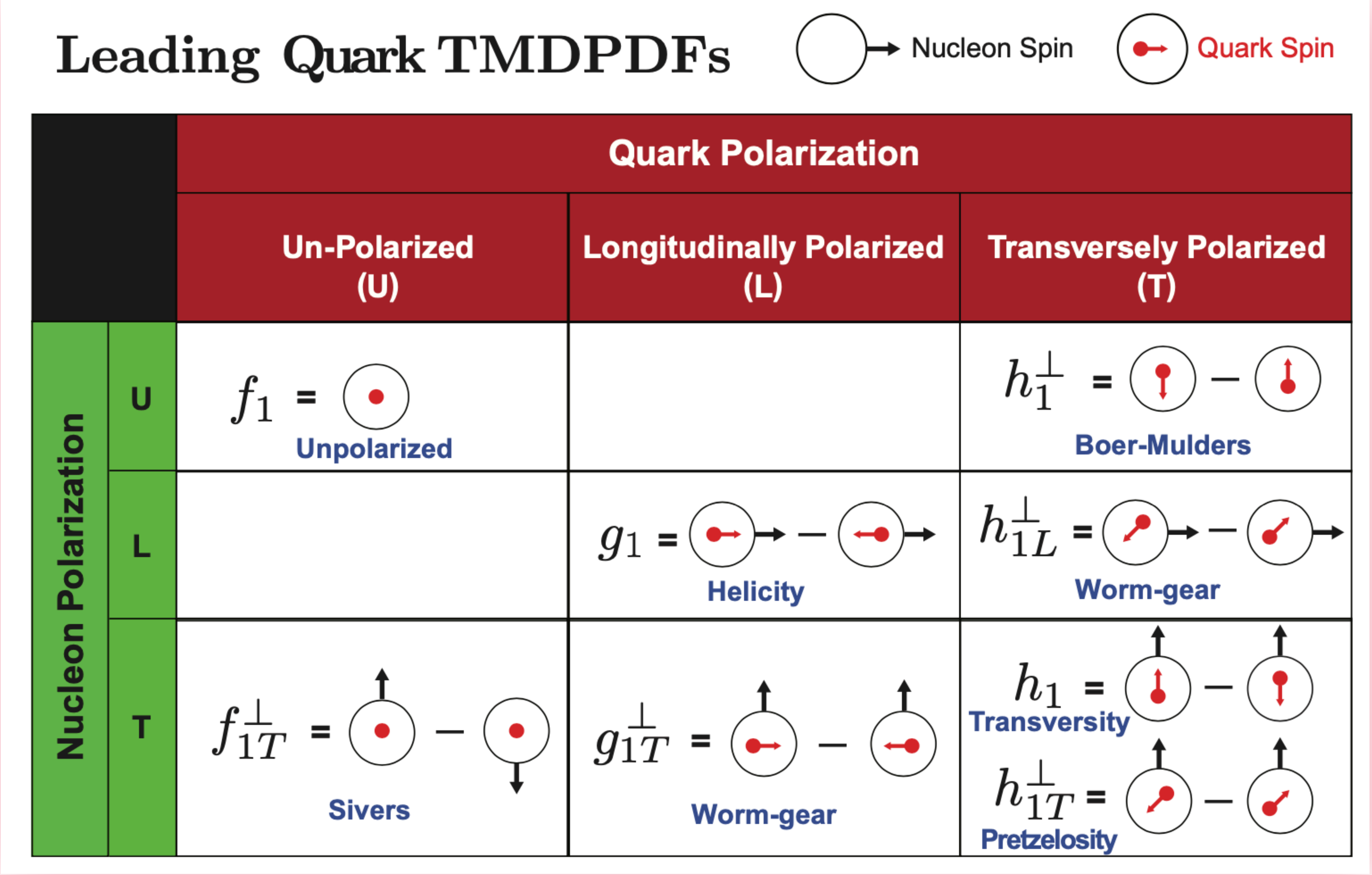
$$W^{(2)}_{\mu\nu} = \frac{1}{N_c}\sum_{a,b}\mathrm{Tr}\left[\gamma^\mu\,\bar{\Gamma}^a\,\gamma^\nu\,\bar{\Gamma}^b\right]\mathcal{C}^{\mathrm{DIS}}\left[\Phi^{[\Gamma^a]}(x,\boldsymbol{k}_\perp,\boldsymbol{S})\;\Delta^{[\Gamma^b]}(z,\boldsymbol{p}_\perp,\boldsymbol{S}_h)\right]$$

$$\mathcal{C}^{\mathrm{DIS}}[A\,B] = \sum_q e_q^2\int d^2\boldsymbol{k}_\perp\,d^2\boldsymbol{p}_\perp\,\delta^{(2)}(\boldsymbol{q}_\perp+\boldsymbol{k}_\perp+\boldsymbol{p}_\perp/z)\;\times\;A_{q/P}(x,\boldsymbol{k}_\perp,\boldsymbol{S})\,B_{h/q}(z,\boldsymbol{p}_\perp,\boldsymbol{S}_h)$$

By organizing the operators by their twists,
we arrive at the well known expression for the LP and NLP correlation functions

$$\Phi(x, k_T)$$

$$\Phi^{(2)}_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \left(f_1 - \frac{\epsilon_\perp^{ij} k_{\perp i} S_{\perp j}}{M} f_{1T}^\perp\right)\frac{\not{\bar{n}}}{4} + \left(\lambda g_{1L} - \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} g_{1T}\right)\frac{\gamma^5 \not{\bar{n}}}{4}$$
$$+ \left(S_\perp^i h_1 + \frac{\lambda k_\perp^i}{M} h_{1L}^\perp - \frac{\epsilon_\perp^{ij} k_{\perp j}}{M} h_1^\perp - \frac{k_\perp^i k_\perp^j - \frac{1}{2} k_\perp^2 g_\perp^{ij}}{M^2} S_{\perp j} h_{1T}^\perp\right)\frac{i\gamma^5 \sigma_{-i}}{4}$$



Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}$, $\frac{1}{4}\not{\bar{n}}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}$, $\frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5$, $\frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5$, $\frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5$, $\frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k$, $\frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5$, $\frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5$, $\frac{i}{4}\gamma^5\sigma_{+-}$	

✦Mulders Tangerman NPB1995
✦Goeke Metz Schlegel PLB 2005
✦Bacchetta et al 2007 JHEP

Intrinsic

By organizing the operators by their twists,
we arrive at the well known expression for the LP and NLP correlation functions

$$\Phi(x, \boldsymbol{k}_T)$$

Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}\,,\ \frac{1}{4}\not{\bar{n}}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}\,,\ \frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5,\ \frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5,\ \frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5,\ \frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k,\ \frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5\,,\ \frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5,\ \frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{2}\sigma^{+-}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{+-}$	

✦Mulders Tangerman NPB1995
✦Goeke Metz Schlegel PLB 2005
✦Bacchetta et al 2007 JHEP

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^{\perp},\ g^{\perp}$	$e\ ,\ h$
	L	$f_L^{\perp},\ g_L^{\perp}$	$e_L,\ h_L$
	T	$f_T,\ f_T^{\perp},\ g_T,\ g_T^{\perp}$	$e_T,\ e_T^{\perp},\ h_T,\ h_T^{\perp}$

$$\begin{aligned}\Phi^{(3)}_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \frac{M}{P^+}\Bigg[&\left(e - \frac{\epsilon_\perp^{ij} k_{\perp i} S_{\perp j}}{M} e_T^\perp\right)\frac{1}{2} - i\left(\lambda_g e_L - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M} e_T\right)\frac{\gamma^5}{2} \\ &+ \left(\frac{k_\perp^i}{M} f^\perp - \epsilon_\perp^{ij} S_{\perp j} f_T' - \frac{\epsilon_\perp^{ij} k_{\perp j}}{M}\left(\lambda_g f_L^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M} f_T^\perp\right)\right)\frac{\gamma_i}{2} \\ &+ \left(g_T' S_\perp^i - \frac{\epsilon_\perp^{ij} k_{\perp j}}{M} g^\perp + \frac{k_\perp^i}{M}\left(\lambda_g g_L^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M} g_T^\perp\right)\right)\frac{\gamma^5\gamma_i}{2} \\ &+ \left(\frac{S_\perp^i k_\perp^j}{M} h_T^\perp\right)\frac{i\gamma^5\sigma_{ji}}{4} + \left(h + \lambda_g h_L - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M} h^\perp\right)\frac{i\gamma^5\sigma_{+-}}{4}\Bigg]\end{aligned}$$

Factorization at sub-leading power … 3 partons

- "TMD" region

$$W_{\mu\nu} = \frac{1}{(2\pi)^4} \sum_X \int d^4x\, e^{-iqx} \langle P | J^\dagger_\mu(0) | h, X \rangle \langle h, X | J_\nu(x) | P \rangle ,$$

$$J_\mu(x) = J^{(2)}_\mu(x) + J^{(3)}_\mu(x)$$

Consider 3 partons entering from one hadron: transverse gluon leads to power suppression of order λ

$$W^{(3)}_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4x\, e^{-iqx} \left\langle P_1 , P_2 \middle| \left(J^{(3)}_\mu(0)\, J^{(2)\,\dagger}_\nu(x) + J^{(2)}_\mu(0)\, J^{(3)\,\dagger}_\nu(x) \right) \middle| P_1 , P_2 \right\rangle$$

$$W^{(3)}_{\mu\nu} = -\frac{1}{N_c C_F} \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp\, \delta^{(2)}\left(\boldsymbol{q}_\perp + \boldsymbol{k}_\perp + \boldsymbol{p}_\perp / z\right)$$

$$\times \Bigg[\int dk_g^+ \,\mathrm{Tr}\left[\Phi^i_{A\,q/P_1}(x, x_g, \boldsymbol{k}_\perp, \boldsymbol{S}) \gamma^\mu \Delta_{h/q}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h) \gamma_i \frac{\not{p} - \not{k}_g}{(p - k_g)^2 + i\epsilon} \gamma^\nu \right]$$

$$+ \int dp_g^- \,\mathrm{Tr}\left[\Delta^i_{A\,h/q}(z, z_g, \boldsymbol{p}_\perp, \boldsymbol{S}_h) \gamma^\nu \frac{\not{k} - \not{p}_g}{(k - p_g)^2 + i\epsilon} \gamma_i \Phi_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) \gamma^\mu \right] + \mathrm{h.c.} \Bigg]$$

Similar Fierzing algorithm
Get factorized Hadronic tensor

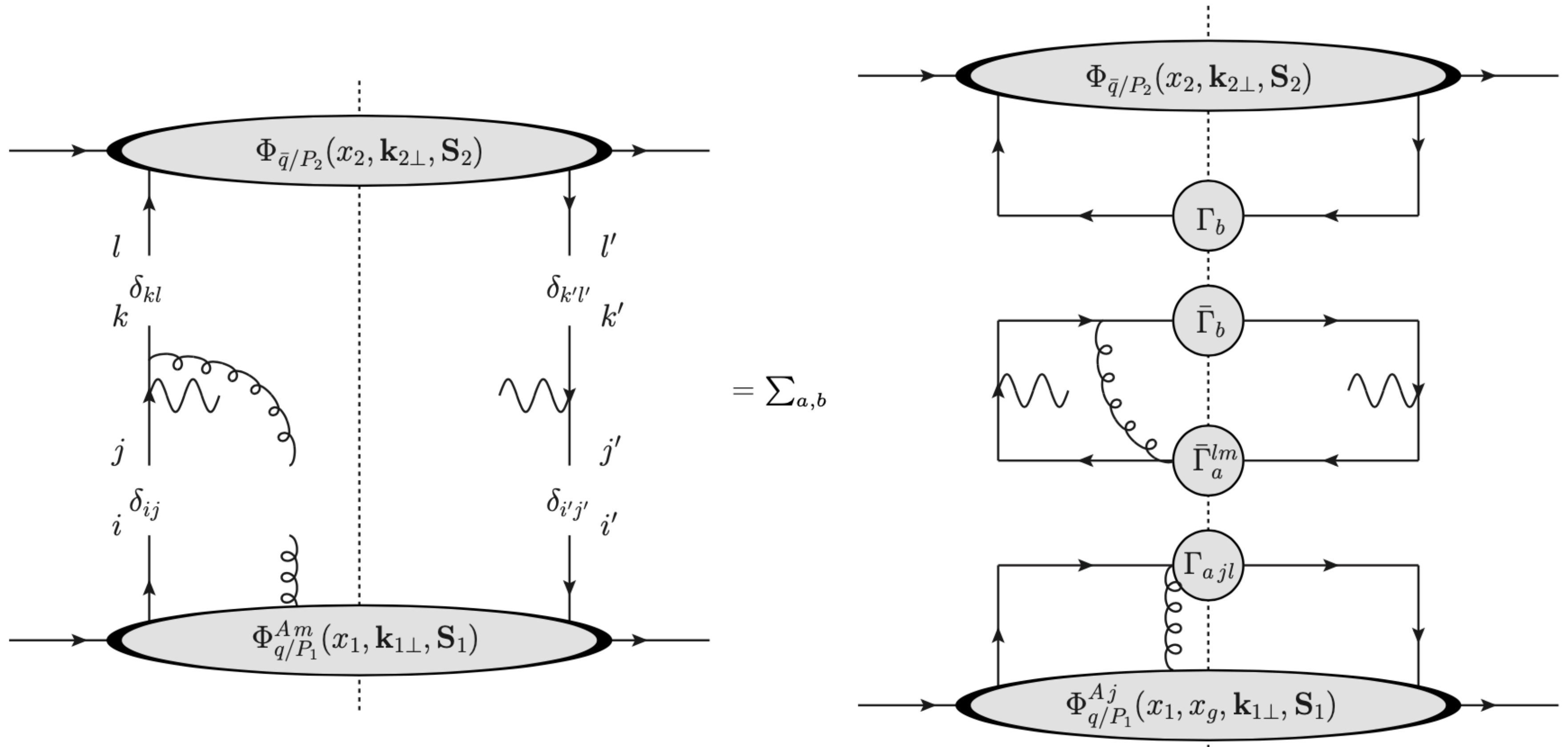


FIG. 4. Fierz decomposition of the dynamic sub-leading contribution to the cross section. In this graph, m represents a transverse Lorentz index.

DY/SIDIS tree-level diagrams relevant for sub-leading-power observables diagrams "dynamical" qgq contributions

$$\Phi^i_A(x, x_g, \boldsymbol{k}_\perp, \boldsymbol{S}) = \frac{1}{x_g P^+} \Phi^i_F(x, x_g, \boldsymbol{k}_\perp, \boldsymbol{S})$$

Dynamical

$$\Phi_A^i(x, x_g, \boldsymbol{k}_T, S)$$

SIDIS tree-level diagrams relevant for sub-leading-power observables.
diagrams "*dynamical*" contributions with

Generalization of

- ✦Mulders Tangerman NPB1995
- ✦ Boer Pijlman Mulders NPB 2003
- ✦Bacchetta et al 2007 JHEP

Subleading Quark-Gluon-Quark TMDPDFs

Nucleon Polarization	Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
U	$\tilde{f}^\perp,\ \tilde{g}^\perp$	$\tilde{e}\ ,\ \tilde{h}$
L	$\tilde{f}_L^\perp,\ \tilde{g}_L^\perp$	$\tilde{e}_L,\ \tilde{h}_L$
T	$\tilde{f}_T,\ \tilde{f}_T^\perp,\ \tilde{g}_T,\ \tilde{g}_T^\perp$	$\tilde{e}_T,\ \tilde{e}_T^\perp,\ \tilde{h}_T,\ \tilde{h}_T^\perp$

$$
\begin{aligned}
& x_g P^+\, \Phi_A^i\left(x, x_g, \boldsymbol{k}_\perp, \boldsymbol{S}\right) = \\
& \frac{xM}{2}\Bigg\{\Bigg[\left(\tilde{f}^\perp - i\tilde{g}^\perp\right)\frac{k_\perp^i}{M} - \left(\tilde{f}_T' + i\tilde{g}_T'\right)\epsilon_{\perp\, jl} S_\perp^l \\
& \quad - \left(\lambda\tilde{f}_L^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{f}_T^\perp\right)\frac{\epsilon_{\perp\, jl}k_\perp^l}{M} - i\left(\lambda\tilde{g}_L^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{g}_T^\perp\right)\frac{\epsilon_{\perp\, jl}k_\perp^l}{M}\Bigg]\left(g_\perp^{ij} - i\epsilon_\perp^{ij}\gamma_5\right) \\
& \quad - \left[\left(\lambda\tilde{h}_L^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{h}_T^\perp\right) + i\left(\lambda\tilde{e}_L^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{e}_T^\perp\right)\right]\gamma_\perp^i\gamma_5
\end{aligned}
$$

Challenges of *SLP/NLP* TMDs

Various sources for power suppressed terms identified and discussed in the literature from

Tree level Studies, Mulders, Tangerman (1996), Bacchetta et al. JHEP (2007)

- This includes corrections associated to kinematic prefactors involving contractions between the leptonic and hadronic tensors, referred to as ***kinematic power corrections***

- Another involve subleading terms in quark-quark correlators involving Dirac structures that differ from LP ones referred to as ***intrinsic power corrections***— *e.g. Cahn function* $f^{\perp}(x, k_T)$, $e(x, k_T)$ …

- Another from hadronic matrix elements of (interaction dependent) quark-gluon-quark operators, referred to ***dynamic power corrections*** multi-parton *qgq* correlators

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) + \Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S})$$

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Next step Factorization: express in terms of good and bad LC fields @ tree level

- Terms of the correlation function, traces of quark correlation functions with the Γ^a operators entered,

$$\Phi^{\Gamma^a}\left(x_1, \boldsymbol{k}_T, \boldsymbol{S}\right) \equiv \mathrm{Tr}\left[\Phi\left(x_1, \boldsymbol{k}_T, \boldsymbol{S}\right)\Gamma^a\right]$$

where $$\Phi_{q/P_1\, jj'}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \int \frac{d^4\xi}{(2\pi)^3} e^{ik\cdot\xi}\, \delta\left(\xi^+\right) \left\langle P, \boldsymbol{S} \left| \bar{\psi}^c_{j'}(0)\, \mathcal{U}^{\bar{n}}_{\llcorner}(0)\, \mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\, \psi^c_j(\xi) \right| P, \boldsymbol{S} \right\rangle$$

- To separate the contributions of hadronic tensor at LP & SLP, employ light-cone projections of the Dirac fields, "good" and "bad (power surpressed)" $\lambda = q_\perp / Q$ light-cone components

$$\psi^c = \chi^c + \phi^c$$

$$\chi^c(x) = \frac{\bar{n}\!\!\!/ \, n\!\!\!/}{4}\psi^c(x)\,, \qquad \varphi^c(x) = \frac{n\!\!\!/ \, \bar{n}\!\!\!/}{4}\psi^c(x)$$

- Upon expressing ψ^c in terms of ϕ^c and χ^c in the correlation function, four field configurations enter into the position space matrix elements,

2 good twist 2 — 1 good 1 bad twist 3 — 2 bad twist 4

$$\langle P, \boldsymbol{S}|\bar{\chi}^c_{j'}\,\chi^c_j|P, \boldsymbol{S}\rangle,\ \langle P, \boldsymbol{S}|\bar{\varphi}^c_{j'}\,\chi^c_j|P, \boldsymbol{S}\rangle,\ \langle P, \boldsymbol{S}|\bar{\chi}^c_{j'}\,\varphi^c_j|P, \boldsymbol{S}\rangle,\ \text{and}\ \langle P, \boldsymbol{S}|\bar{\varphi}^c_{j'}\,\varphi^c_j|P, \boldsymbol{S}\rangle$$

EOMs and kinematic (Suppressed) Distributions

- Employ the QCD equations of motion to demonstrate the appearance of the "kinematic sub-leading distributions"

$$\frac{i\not{D}_\perp(\xi)}{in\cdot D(\xi)}\frac{\not{n}}{2}\chi^c(\xi) = \varphi^c(\xi)$$

$$\Phi^{\mathrm{int}\,[\Gamma^a]}_{q/P}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\langle P,\boldsymbol{S}\,|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\varphi^c(\xi)|\,P,\boldsymbol{S}\rangle + \langle P,\boldsymbol{S}\,|\bar{\varphi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\chi^c(\xi)|\,P,\boldsymbol{S}\rangle\right]$$

$$\Phi^{\mathrm{int}\,[\Gamma^a]}_{q/P\,\mathrm{A}}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \frac{1}{k^+}\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left\langle P,\boldsymbol{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,i\not{D}_\perp(\xi)\frac{\not{n}}{2}\chi^c(\xi)\right|P,\boldsymbol{S}\right\rangle$$

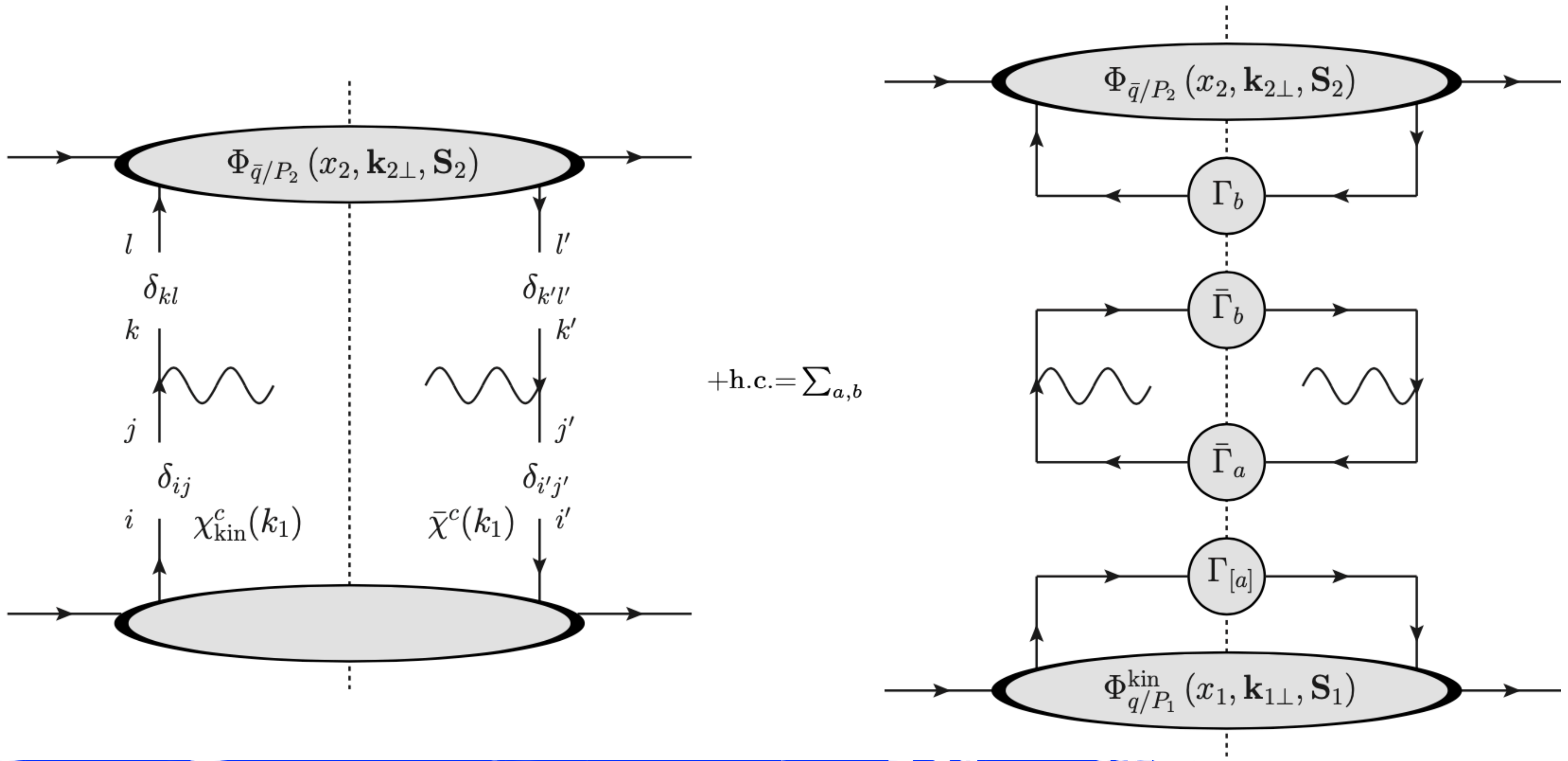
$$\Phi^{\mathrm{int}\,[\Gamma^a]}_{q/P\,\mathrm{A}}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \frac{1}{k^+}\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left\langle P,\boldsymbol{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\not{k}_\perp\frac{\not{n}}{2}\chi^c(\xi)\right|P,\boldsymbol{S}\right\rangle + \frac{ig}{k^+}\int d\eta^-\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left\langle P,\boldsymbol{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\eta)\,F^{j+}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-;\xi^+,\boldsymbol{\xi}_\perp\right)\gamma_j\frac{\not{n}}{2}\chi^c(\xi)\right|P,\boldsymbol{S}\right\rangle$$

$$\Phi^{\mathrm{kin}}_{q/P\,jj'}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\langle P,\boldsymbol{S}\,|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\chi^c_{\mathrm{kin\,j}}(\xi) + \bar{\chi}^c_{\mathrm{kin\,j'}}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\chi^c_j(\xi)|\,P,\boldsymbol{S}\rangle\right]$$

$$\chi^c_{\mathrm{kin}}(\xi) = \frac{\not{k}_\perp}{k^+}\frac{\not{n}}{2}\chi^c(\xi)$$

EOMs and kinematic Suppressed Distributions

- Employ the QCD equations of motion to demonstrate the appearance of the "kinematic sub-leading distributions"



$$\Phi^{\rm kin}_{q/P\,jj'}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \sum_a \bar\Gamma^a_{jj'} \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left\langle P,\boldsymbol{S}\left|\bar\chi^c(0)\,\mathcal{U}^{\bar n}_{\rm L}(0)\,\Gamma^{[a]}\,\mathcal{U}^{\bar n\,\dagger}_{\rm L}(\xi)\,\chi^c(\xi)\right|P,\boldsymbol{S}\right\rangle$$

where $\Gamma^{[a]} = [\Gamma^a, \not k_\perp \not n/2k^+]$

Subleading fields and correlator(s) Summary

Three possible sub-leading field configurations. They are related through the QCD EOM

$$\varphi^c(x) = \frac{\bar{\not n}\not n}{4}\psi^c(x) \qquad \chi^c(x) = \frac{\not n\bar{\not n}}{4}\psi^c(x) \qquad \varphi^c(x) = -\frac{\not n}{2}\frac{\not D_\perp}{n\cdot D}\chi^c(x)$$

Using properties of the Wilson lines, the relevant collinear functions are given by

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta(\xi^+)\left[\langle P,\mathbf{S}\,|\bar\chi^c_{j'}(0)\,\mathcal{U}^{\bar n}_{\llcorner}(0)\,\mathcal{U}^{\bar n\,\dagger}_{\llcorner}(\xi)\,\varphi^c_j(\xi)|\,P,\mathbf{S}\rangle + \text{h.c.}\right]$$

$$\Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \frac{ig}{k^+}\int d\eta^-\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta(\xi^+)$$
$$\times\left[\left\langle P,\mathbf{S}\left|\bar\chi^c(0)\,\mathcal{U}^{\bar n}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar n\,\dagger}_{\llcorner}(\eta)\,F^{i+}(\eta)\,\mathcal{U}^{\bar n}(\eta^-,\xi^-;\xi^+,\xi_\perp)\,\gamma_i\frac{\not n}{2}\chi^c(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

$$\Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta(\xi^+)\left[\left\langle P,\mathbf{S}\left|\bar\chi^c_{j'}(0)\,\mathcal{U}^{\bar n}_{\llcorner}(0)\,\frac{i\partial^i_\perp}{in\cdot D}\,\mathcal{U}^{\bar n\,\dagger}_{\llcorner}(\xi)\,\frac{\bar{\not n}}{2}\gamma^\perp_i\chi^c_{\rm kin\,j}(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) + \Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S})$$

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Subleading fields and correlator(s) Alternative

Three possible sub-leading field configurations. They are related through the QCD EOM

$$\varphi^c(x) = \frac{\not{n}\not{\bar{n}}}{4}\psi^c(x) \qquad \chi^c(x) = \frac{\not{\bar{n}}\not{n}}{4}\psi^c(x) \qquad \varphi^c(x) = -\frac{\not{n}}{2}\frac{\not{D}_\perp}{n\cdot D}\chi^c(x)$$

Using properties of the Wilson lines, the relevant collinear functions are given by

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\varphi^c_j(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$ [9]

$$\Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \frac{ig}{k^+}\int d\eta^-\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)$$
$$\times\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\eta)\,F^{i+}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-;\xi^+,\xi_\perp\right)\,\gamma_i\frac{\not{n}}{2}\chi^c(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

$$\Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\frac{i\partial^i_\perp}{in\cdot D}\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\frac{\not{\bar{n}}}{2}\gamma^\perp_i\,\chi^c_{{\rm kin}\,j}(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) + \Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S})$$

[9]Ebert, Gao, Stewart 2021

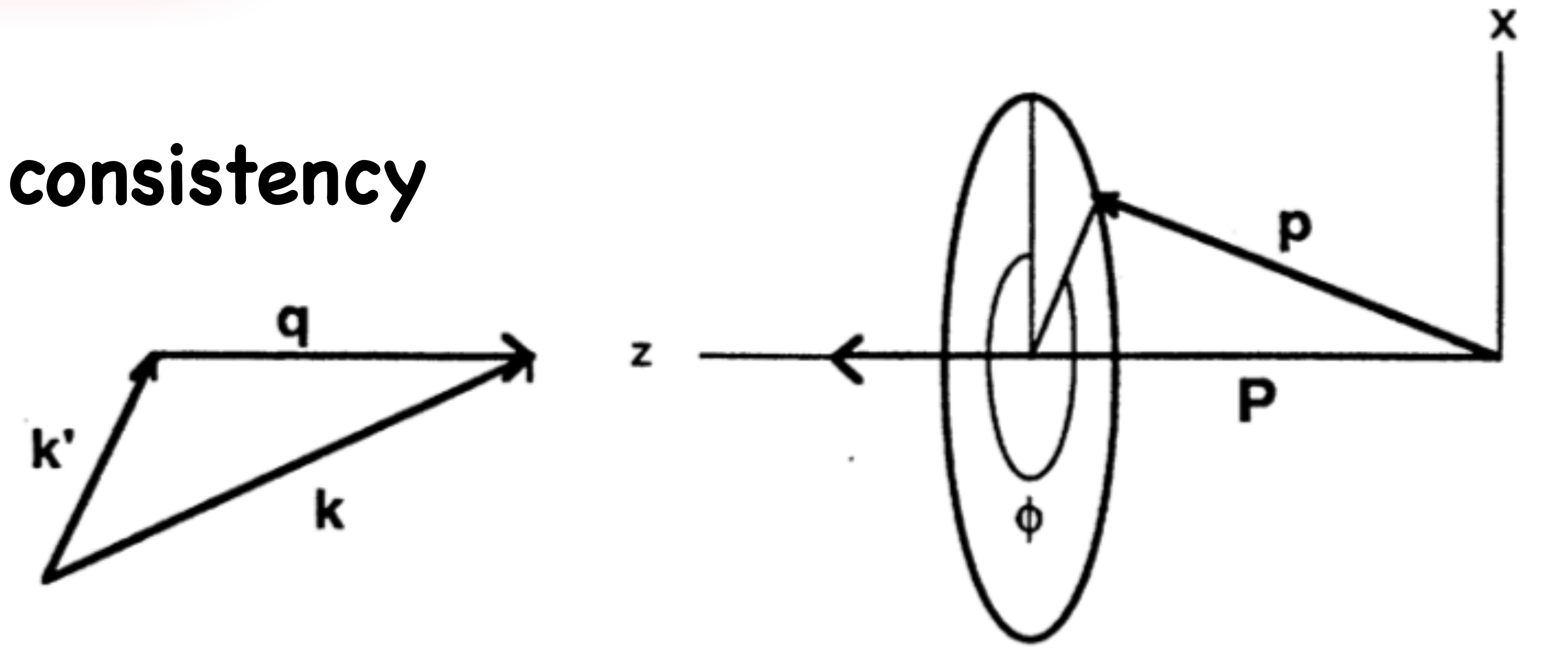
Tree level factorization sub-leading power

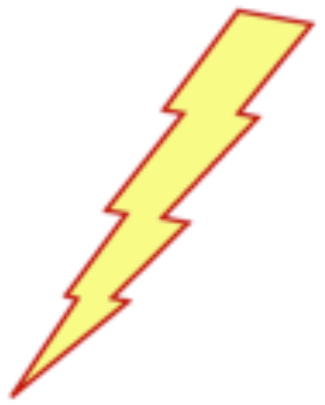
Combining these contributions and multiplying by leptonic tensor get factorized
Cahn and more .... Includes dynamical "tilde" contributions
Using "intrinsic & dynamical" basis

$$F^3_{\rm DIS}(x,z,\boldsymbol{P}_{h\perp}) = \mathcal{C}^{\rm DIS}\left[\frac{q_\perp}{Q} f_1\, D_1\right] - \mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp\right) D_1 - f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} D^\perp\right)\right] - \int\frac{dx_g}{x_g}\mathcal{C}^{\rm DIS}_{\rm dyn\, x_g}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp\right) D_1\right] + \int\frac{dz_g}{z_g}\mathcal{C}^{\rm DIS}_{\rm dyn\, z_g}\left[f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}\tilde{D}^\perp\right)\right],$$

Cahn and more intrinsic k_T

Slightly different setup then Bacchetta et al 2007 allows us to check RG consistency
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209

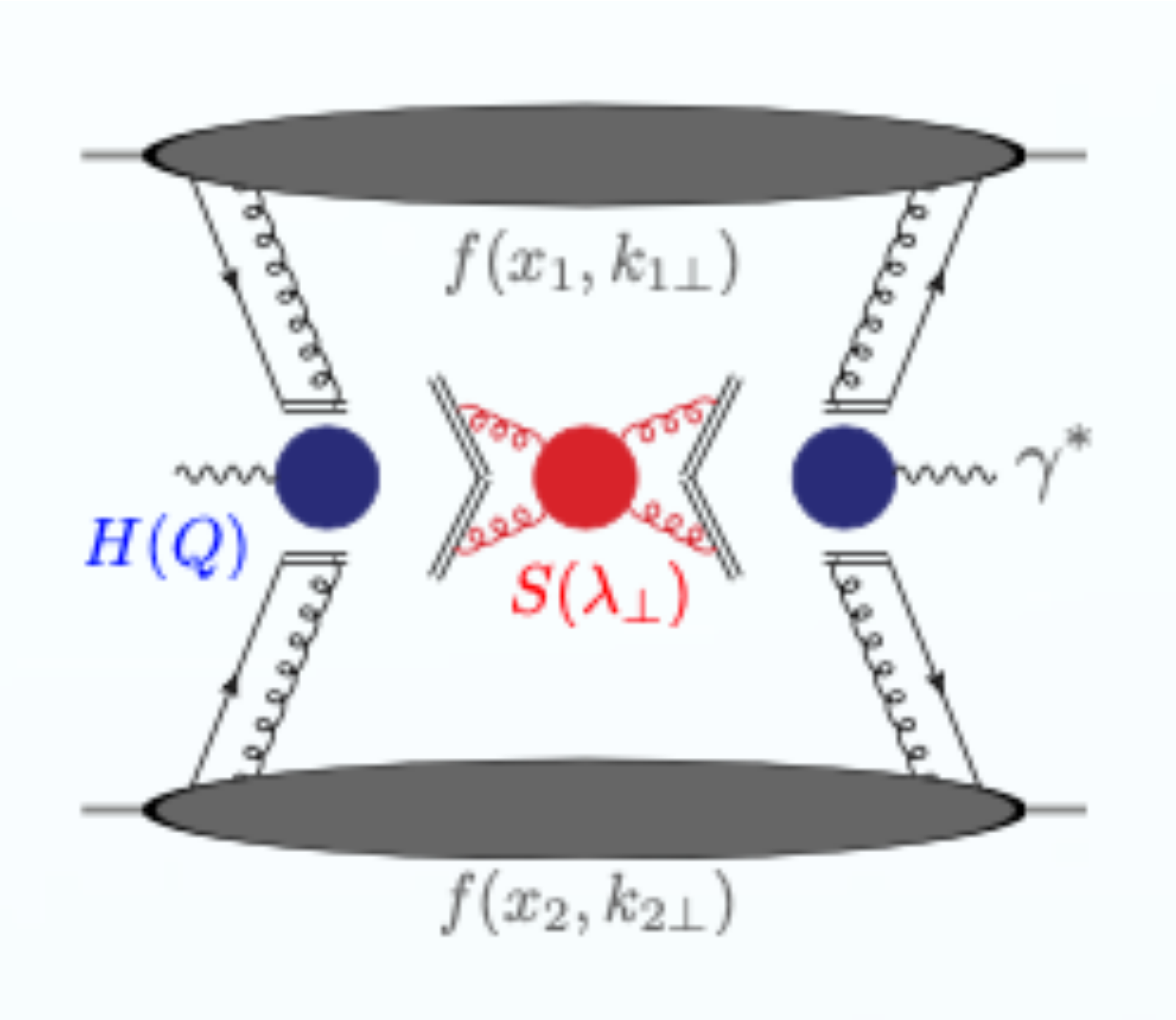


To understand appreciate the subtleties  review
Tree level TMD @ LP and NLP factorization

In reviewing will remind about the utility of using
Fierz decamp & "good and bad" LC quark fields "

Then onto Factorization at NLO address soft factor calculation

TMD factorization at NLO and NLP



$q_T \sim k_T \ll Q$

TMD Factorization beyond LO in QCD

✦*Collins Soper Sterman NPB 1985*

✦*Ji Ma Yuan PRD PLB …2004, 2005*

✦*Aybat Rogers PRD 2011*

✦*Collins 2011 Cambridge Press*

✦*Echevarria, Idilbi, Scimemi JHEP 2012, …*

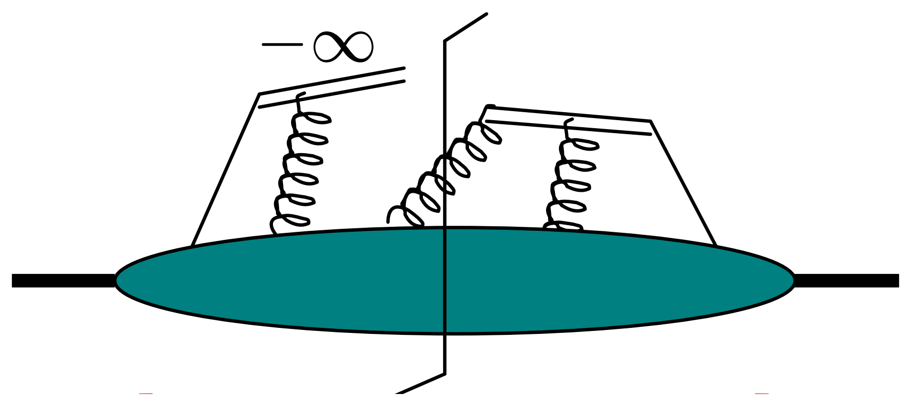
✦*SCET Becher & Neubert, 2011 EJPC*

$$\frac{d\sigma^W}{dQ^2\, dx_F\, dp_T^2} = \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{p}_T\cdot \boldsymbol{b}_T}\, \tilde{W}(x_F, b_T, Q)$$

$$\tilde{W}(x_F, b_T, Q) = \sum_j H_{j\bar{j}}^{\mathrm{DY}}(Q, \mu, a_s(\mu))\, \tilde{f}_{j/A}(x_A, b_T; \zeta_A, \mu)\, \tilde{f}_{\bar{j}/B}(x_B, b_T; \zeta_B, \mu)$$

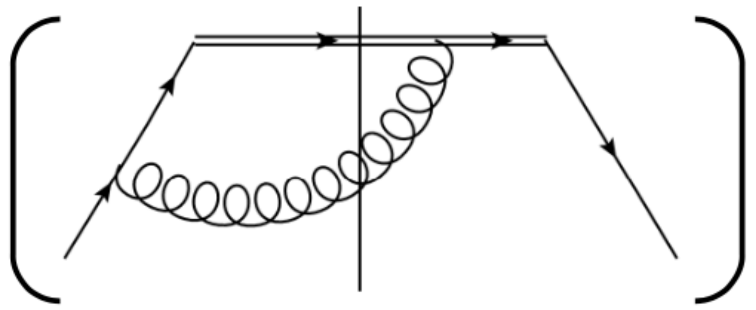
Each factor is regularized where the total cs is independent of of UV and rapidity μ , ζ reg. scales

Rapidity Divergence TMD



Unregulated divergence when $k^+ = P^+$

$$\int dk^- \frac{\mathrm{Tr}_C}{N}\frac{\mathrm{Tr}_D}{4}$$

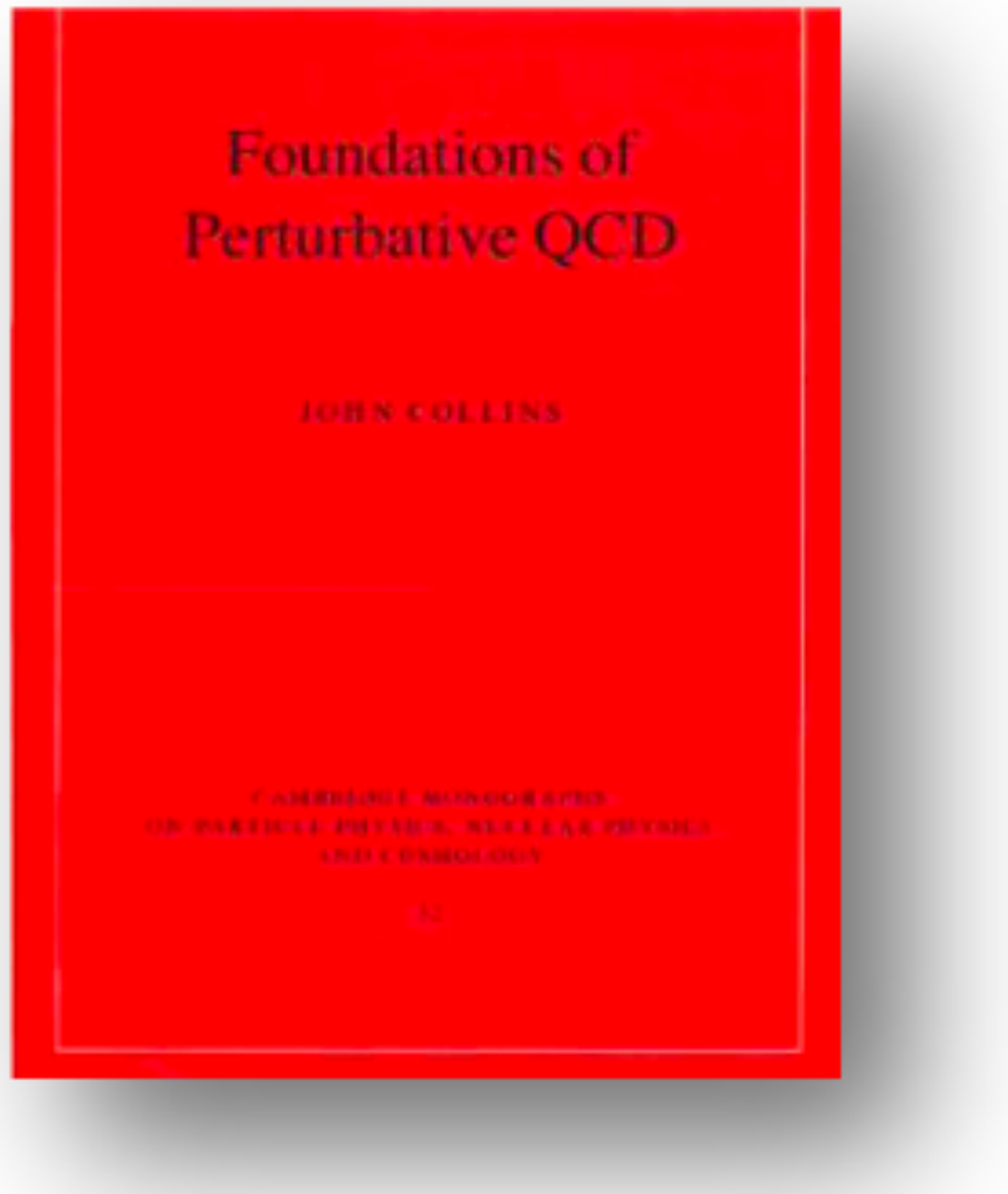


$$\sim a_S \frac{1}{k_T^2}\frac{1}{1-\xi}$$

$$\tilde{f}^{\mathrm{sub}}_{j/H}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}^{\mathrm{unsub}}_{j/H}(x, b_T; \mu, y_P - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

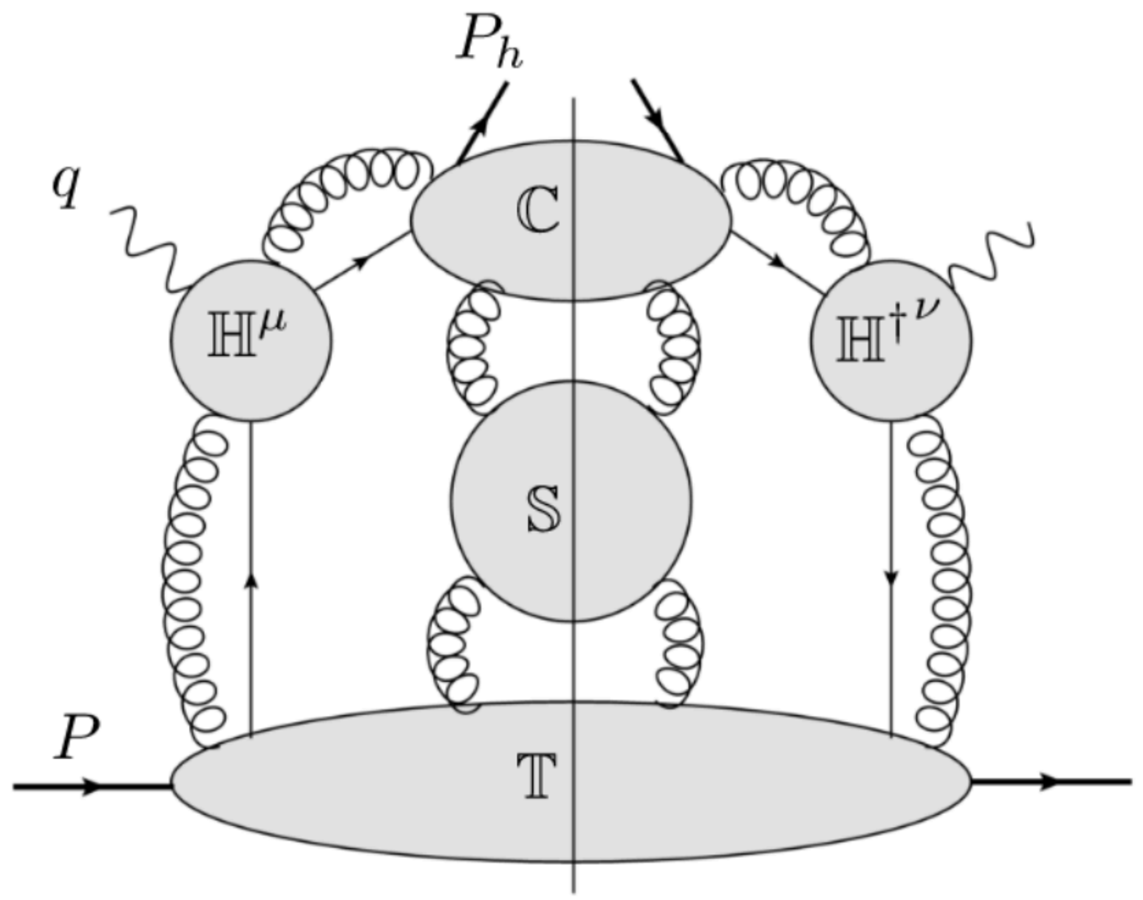
$$\Updownarrow$$

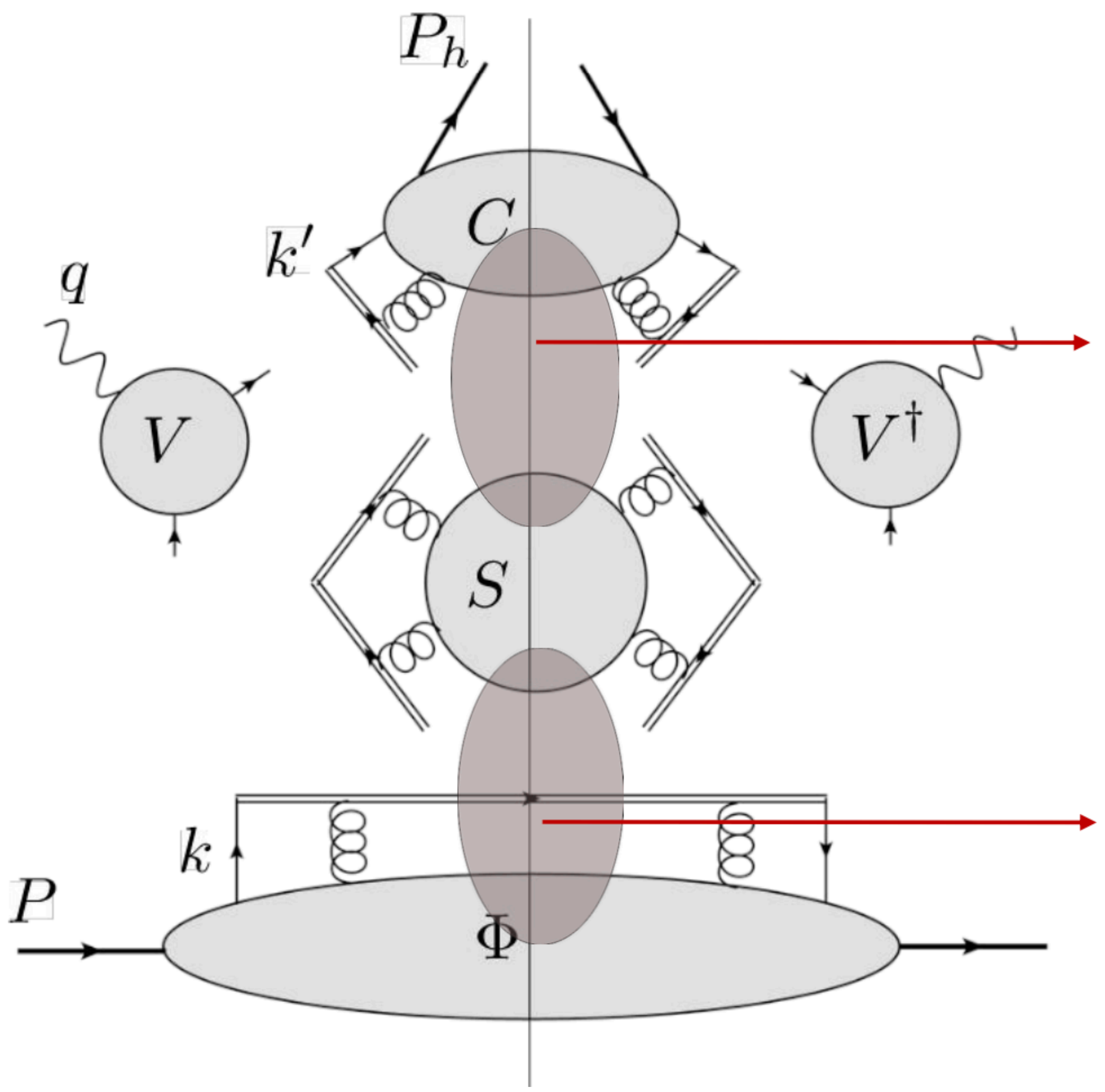
$$\tilde{f}^{\mathrm{unsub}}_{j/H}(x, b_T; \mu, y_P - y_B) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}(0)\gamma^+\mathcal{U}_{[0,b]}\psi(b)|P\rangle\big|_{b^+=0}$$



TMD factorization at NLO @LP

$$\frac{d\sigma^W}{dQ^2\,dx_F\,dp_T^2} = H_{\text{SIDIS}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T} \frac{\Phi^{[\Gamma]}(x,b_T)}{S(b_T,y_1-(-\infty))} S(b_T,y_1-y_2) \frac{C^{[\Gamma']}(z,b_T)}{S(b_T,+\infty-y_2)}$$





$S(b_T,+\infty-y_2) = \langle 0|W_C(b_T,+\infty-y_2)|0\rangle$

$S(b_T,y_1-(-\infty)) = \langle 0|W_C(b_T,y_1-(-\infty))|0\rangle$

Pics courtesy of Andrea Simonelli

Renormalization and TMD Evolution- $\{\zeta, \mu\}$

- **Collins Soper Eq.**

$$\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

$$\tilde{K}(b_T, \mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, -\infty)}$$

- **RGE for C.S. kernel**

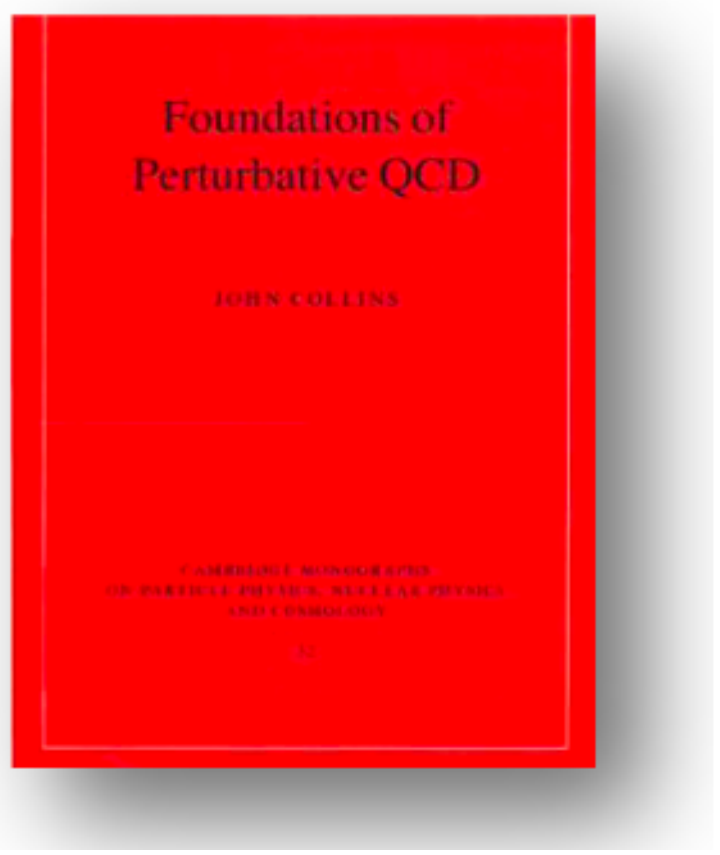
$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_k(\alpha_s(\mu))$$

- **RGE for TMD**

$$\frac{d \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(\alpha_s(\mu), \zeta/\mu)$$

Solve simultaneously and get evolved renormalized TMD $\rightarrow \zeta = Q^2, \quad \mu = \mu_Q \sim Q$

See Marco Radici's and Yong Zhao talks



$$\tilde{f}_1(x, b_T, Q^2, \mu_Q) \sim \left[\tilde{C}^{f_1}\left(x/\hat{x}, \boldsymbol{b}_*; \mu_{b_*}^2, \mu_{b_*}, \alpha(\mu_{b_*})\right) \otimes f_1(\hat{x}, \mu_{b_*})\right]$$

$$\times \exp\left[-S_{pert}(\mu_{b_*}(b_T); \mu_{b_*}, Q^2) - S_{NP}(b_T, Q)\right]$$

Factorization & resummation at NLO and NLP

Beyond tree level

Note first attempt Bacchetta Boer Diehl Mulders JHEP 2008

- We perform one loop calculation
- & attempt to establish renormalization group consistency: Regions hard,soft,collinear

$$F^3_{\mathrm{DIS}}\left(x,z,\boldsymbol{P}_{h\perp}\right)=\mathcal{C}^{\mathrm{DIS}}\left[\frac{q_\perp}{Q}f_1\,D_1\right]-\mathcal{C}^{\mathrm{DIS}}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}f^\perp\right)D_1-f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}D^\perp\right)\right]$$
$$-\int\frac{dx_g}{x_g}\mathcal{C}^{\mathrm{DIS}}_{\mathrm{dyn\,x_g}}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp\right)D_1\right]+\int\frac{dz_g}{z_g}\mathcal{C}^{\mathrm{DIS}}_{\mathrm{dyn\,z_g}}\left[f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}\tilde{D}^\perp\right)\right],$$

$$F^3_{\mathrm{DIS}}\left(x,z,\boldsymbol{P}_{h\perp}\right)=H^{\mathrm{LP}}_{\mathrm{DIS}}(Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\frac{q_\perp}{Q}f_1\,D_1\,\mathcal{S}^{\mathrm{LP}}\right]$$
$$\dots\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}f^\perp D_1-\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}f_1\,D^\perp\right)\mathcal{S}^{\mathrm{int}}\right]$$
$$\dots\int\frac{\dots}{x_g}\dots,Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp D_1\,\mathcal{S}^{\mathrm{dyn}}\right]$$
$$+\int\frac{dz_g}{z_g}H^{\mathrm{dyn}}_{\mathrm{DIS}}(z_g,Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}f_1\,\tilde{D}^\perp\mathcal{S}^{\mathrm{dyn}}\right].$$

Factorization & resummation at NLO and NLP

Beyond tree level

Note first attempt Bacchetta Boer Diehl Mulders JHEP 2008

- We perform one loop calculation

  & attempt to establish renormalization group consistency:

  Regions hard,soft,collinear

$$
\begin{aligned}
F^3_{\mathrm{DIS}}(x,z,\boldsymbol{P}_{h\perp}) = {} & H^{\mathrm{LP}}_{\mathrm{DIS}}(Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\frac{q_\perp}{Q} f_1\, D_1\, \mathcal{S}^{\mathrm{LP}}\right] \\
& - H^{\mathrm{int}}_{\mathrm{DIS}}(Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, D^\perp\right)\mathcal{S}^{\mathrm{int}}\right] \\
& - \int \frac{dx_g}{x_g} H^{\mathrm{dyn}}_{\mathrm{DIS}}(x_g,Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp D_1\, \mathcal{S}^{\mathrm{dyn}}\right] \\
& + \int \frac{dz_g}{z_g} H^{\mathrm{dyn}}_{\mathrm{DIS}}(z_g,Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, \tilde{D}^\perp \mathcal{S}^{\mathrm{dyn}}\right].
\end{aligned}
$$

- H^{LP}, H^{int} and H^{dynam} represent LP, intrinsic NLP, and dynamic NLP hard functions.
- Additionally, S^{LP}, S^{int} and S^{dyn} denote the LP, intrinsic sub-leading power, and dynamic sub-leading power soft function
- ***NB* if soft factors are different universality of TMDs breaks down. Global analysis w/ NLP observables hopeless**

NLO-calculation-factorization
Necessary but not sufficient condition to establish factorization

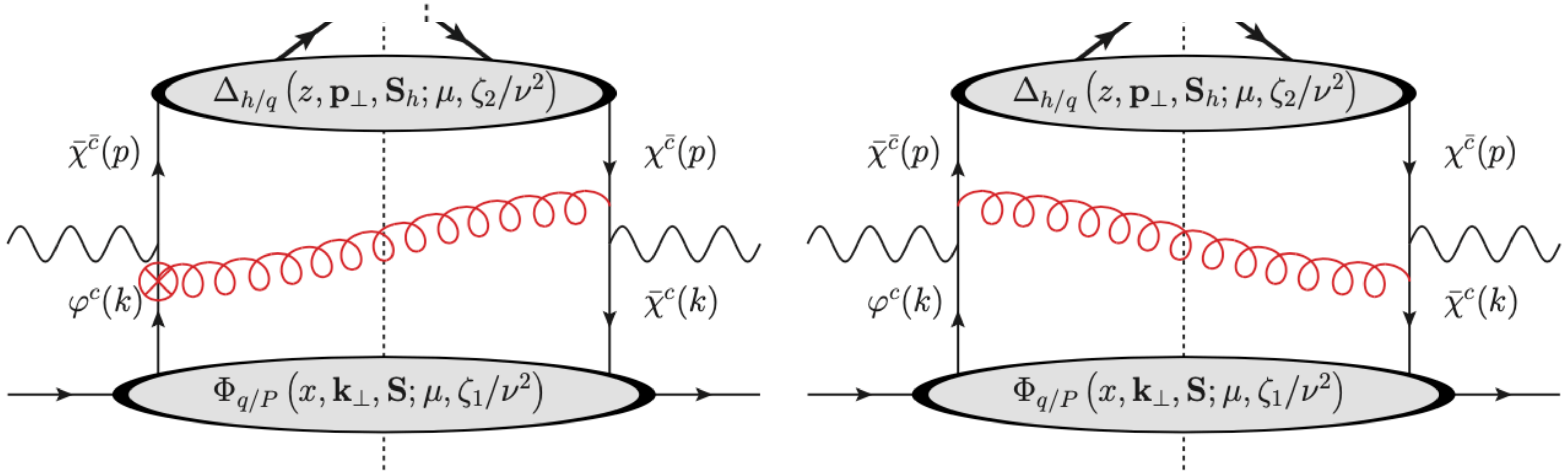
Recipe

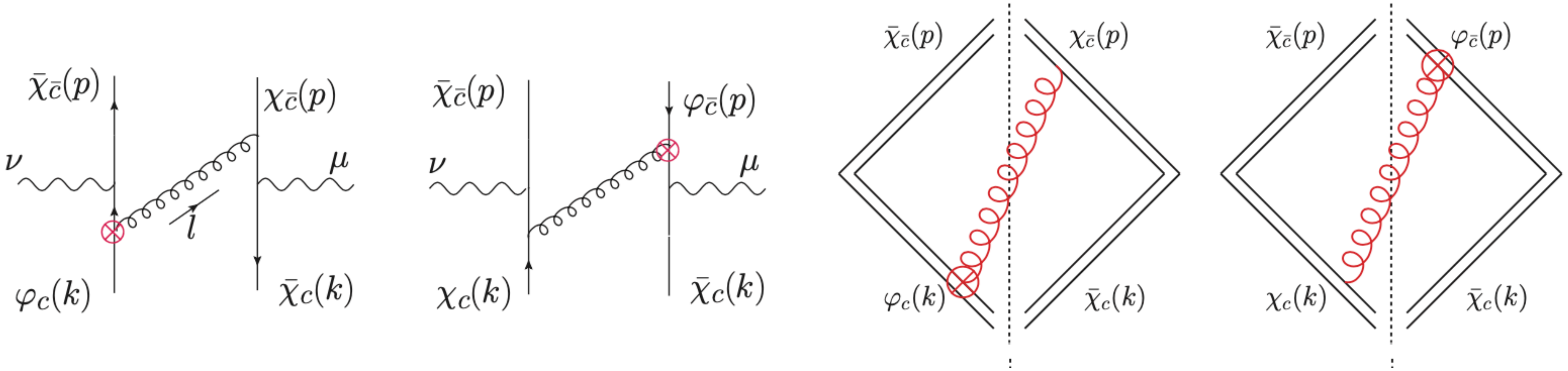
- Calculate: soft, collinear (and anti), & hard
- Renormalize
  - Exploit properties of good and bad fields & power counting
- Check renormalization group consistency

NLO Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section





$$\hat{\mathcal{S}}^{\rm LP}(b;\mu,\nu) = Z_{\mathcal{S}\,\rm LP}(b;\mu,\nu)\mathcal{S}^{\rm LP}(b;\mu,\nu)$$

$$\hat{\mathcal{S}}^{\rm NLP}(b;\mu,\nu) = Z_{\mathcal{S}\,\rm NLP}(b;\mu,\nu)\mathcal{S}^{\rm NLP}(b;\mu,\nu)$$

$$\frac{\partial}{\partial \ln\mu}\mathcal{S}^{\rm NLP}(b,\mu,\nu) = \Gamma^{\mu}_{\mathcal{S}\,NLP}\mathcal{S}^{\rm NLP}(b,\mu,\nu)$$

$$\frac{\partial}{\partial \ln\nu}\mathcal{S}^{\rm NLP}(b,\mu,\nu) = \Gamma^{\nu}_{\mathcal{S}\,NLP}\mathcal{S}^{\rm NLP}(b,\mu,\nu)$$

$$\Gamma^{\nu}_{\mathcal{S}\,\rm int} = \frac{\partial}{\partial \ln\nu} Z_{\mathcal{S}\,NLP}(b;\mu,\nu)$$

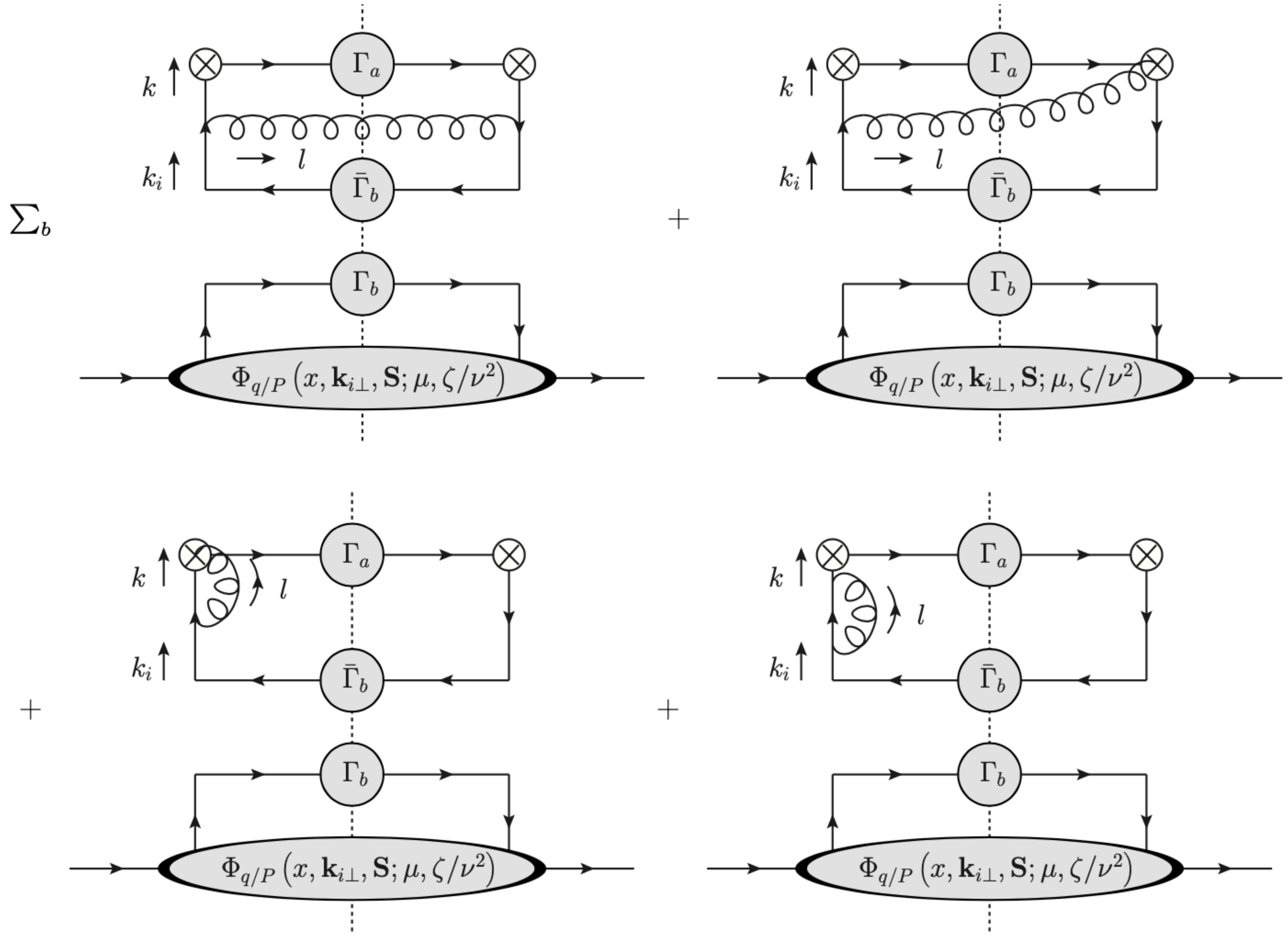
Gamberg, Kang, Shao, Terry, Zhao
arXiv: e-Print:221.13209

Soft emission from sub-leading fields vanish —> NLO + NLP soft function is half the LP one

$$\Gamma^{\mu}_{\mathcal{S}\,\rm int} = \frac{1}{2}\Gamma^{\mu}_{\mathcal{S}\,\rm LP}\,, \quad \Gamma^{\nu}_{\mathcal{S}\,\rm int} = \frac{1}{2}\Gamma^{\nu}_{\mathcal{S}\,\rm LP}$$

NLO Ingredients collinear factor

Diagrams associated with the evolution of the collinear region



Renormalize TMDs: soft & UV subtraction

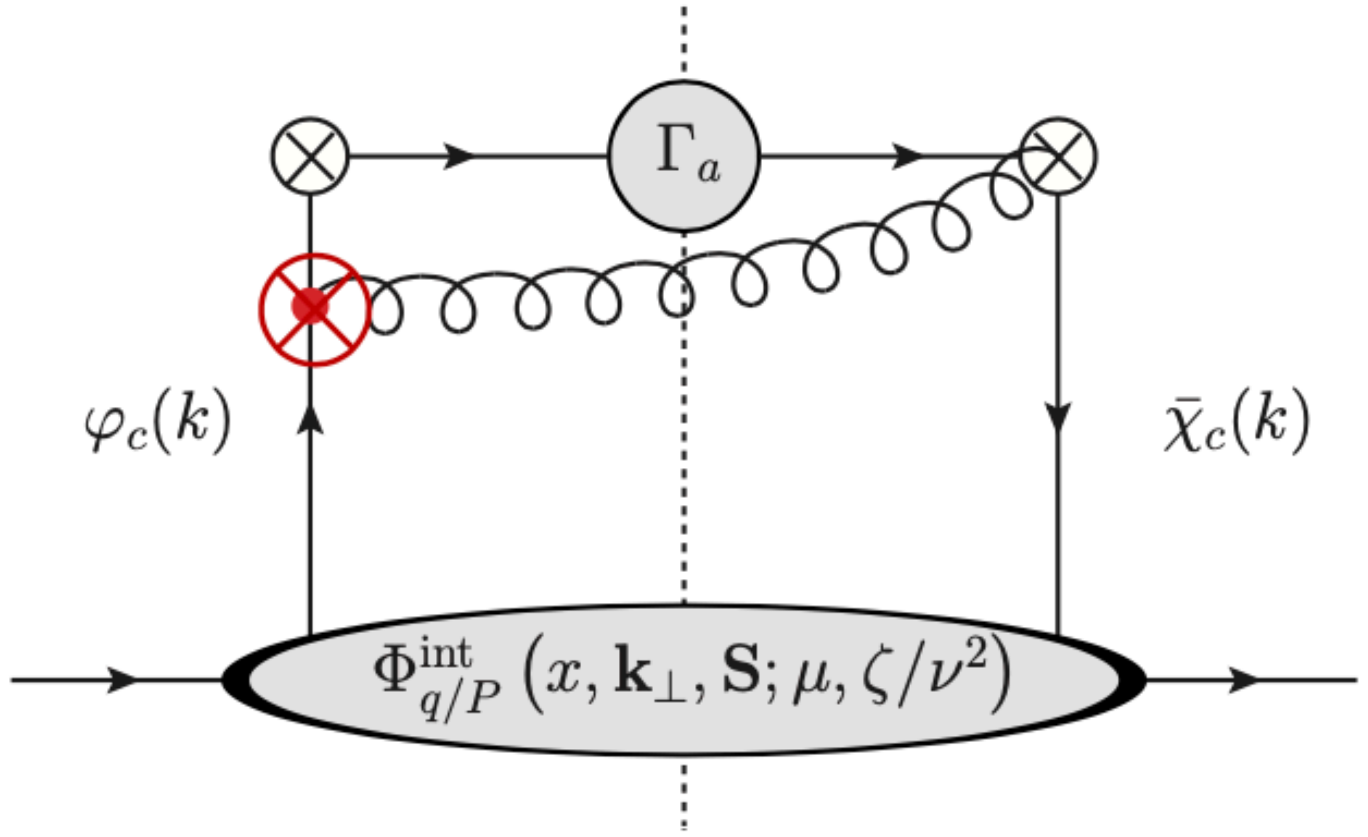
$$\hat{\Phi}^{[\Gamma^a]}\left(x, \boldsymbol{b}, \boldsymbol{S}; \mu, \zeta/\nu^2\right) = Z_{\Gamma^a\,\Gamma^b}\left(b, \mu, \zeta/\nu^2\right)\Phi^{[\Gamma^b]0}\left(x, \boldsymbol{b}, \boldsymbol{S}; xP^+\right)$$

$$\Gamma_3^\nu = \frac{\partial}{\partial \ln \nu} Z_{NLP}(b; \mu, \nu)$$

NLO Ingredients collinear factor

Differences from LP TMDs

Study the interaction of the sub-leading fields with the Wilson lines



$$\frac{\not{n}}{2}\varphi_c(k) = 0$$

$$\Gamma_3^\nu = \frac{\partial}{\partial \ln \nu} Z_{NLP}(b; \mu, \nu)$$

Can show that these interactions vanish trivially

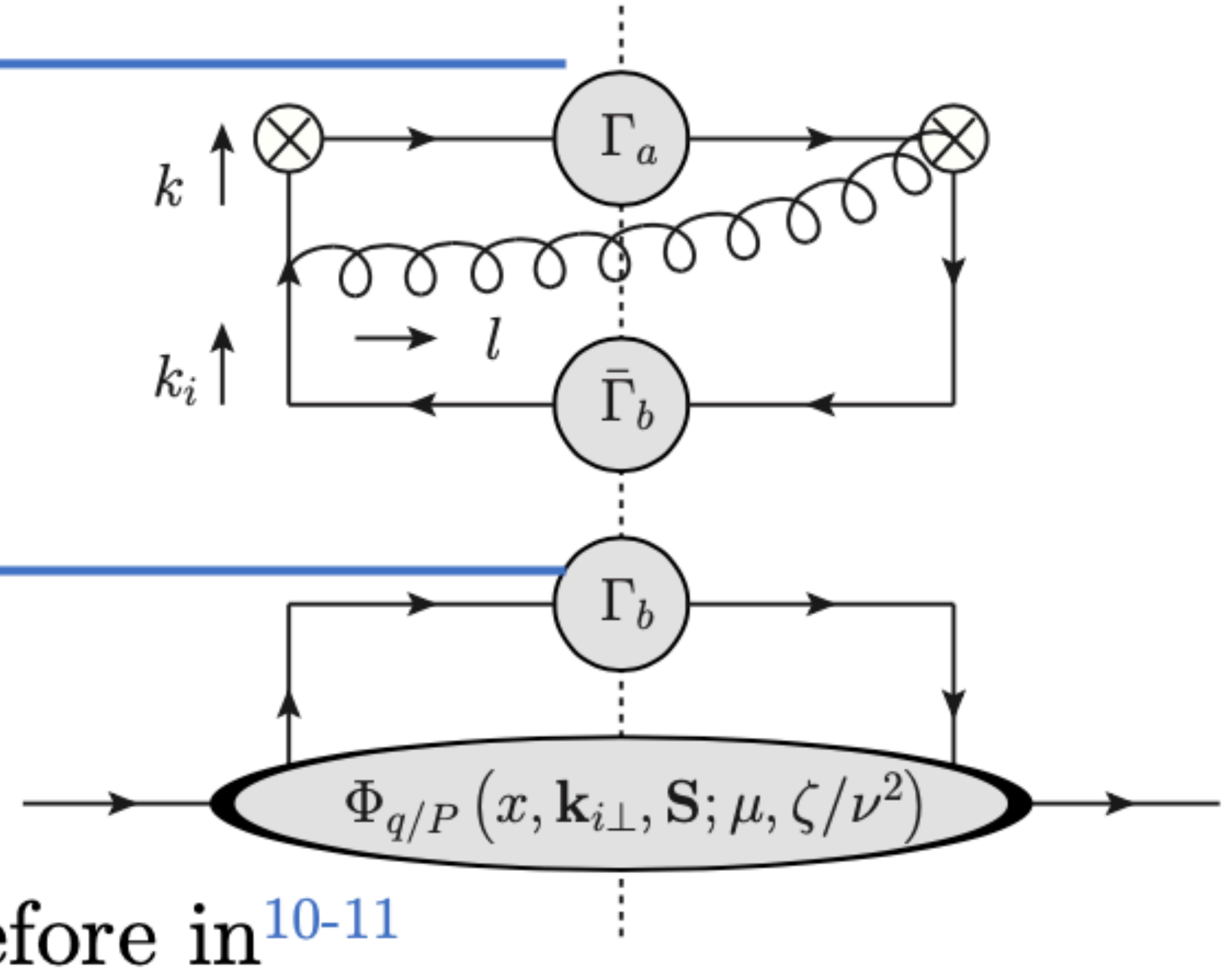
Resulting in, $\Gamma^\nu_{3\,\text{int}}(\mu, \nu, \zeta) = \frac{1}{2}\Gamma^\nu_2(\mu, \nu, \zeta)$

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Anomalous dimension matrices

Evolution equations naturally enter as matrices due to mixing

$$\frac{\partial}{\partial \ln \mu}\begin{bmatrix}\Phi^{[\not{n}]}\\ \Phi^{[\not{n}\gamma^5]}\\ \Phi^{[i\sigma^{i+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^i]}\\ \Phi^{[\gamma^i\gamma^5]}\\ \Phi^{[i\sigma^{ij}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix} = \mathbf{\Gamma}^{\mu}\begin{bmatrix}\Phi^{[\not{n}]}\\ \Phi^{[\not{n}\gamma^5]}\\ \Phi^{[i\sigma^{l+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^l]}\\ \Phi^{[\gamma^l\gamma^5]}\\ \Phi^{[i\sigma^{lm}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix} \qquad \frac{\partial}{\partial \ln \nu}\begin{bmatrix}\Phi^{[\not{n}]}\\ \Phi^{[\not{n}\gamma^5]}\\ \Phi^{[i\sigma^{i+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^i]}\\ \Phi^{[\gamma^i\gamma^5]}\\ \Phi^{[i\sigma^{ij}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix} = \mathbf{\Gamma}^{\nu}\begin{bmatrix}\Phi^{[\not{n}]}\\ \Phi^{[\not{n}\gamma^5]}\\ \Phi^{[i\sigma^{l+}\gamma^5]}\\ \Phi^{[1]}\\ \Phi^{[\gamma^5]}\\ \Phi^{[\gamma^l]}\\ \Phi^{[\gamma^l\gamma^5]}\\ \Phi^{[i\sigma^{lm}\gamma^5]}\\ \Phi^{[i\sigma^{+-}\gamma^5]}\end{bmatrix}.$$



We find operator mixing in the Collins-Soper equation. Seen before in[10-11]

$$\mathbf{\Gamma}^{\mu} = \begin{bmatrix} \Gamma_2^{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \Gamma_2^{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \Gamma_2^{\mu}\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \Gamma_3^{\mu} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \Gamma_3^{\mu} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}\delta_l^i & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu}\delta_l^i & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\Gamma_3^{\mu}\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^{\mu} \end{bmatrix}$$

$$\mathbf{\Gamma}^{\nu} = \frac{\alpha_s C_F}{2\pi}\begin{bmatrix} 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 2L\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & L & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & L & 0 & 0 & 0 & 0\\ \frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0 & 0\\ 0 & \frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0\\ 0 & 0 & \frac{i}{xP^+}\frac{\partial L}{\partial b^2}\left(b^j\delta_l^i - b^i\delta_l^j\right) & 0 & 0 & 0 & 0 & L\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0\\ 0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & 0 & L \end{bmatrix}$$

LP to LP

LP to NLP

NLP to NLP

Necessary condition rapidity RG Consistency

Review Leading power

$$f_1(x,b;\mu,\zeta_1) = f_1(x,b;\mu,\zeta_1/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)}$$

$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)}$$

$$\Gamma_2^\nu + \frac{1}{2}\Gamma_S^\nu = 0\,, \qquad \Gamma_2^\nu + \frac{1}{2}\Gamma_S^\nu = 0$$

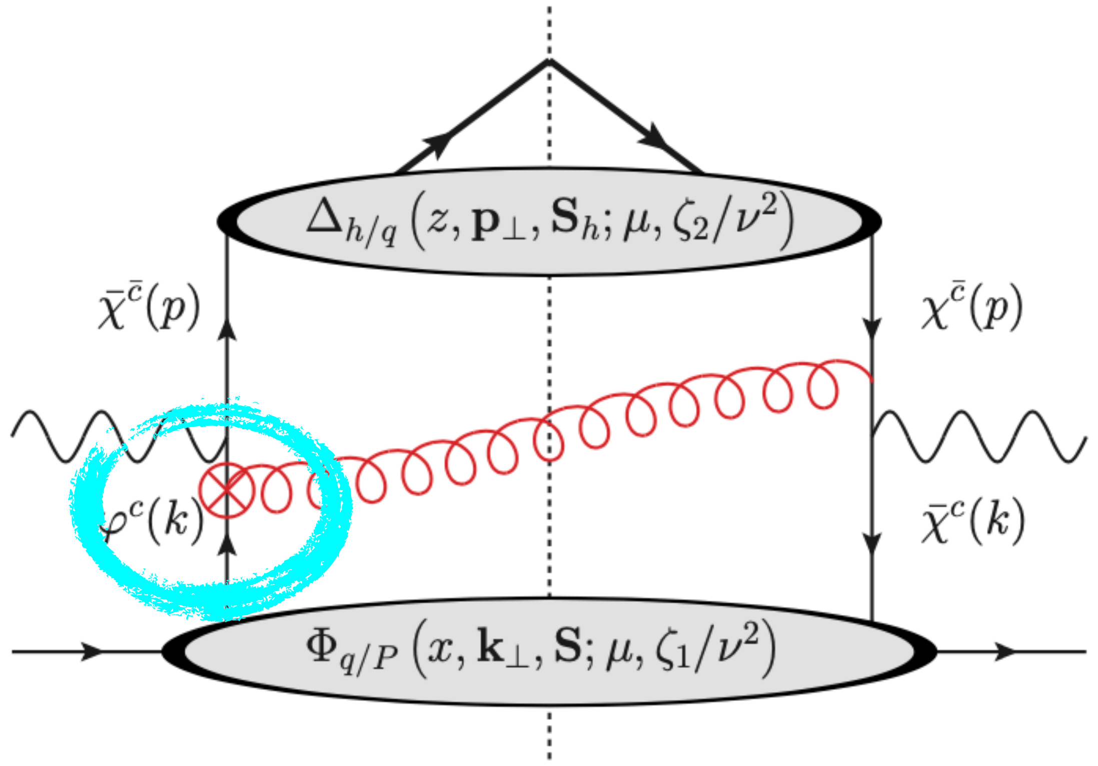
Next to leading power

$$-\,H_{\rm DIS}^{\rm int}(Q;\mu)\,\mathcal{C}^{\rm DIS}\left[\left(x\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}f_1 D^\perp\right)\mathcal{S}^{\rm int}\right]$$

$$ib^\mu M^2 f^{\perp\,(1)}(x,b;\mu,\zeta_1) = ib^\mu M^2 f^{\perp\,(1)}(x,b;\mu,\zeta_1/\nu^2)\sqrt{\mathcal{S}^{\rm int}(b;\mu,\nu)}$$

$$\Gamma_{3\,\rm int}^\nu + \frac{1}{2}\Gamma_{S\,\rm int}^\nu = 0$$

Non-trivial result



However for cross section

$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)}$$

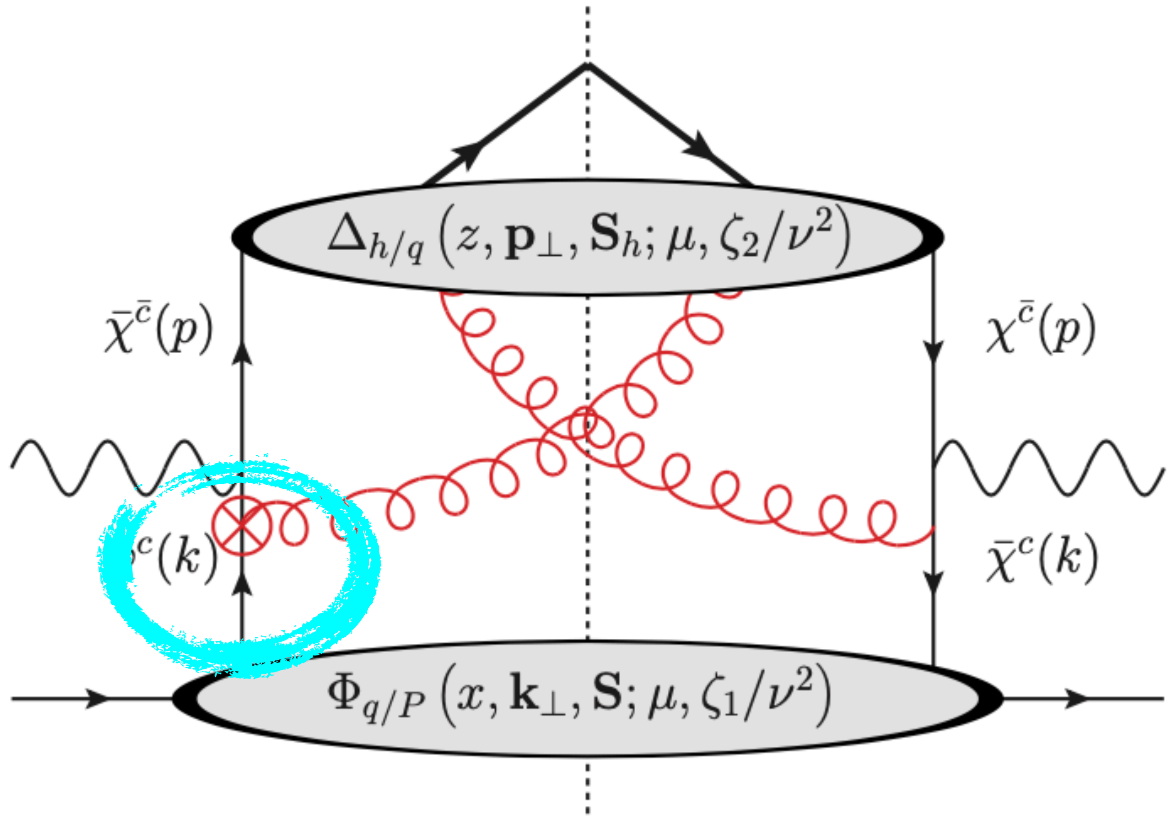
!!

$$\Gamma_2^\nu + \frac{1}{2}\Gamma_{S\,\rm int}^\nu \neq 0$$

!!

$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu)\sqrt{S^{int}}\,??$$

$$\Gamma_{2\,\rm mod}^\nu + \frac{1}{2}\Gamma_{S\,\rm int}^\nu = 0$$



Necessary condition rapidity RG Consistency

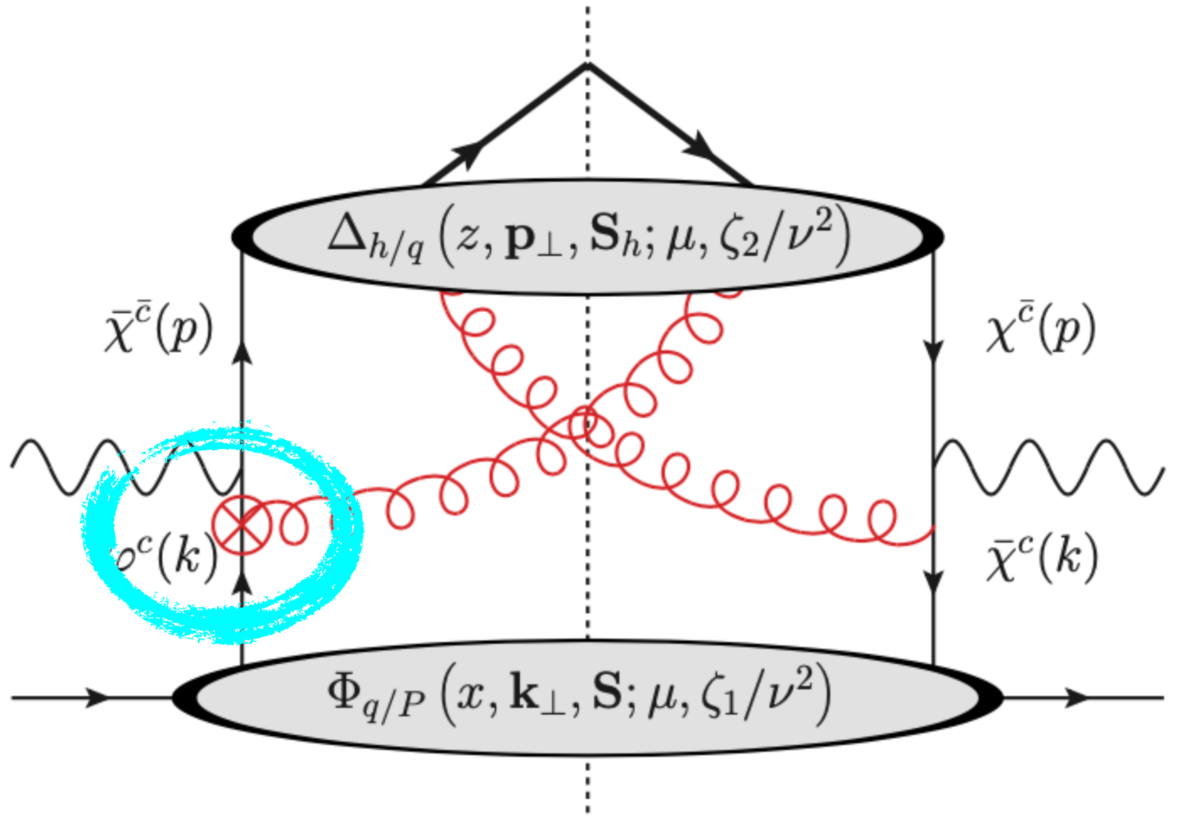
Next to leading power $\frac{d\sigma}{d\ln\nu} = 0$ & $\frac{d\sigma}{d\ln\mu} = 0$

$$- H^{\text{int}}_{\text{DIS}}(Q;\mu)\, \mathcal{C}^{\text{DIS}} \left[\left(x\, \frac{\boldsymbol{k}_\perp \cdot \hat{x}}{Q} f^\perp D_1 - \frac{\boldsymbol{p}_\perp \cdot \hat{x}}{zQ} f_1 D^\perp \right) \mathcal{S}^{\text{int}} \right]$$

!!

$$\Gamma^\nu_{\mathcal{S}\,\text{int}} + \Gamma^\nu_{3\,\text{int}} + \Gamma^\nu_{2\,\text{mod}} = 0$$

Have shown ...



Problem: Breakdown of universality different soft function for D_1 ?!

$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu)\sqrt{S^{int}}$$!!

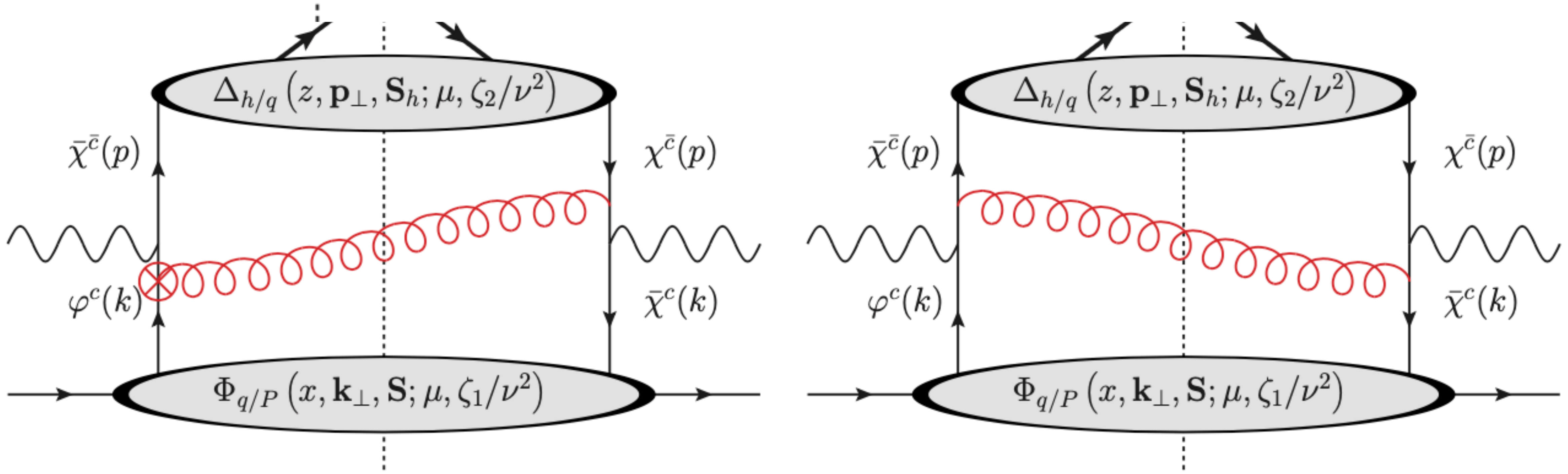
$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu^2)\, \sqrt{\mathcal{S}^{\text{LP}}(b; \mu, \nu)}$$

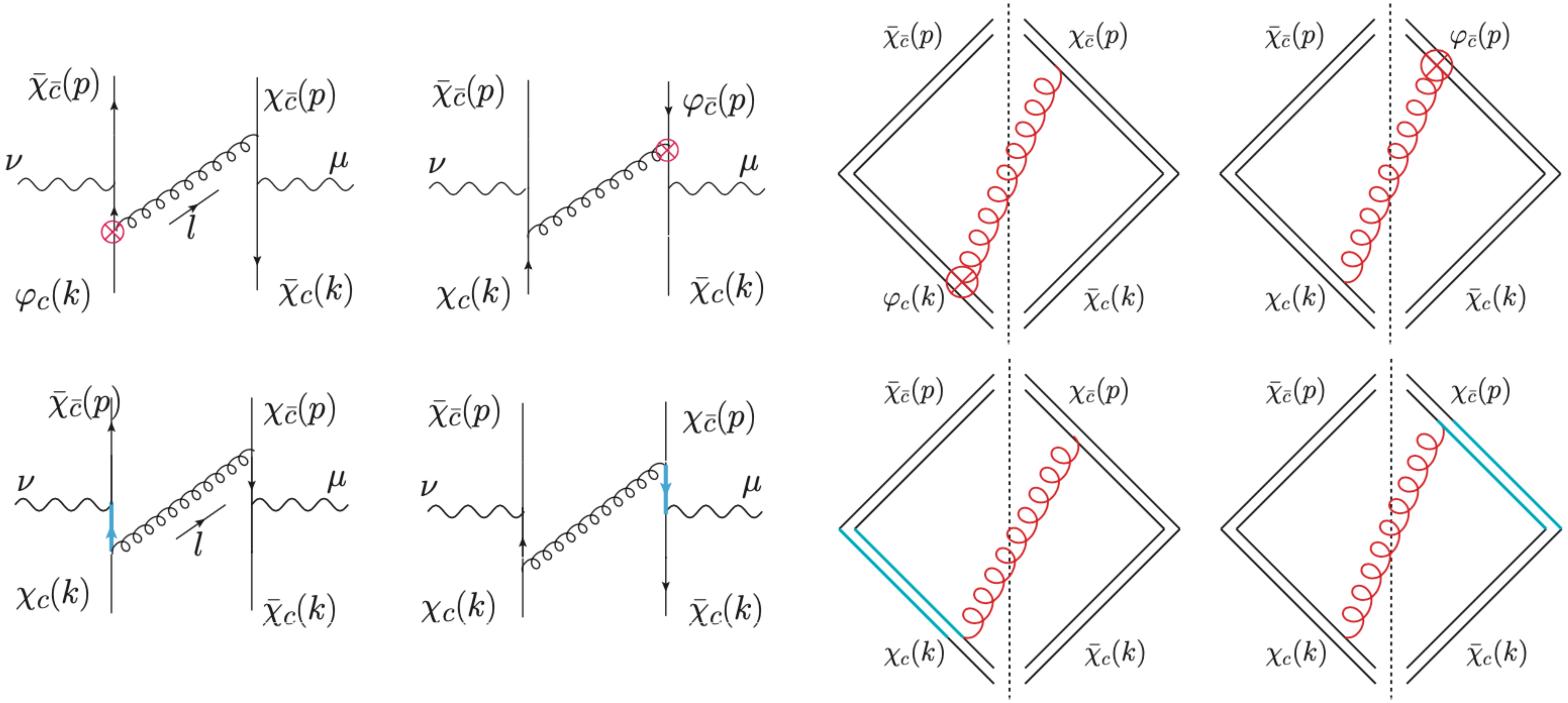
NLP **LP**

Other contributions ? Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section

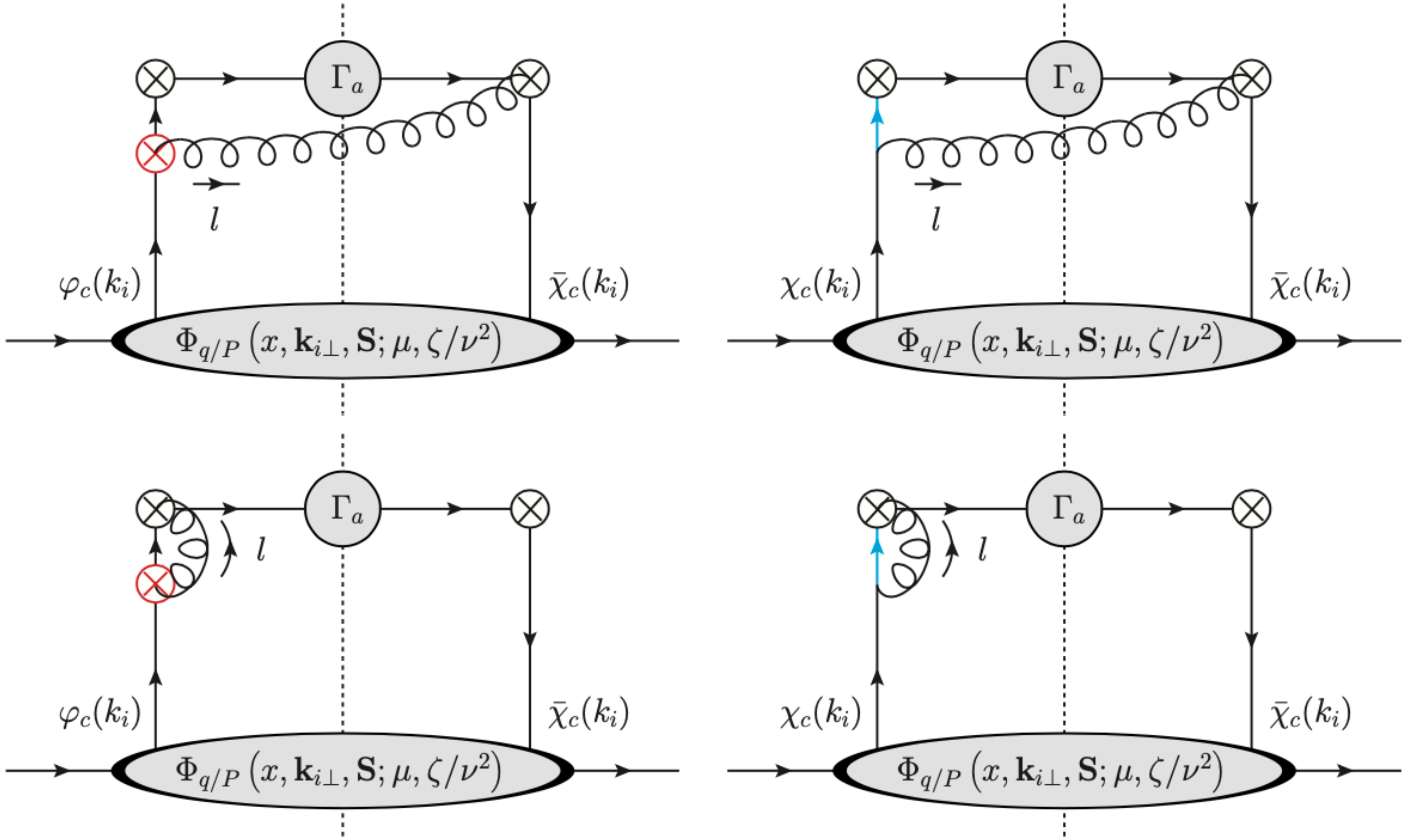




Progress Report
Stay tuned ...

Contributions to the soft factor after applying the eikonal approximation and including the effect from the transverse momentum contributions from the quark propagators.

NLO Ingredients collinear factor



Contributions to the collinear factor from kinematic power corrections ie including the effect from the transverse momentum contributions from the transverse momentum of the quark propagators

$$\Gamma^{\nu}_{3\,\mathrm{int}}(\mu,\nu,\zeta) = \Gamma^{\nu}_{2}(\mu,\nu,\zeta) \qquad ??$$

Necessary condition RRG Consistency

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$

Taking into account this aforementioned modification of leading distribution by the presence of the sub- leading field, we explore iff there are other contributions to rescue the renormalization group consistency at one loop for RRG

$$\Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\nu}_{3\,\mathrm{int}} + \Gamma^{\nu}_{2\,\mathrm{mod}} = 0 \longrightarrow$$

$$\Gamma^{\nu}_{3\,\mathrm{int}}(\mu,\nu,\zeta) + \Gamma^{\nu}_{3\,\perp}(\mu,\nu,\zeta) + \Gamma^{\nu}_{2\,\mathrm{mod}}(\mu,\nu,\zeta) + \Gamma^{\nu}_{2\,\perp}(\mu,\nu,\zeta) + \Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\nu}_{\mathcal{S}\,\perp} = 0 \;??$$

Necessary condition RG Consistency

$$\Gamma^{\mu}_{3\,\mathrm{hard}}(\mu,\nu,\zeta) + \Gamma^{\mu}_{3\,\mathrm{int}}(\mu,\nu,\zeta) + \Gamma^{\mu}_{2\,\mathrm{mod}}(\mu,\nu,\zeta) + \Gamma^{\mu}_{\mathcal{S}\,\mathrm{int}} = 0$$

Summary

We explore sub-leading power Λ_{QCD}/Q TMDs in the context of factorization theorem

- NLP factorization based on "*TMD formalism*"

  —extend the tree level Amsterdam formalism and beyond leading order

  CSS, Ji Ma Yuan, Abyat Rogers, framework vs. SCET and Background Field Methods

- Revisit "Cahn effect" & matching related to early picture of importance intrinsic k_T
  - *"Intrinsic"NLP TMDs related thru EOM in terms "kinematic" & "dynamical"*
- Consider RG consistency of matching to collinear factorization
  - Bacchetta, Boer, Diehl, Mulders JHEP 2008, Bacchetta et al. PLB 2019
- Report progress in this *necessary condition* NLP factorization (not yet sufficient)

- In doing so, we provide the basis for performing global analysis & phenomenology of one the earliest observables used to study intrinsic 3-D momentum structure of the nucleon—Opportunity for SoLID study of nucleon/pion

Thank You