Combined Study of QED and QCD for Lepton-Hadron Scattering including DIS, SIDIS, and Parity Violating Processes

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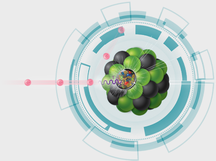
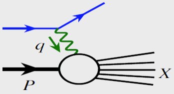
Jia-Yue Zhang

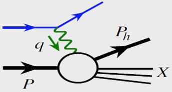
Jefferson Lab

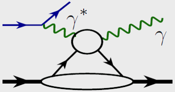
SOLENOIDAL LARGE INTENSITY DEVICE

Lepton-Hadron Scattering and Hadron Structure



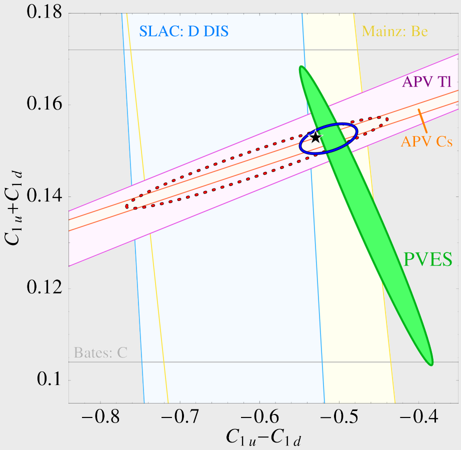






- Virtual photon as a hard probe of hadron structure;
- Multiple processes: DIS, SIDIS, exclusive scattering, etc;
- Main source of information about the parton structure of hadrons;
- Hard collision induces both QCD and QED radiation;
- QCD factorization have been very successful in treating QCD radiations;
- The precision of the hard probe depends on how precise we were able to treat collision-induced QED radiation.

Precision Standard Model Measurements



Young, Carlini, Thomas, Roche, PRL2007

- SM is very successful;
- Hard to find BSM by increasing energy;
- High precision measurements are needed;

→ Systematic improvement of radiation corrections could be important.

Inclusive ep deep inelastic scattering

- Due to QED radiation from the incoming/outgoing leptons, the momentum transfer q is not fixed. Hadron is probed by the virtual photon with momentum $\hat{q}$ instead of q.

$$q \to \hat{q}$$

$$Q^2 := -q^2 \to \hat{Q}^2 := -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P \cdot q} \to \hat{x}_B := \frac{\hat{Q}^2}{2P \cdot \hat{q}}$$

- Measurement with fixed Q^2 and x_B could cover a kinematic range of $\hat{Q}^2$ and $\hat{x}_B$ even with the approximation of one-photon exchange.

$$x_B \to \hat{x}_B \in [x_B, 1], \quad Q^2 \to \hat{Q}^2 \in \left[\frac{Q^2(1-y)}{1-x_B y}, \frac{Q^2}{1-y+x_B y}\right], \quad y := \frac{P \cdot q}{P \cdot l}$$

- Traditionally, a simple RC factor is applied to correct the measured cross section to the Born level.

$$\sigma_{\text{measured}} = \mathrm{RC} \otimes \sigma_{\text{Born}}$$

Joint QED and QCD Factorization For DIS

- Factorization Formula for the Inclusive DIS $e(l)p(P) \to e(l') + X$ at leading power:

  Liu, Melnitchouk, Qiu, Sato, PRD2021; JHEP2021

$$E' \frac{\mathrm{d}\sigma}{\mathrm{d}^3 l'} \approx \frac{1}{2s} \sum_{ija} \int_{\zeta_{\mathrm{min}}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} \int_{\xi_{\mathrm{min}}}^{1} \frac{\mathrm{d}\xi}{\xi} D_{e/j}(\zeta, \mu^2) f_{i/e}(\xi, \mu^2) \int_{x_{\mathrm{min}}}^{1} \frac{\mathrm{d}x}{x} f_{a/p}(x, \mu^2) \hat{H}_{ia\to jX}\left(\xi l, xP, \frac{l'}{\zeta}, \mu^2\right)$$

- $D_{e/j}$: **universal** lepton fragmentation function (LFF),
- $f_{i/e}$: **universal** lepton/parton distribution function (LDF/PDF),
- $\hat{H}_{ia\to jX}$: perturbative calculable **IR&CO-safe** hard scattering coefficient

$$\hat{H}_{ia\to jX}\left(\xi l, xP, \frac{l'}{\zeta}, \mu^2\right) = \sum_{m,n} \alpha^m \alpha_s^n \hat{H}^{(m,n)}_{ia\to jX}\left(\xi l, xP, \frac{l'}{\zeta}, \mu^2\right)$$

- The cross section without QED radiation can be recovered by setting

$$D_{e/j}(\zeta, \mu^2) = \delta(1-\zeta), \quad f_{i/e}(\xi, \mu^2) = \delta(1-\xi), \quad m = 0 \text{ for } \sum_{m,n}$$

Factorized QED Contributions to DIS

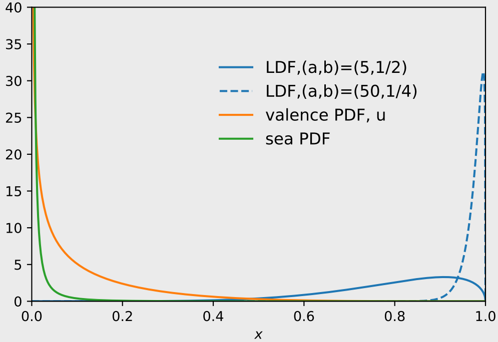
- Model distributions -analytic and everyone can test and verify without numerical complications
  - LDF&LFF:

$$f_{e/e}(x) \approx D_{e/e}(x) = \frac{x^a(1-x)^b}{B(a+1,b+1)}$$

  - PDF:

$$f_{q/p}(x) \approx \begin{cases} N_q \dfrac{x^{-1/2}(1-x)^{7/2}}{B(1/2,9/2)} & \text{valence quark,} \\ N_q \dfrac{x^{-3/2}(1-x)^{5}}{B(-3/2,6)} & \text{sea quark,} \end{cases}$$

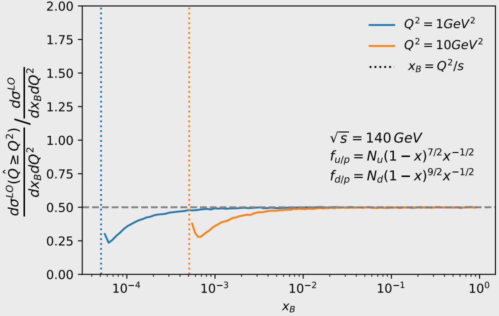
$$N_u = 2, \quad N_d = 1, \quad N_s = \frac{1}{2}$$

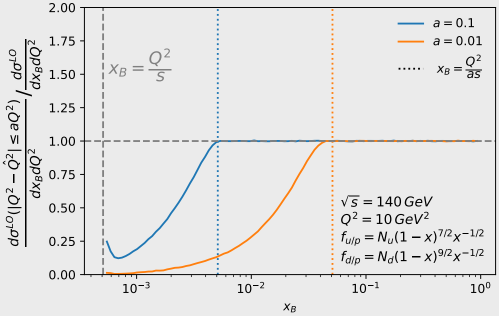


Resummed Collinear Contribution:

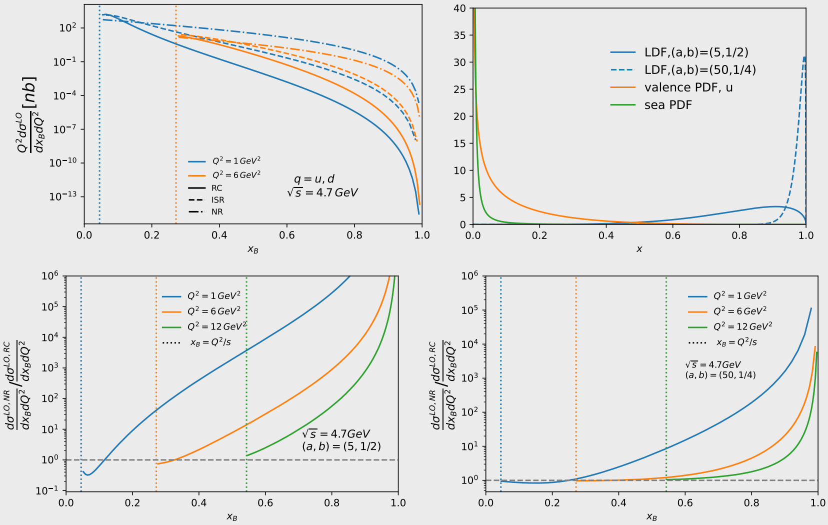
Recall $\hat{Q}^2 \in \left[\frac{Q^2(1-y)}{1-x_B y}, \frac{Q^2}{1-y+x_B y}\right].$

- More than $1/2$ cross sections are from $\hat{Q}^2 \leq Q^2$.
- Very significant events are NOT from the region where $\hat{Q}^2 \sim Q^2$ when x_B is small.





Resummed Collinear Contribution



NLO QED Corrections

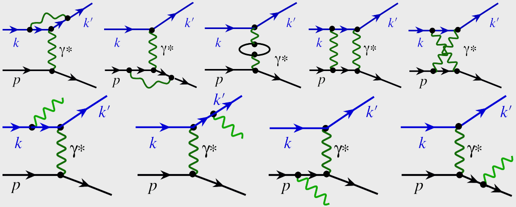
- Joint QED and QCD factorization allows a systematic expansion in α_s and α.

$$\hat{\sigma}^{(1)} = D_{e/e}^{(0)} \otimes f_{e/e}^{(0)} \otimes f_{q/p}^{(0)} \otimes \hat{H}_{eq\to eX}^{(1)} + D_{e/e}^{(1)} \otimes f_{e/e}^{(0)} \otimes f_{q/p}^{(0)} \otimes \hat{H}_{eg\to eX}^{(0)} + D_{e/e}^{(0)} \otimes f_{e/e}^{(1)} \otimes f_{q/p}^{(0)} \otimes \hat{H}_{eq\to eX}^{(0)}$$
$$+ D_{e/e}^{(0)} \otimes f_{e/e}^{(0)} \otimes f_{q/p}^{(1)} \otimes \hat{H}_{eq\to eX}^{(0)} + D_{e/e}^{(0)} \otimes f_{e/e}^{(0)} \otimes f_{\gamma/p}^{(1)} \otimes \hat{H}_{\gamma q\to eX}^{(0)}$$

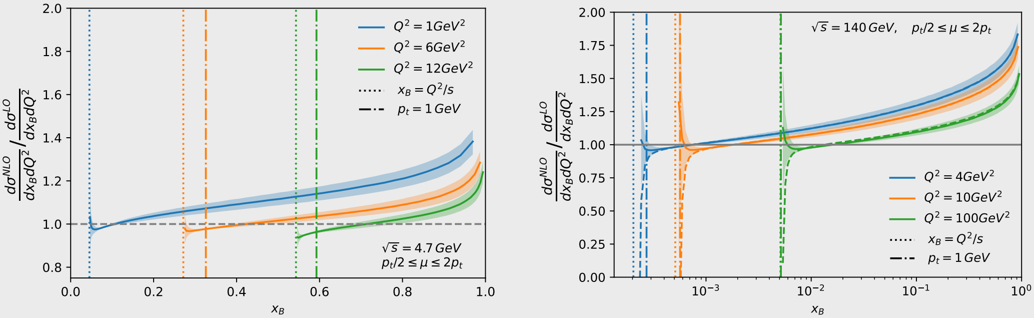
Hadron's parton distributions are not pure QCD.

→ Matching condition of hard scattering coefficient:

$$\hat{H}_{eq\to eX}^{(1)} = \hat{\sigma}^{(1)} - D_{e/e}^{(1)} \otimes \hat{H}_{eg\to eX}^{(0)} - f_{e/e}^{(1)} \otimes \hat{H}_{eq\to eX}^{(0)} - f_{q/p}^{(1)} \otimes \hat{H}_{eq\to eX}^{(0)} - f_{\gamma/p}^{(1)} \otimes \hat{H}_{\gamma q\to eX}^{(0)}$$



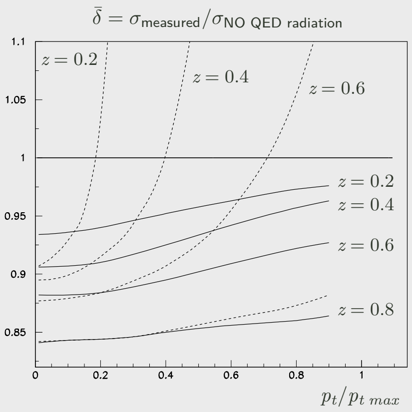
NLO QED Corrections



Remark

$$p_t^2 = Q^2(1-y)$$

No Simple Radiative Correction for SIDIS



Radiative effects in the processes of hadron electroproduction

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Abstract. An approach to calculate radiative corrections to the unpolarized cross section of semi-inclusive electroproduction is developed. Explicit formulae for the lowest order QED radiative correction are presented. A detailed numerical analysis is performed with the kinematics of experiments with fixed targets.

- Radiative correction to SIDIS $e + N \to e + \gamma + h(p) + X$ vs hardronic transverse momentum p_t;
- $\sqrt{S}$=7.19 GeV, x=0.15, Q^2=4 GeV^2.
- Dashed curves: Mulders-Tangerman model

$$b \exp\left(-b p_t^2\right), \quad b := R^2/z^2$$

- Solid curves: $\left(a + bz + p_t^2\right)^{-c-dz}$
- RC factor depends on the hadronic input that we want to probe.

SIDIS

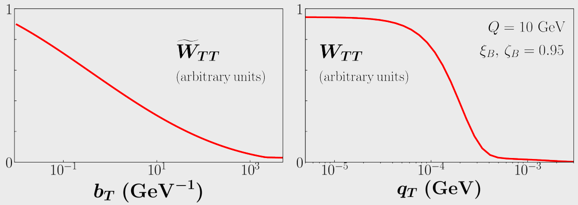
- Assuming one-photon exchange, SIDIS is described by 18 structure functions, which are defined in photon-hadron frame. ***Bacchetta, Diehl, Goeke, Metz,Mulders, Schlege, JHEP2007***

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}+S_{\|}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+S_{\|}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+|\boldsymbol{S}_\perp|\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\Big\},
\end{aligned}
$$

Hybrid Factorization for SIDIS *Liu, Melnitchouk, Qiu, Sato, PRD2021; JHEP2021*

- In the presence of QED radiation, momentum direction of exchanged photon is not fixed.
- When scattered lepton and hadron are back-to-back, TMD factorization is applicable.

Observation QED broadening for lepton $\ll$ typical parton transverse momentum.



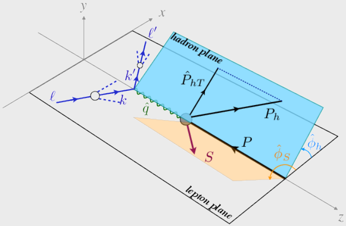
→ Hybrid factorization for SIDIS in the two-scale regime.
- collinear factorization for the two leptons
- TMD factorization for the two hadrons

Hybrid Factorization for SIDIS *Liu, Melnitchouk, Qiu, Sato, PRD2021; JHEP2021*

Factorization Formula for SIDIS

$$E_{\ell'} E_{P_h} \frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{d^3\ell'\, d^3 P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^{1} d\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)$$

$$\times \left[E_{k'} E_{P_h} \frac{d^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{d^3 k'\, d^3 P_h} \right]_{k=\xi\ell,\, k'=\ell'/\zeta}$$

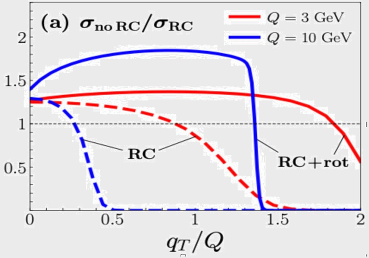


F_{UU}

$$\frac{\mathrm{d}\sigma^h_{\text{SIDIS}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P^2_{hT}} = \int_{\zeta_{\min}}^1 \mathrm{d}\zeta \int_{\xi_{\min}}^1 \mathrm{d}\xi\, D_{e/e}(\zeta) f_{e/e}(\xi) \frac{\hat{x}_B}{x_B\xi\zeta} \frac{(2\pi)^2\alpha}{\hat{x}_B\hat{y}\hat{Q}^2} \frac{\hat{y}}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \hat{y}, \hat{z}, \hat{P}_{hT}).$$

Unpolarized structure function F^h_{UU}

$$F^h_{UU} = x_B \sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{p}_T \mathrm{d}^2\boldsymbol{k}_T \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T)$$
$$\times f_{q/N}(x_B, \boldsymbol{p}_T^2) D_{h/q}(z, \boldsymbol{k}_T^2), \quad \boldsymbol{q}_T := \frac{\boldsymbol{P}_{hT}}{z}.$$



Parity Violating Deep Inelastic Scattering

Parity Violating Lepton-Spin Asymmetry

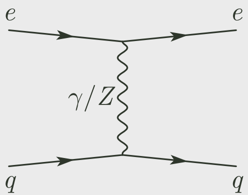
$$A_{\mathsf{PVE}} := \frac{\sigma_{e(\lambda=1)p\to eX} - \sigma_{e(\lambda=-1)p\to eX}}{\sigma_{e(\lambda=1)p\to eX} + \sigma_{e(\lambda=-1)p\to eX}} =: \frac{\Delta\sigma_{ep\to eX}}{\sigma_{ep\to eX}}$$

- Previous framework is hard to extend to full EW&QCD factorization directly.
- For SoLID, Z/W is too heavy to radiate. EW&QCD factorization might be applicable at heavy gauge boson limit.

$$E_{l'}\frac{\mathrm{d}\sigma_{lP\to l'X}}{\mathrm{d}^3 l'} \approx \int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}D_{e/e}\left(\zeta,\mu^2\right)\int_{\xi_{\min}}^{1}\mathrm{d}\xi f_{e(\lambda_k)/e(\lambda_l)}\left(\xi,\mu^2\right)\left(E_{k'}\frac{\mathrm{d}\hat{\sigma}_{kP\to k'X}}{\mathrm{d}^3 k'}\right)_{k=\xi l,\,k'=l'/\zeta}$$

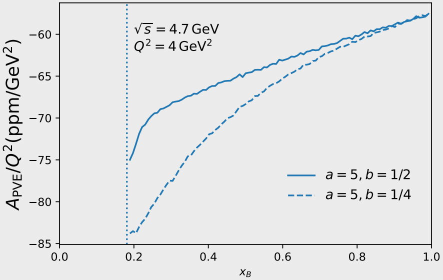
One-Vector-Boson Exchange

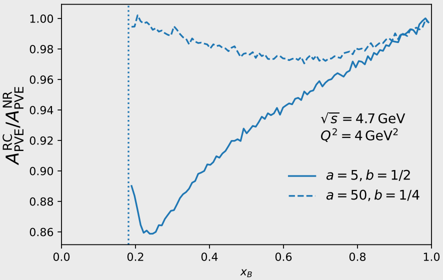
- Only interference between γ and Z exchange contributes to A_{PVE} at LO.



$$\frac{\mathrm{d}\Delta\hat{\sigma}}{\mathrm{d}\hat{y}} = \frac{2\pi\alpha^2 e_q}{m_Z^2 \sin^2\theta_{\mathrm{W}} \cos^2\theta_{\mathrm{W}} \hat{y}} \left\{ e_q \sin^2\theta_{\mathrm{W}} \left[1 + (1-\hat{y})^2\right] + I_3^q \left[2\sin^2\theta_{\mathrm{W}} \left(1 - (1-\hat{y})^2\right) - 1\right] \right\}$$

Resummed Collinear Contribution





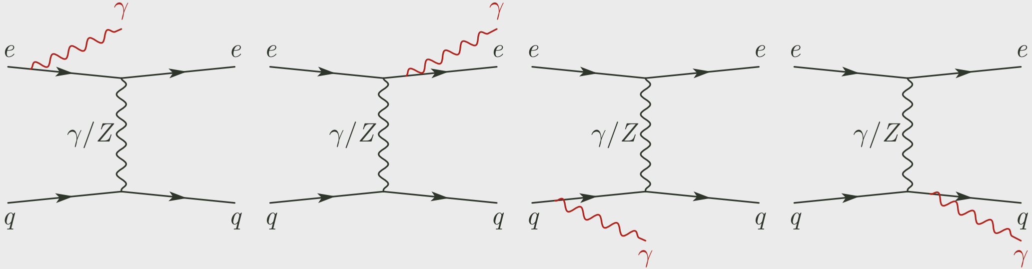
Recall

$$f_{e/e}(x) \approx D_{e/e}(x) = \frac{x^a(1-x)^b}{B(a+1, b+1)}$$

NLO

→ Matching condition of hard scattering coefficient:

$$\hat{H}^{(1)}_{eq\to eX} = \hat{\sigma}^{(1)} - D^{(1)}_{e/e} \otimes \hat{H}^{(0)}_{eg\to eX} - f^{(1)}_{e/e} \otimes \hat{H}^{(0)}_{eq\to eX} - f^{(1)}_{q/p} \otimes \hat{H}^{(0)}_{eq\to eX}$$

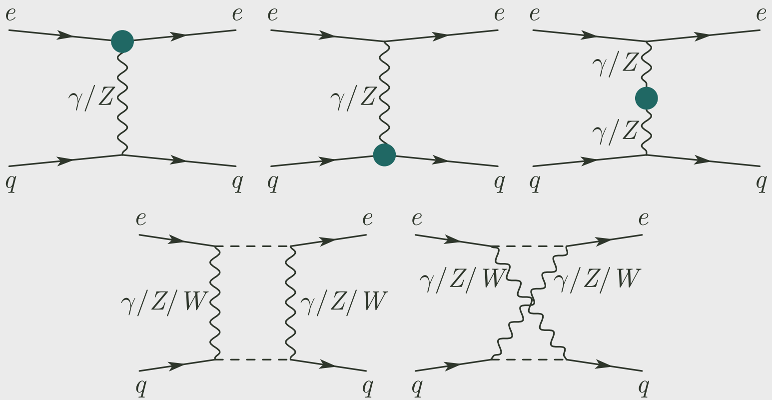


- No divergence when intermediate photon is collinear to the initial parton.

→ **different from the pure QED case for inclusive DIS**

- Conservation of momentum allows expansion of amplitude in m_Z before phase space integration.

NLO



- Joint factorization has been confirmed to NLO by verifying the cancellation of CO divergences.

Next: Finishing up the complete NLO hard part for EW+QCD.

1. Introduction
2. DIS
3. SIDIS
4. PVDIS
5. **Summary**

Summary

- Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton-hadron scattering processes.
- Physical observables are factorized into universal lepton/parton distribution/fragmentation functions and perturbatively calculable hard parts which is IR and CO safe for both QCD and QED.
- No artificial scale introduced for treating QED radiation, other than the standard factorization scale.
- NLO corrections can be calculated in a systematic way for both QCD and QED radiative corrections.
- Joint QED and QCD factorization can help control and qualify the systematic errors for PVDIS.

Thank you