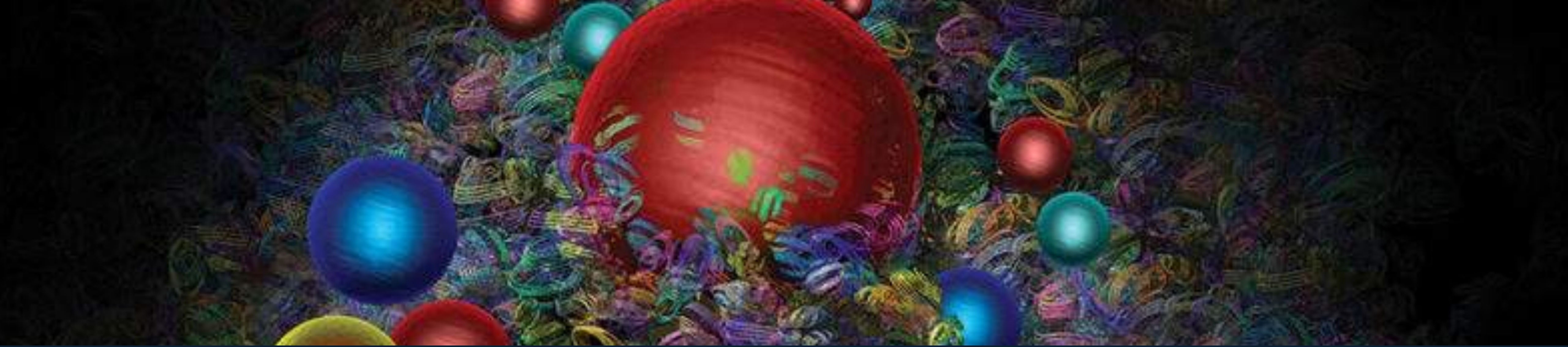

GPDs through Universal Moment Parameterization (GUMP) 1.0
— First global extraction of GPDs

Yuxun Guo

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[arXiv: 2509.08037]

Based on works in collaboration with X. Ji, M. G. Santiago and F. Aslan

QGT Topical Collaboration Meeting 2025

Argonne National Laboratory, Lemont, IL

Sep. 19 – 20th., 2025

BERKELEY LAB

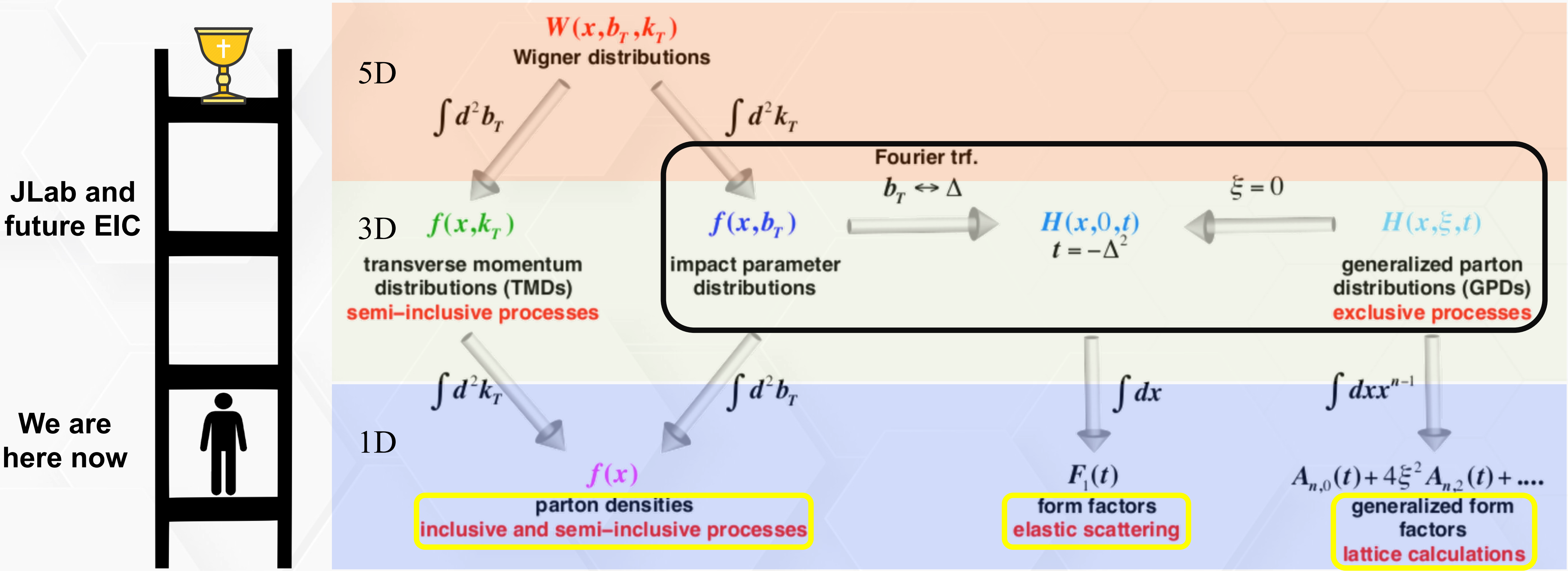
Berkeley UNIVERSITY OF CALIFORNIA

QUARK-GLUON TOMOGRAPHY COLLABORATION

Argonne NATIONAL LABORATORY

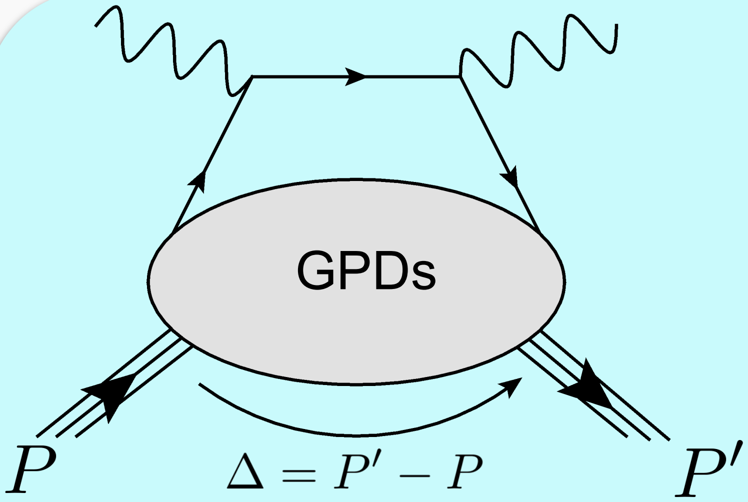
Multi-dimensional nucleon structures

Deciphering the multi-dimensional structures of the nucleons naturally requires a truly global analysis with extensive inputs.



Generalized parton distributions (GPDs)

Generalized parton distributions are parton distributions with momentum transfer



D. Muller et. al. Fortsch.Phys. 42 101 (1994)

X. Ji Phys. Rev. Lett. 78, 610 (1997)

GPDs unify the parton distributions and form factors

$$F(x, \Delta^\mu) = F(x, \xi, t)$$

x : average parton momentum fraction

ξ : skewness – longitudinal momentum transfer $\xi \equiv -n \cdot \Delta/2$

t : total momentum transfer squared $t \equiv \Delta^2$

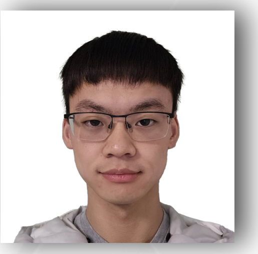
Curse of Dimensionality:

GPDs through Universal Moment Param. (GUMP)

The GPDs through Universal Moment Param.(GUMP) programs aims to obtain the GPDs from global analysis utilizing moment-space parameterization

Goal: To obtain the state-of-the-art phenomenological **Generalized Parton Distributions (GPDs)** through global analysis of both **experimental data** and **lattice QCD simulations**, utilizing a ***universal moment parameterization*** method.

Current Collaborators:



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Temple University

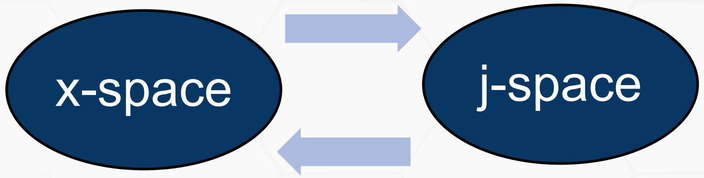
Fatma P. Aslan (Postdoc)
Center for Nuclear Femtography

GPDs parameterized in moments

GPDs can be formally expanded in the conformal moment space:

$$F(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^j p_j(x,\xi)\mathcal{F}_j(\xi,t)$$

D. Mueller and A. Schafer 2005



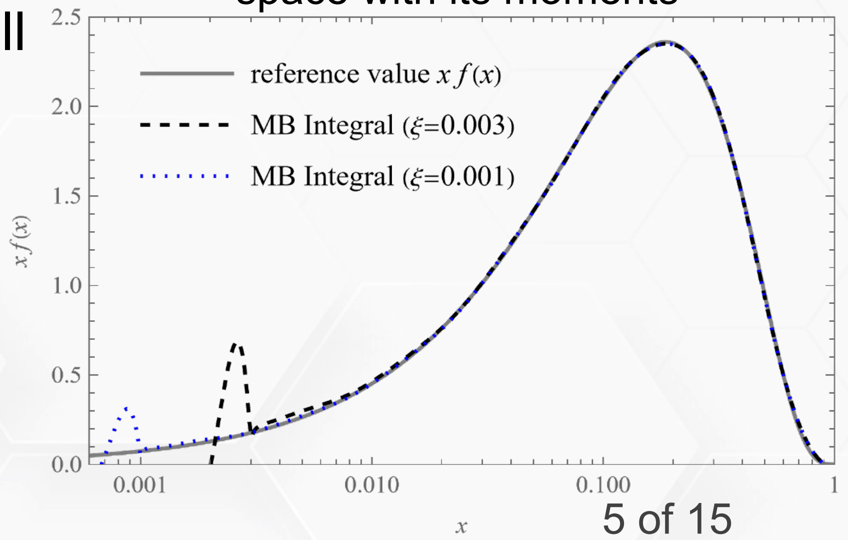
$p_j(x,\xi)$: Orthogonal basis in terms of Gegenbauer polynomials

$\mathcal{F}_j(\xi,t)$: Moments of GPDs to be parameterized

Whereas GPDs in x-space can be reconstructed by resumming all the moments through a complex integral in the moment space.

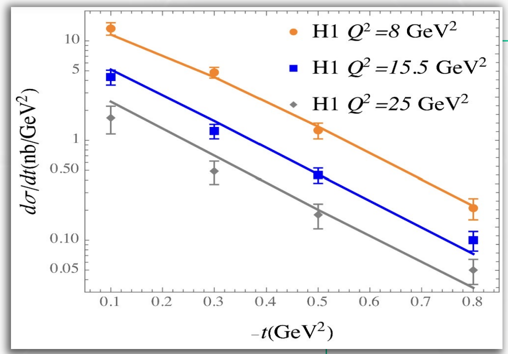
$$F(x,\xi,t)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dj\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)\ ,$$

Example of reconstructed GPD in x-space with its moments



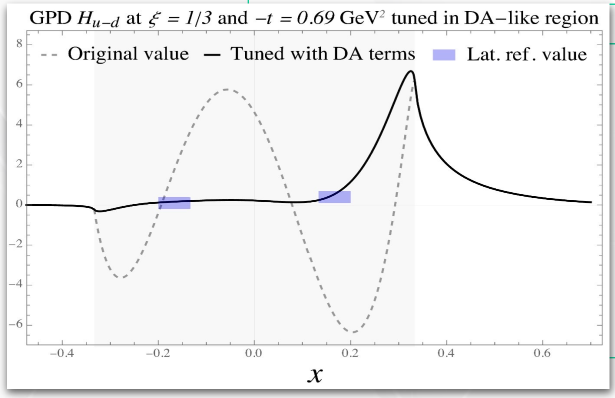
Proof-of-principle analysis (GUMP 0.5)

Previously, we have reported a comprehensive analysis including



Experimental data and constraints

- ❑ *Polarized and unpolarized PDFs from global analysis*
- ❑ *Neutron/ Proton charge form factors from global analysis*
- ❑ Deeply virtual Compton scattering data at JLab and HERA



Lattice QCD simulations

- ❑ Lattice simulations of nucleon generalized form factors
- ❑ Lattice simulations of unpolarized and helicity GPDs at (non-)zero skewness

- ➢ Only leading order in perturbative expansion
- ➢ Lacking meson production constraints
- ➢ Lattice simulations not as abundant

Developments in GUMP

Full next-to-leading(NLO) accuracy

In the past years, we actively include more processes with improved accuracy:

Full GPD evolutions to the next-to-leading order (NLO)

— *Include both evolving-moment and evolving-Wilson-coefficient method*

NLO deeply virtual J/ψ production (DVJ/ψP) with mass corrections

— *In a hybrid framework to also include the mass corrections*

NLO deeply virtual Compton scattering (DVCS) and meson productions (DVMP)

— *Covering most of the existing JLab and HERA measurements of DVCS and ρ productions*

Extend to other observables such as asymmetries measurements.

NLO corrections appear significant for the HERA (future EIC) kinematics !

Developments in lattice simulations

Lattice simulations of Generalized form factors and GPDs have also been fruitful:

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks at Nonzero Skewness

Min-Huan Chu,[1,*] Manuel Colaço,[1] Shohini Bhattacharya,[2] Krzysztof Cichy,[1] Martha Constantinou,[3] Andreas Metz,[3] and Fernanda Steffens[4]

Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[3] Xiang Gao,[4,*] Andreas Metz,[3] Joshua Miller,[3] Swagato Mukherjee,[5] Peter Petreczky,[5] Fernanda Steffens,[6] and Yong Zhao[4]

Gravitational Form Factors of the Proton from Lattice QCD

Daniel C. Hackett,[1,2] Dimitra A. Pefkou,[3,2,4] and Phiala E. Shanahan[2]

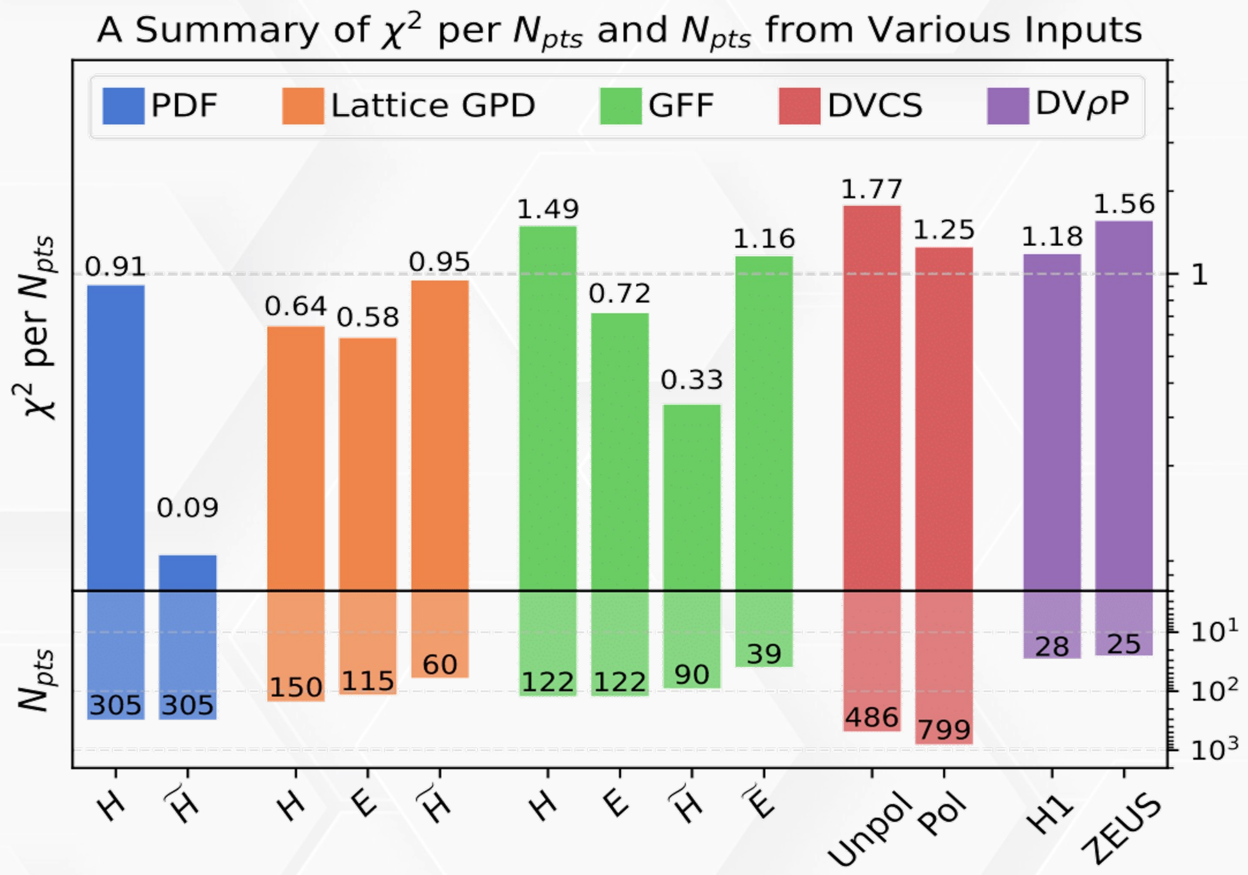
Quark flavor decomposition of the nucleon axial form factors

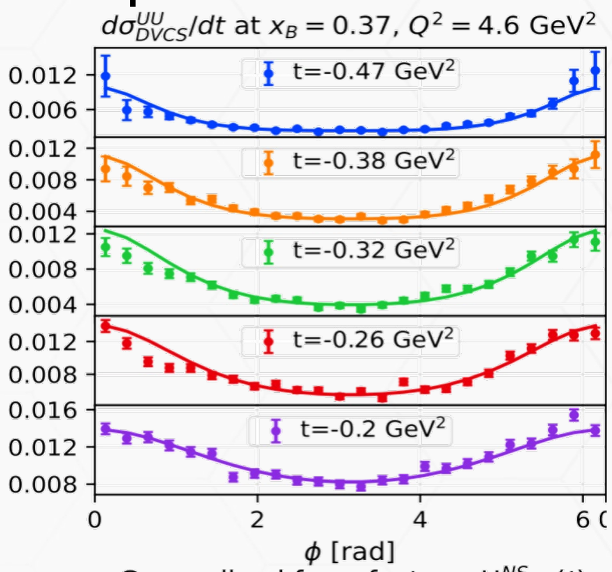
C. Alexandrou,[1,2] S. Bacchio,[2] M. Constantinou,[3] K. Hadjiyiannakou,[1,2] K. Jansen,[4] and G. Koutsou[2]

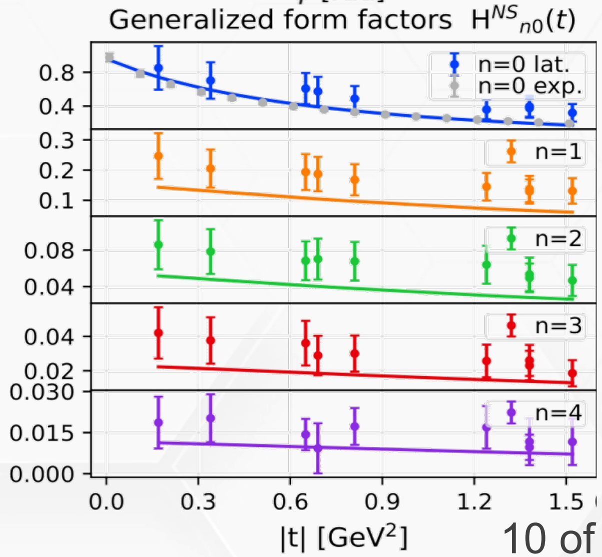
……

Excellent descriptions of exp. and lattices

Generally, we observe excellent agreement across various inputs:



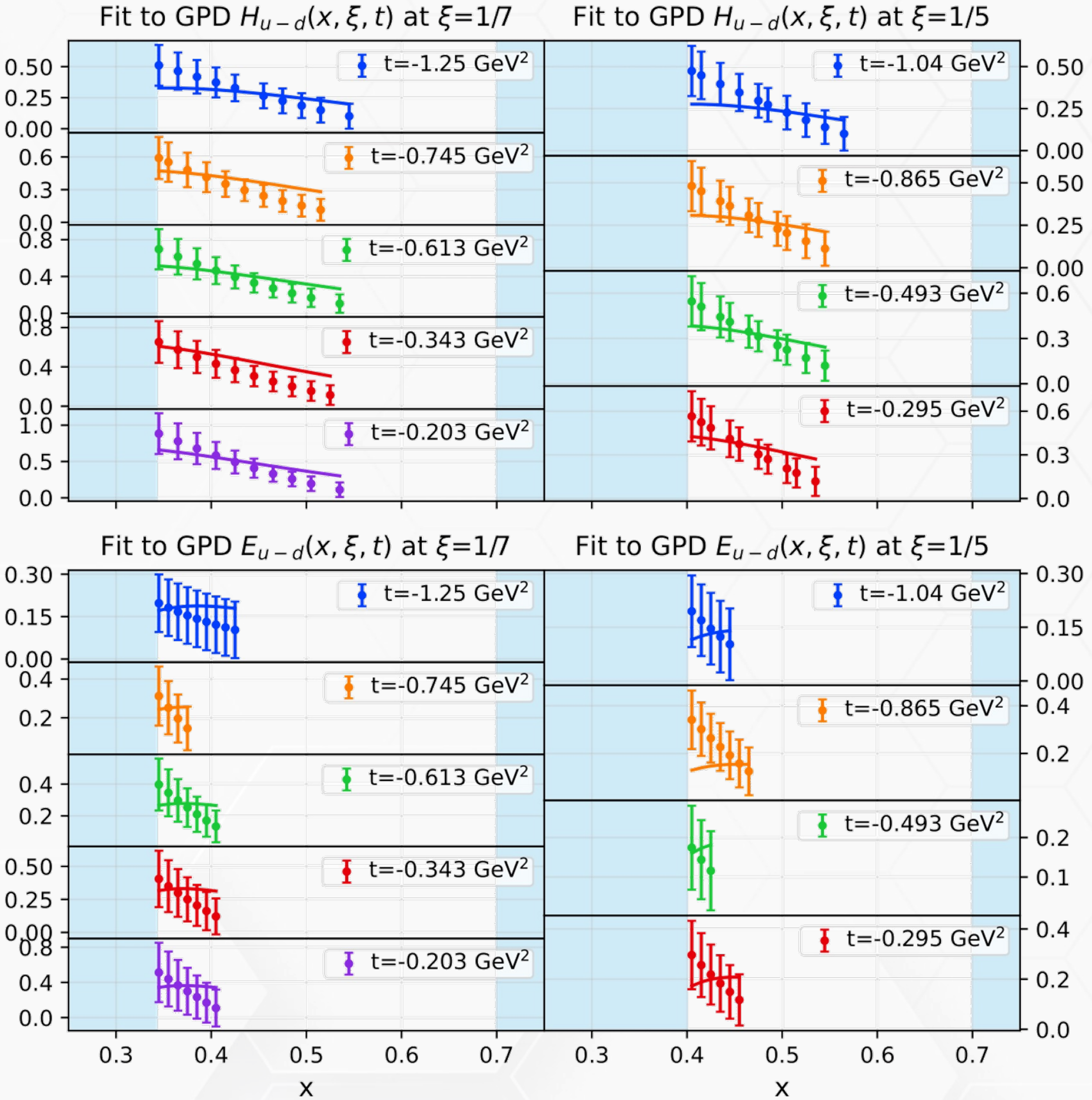




Note: a 30% relative uncertainty is added to all the lattice data

Fits to GPDs at non-zero skewness

Importantly, we also include the recent lattice simulations of GPDs at non-zero skewness.



Significant effects in constraining GPDs at non-zero skewness!

Note: besides the 30% relative uncertainty added to all the lattice data, we have a very conservative selection:

1) Only used the data between 0.3<x<0.7 and |x-xi|>0.2.
2) Exclude region where GPDs get negative or very small (lattice artifacts or intrinsic GPD behaviors?)

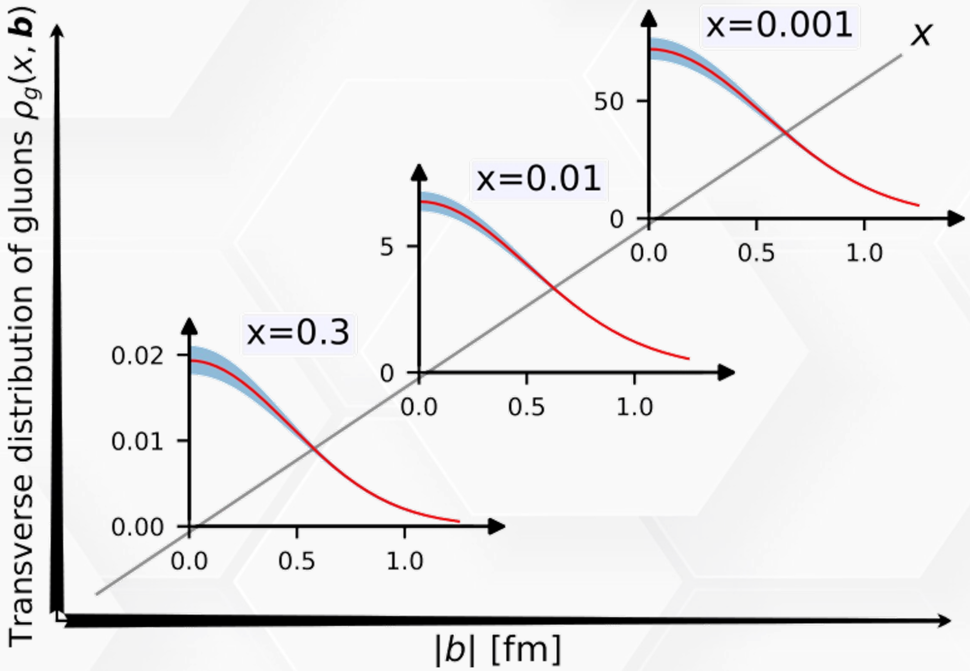
Nucleon tomography with GUMP1.0 GPDs

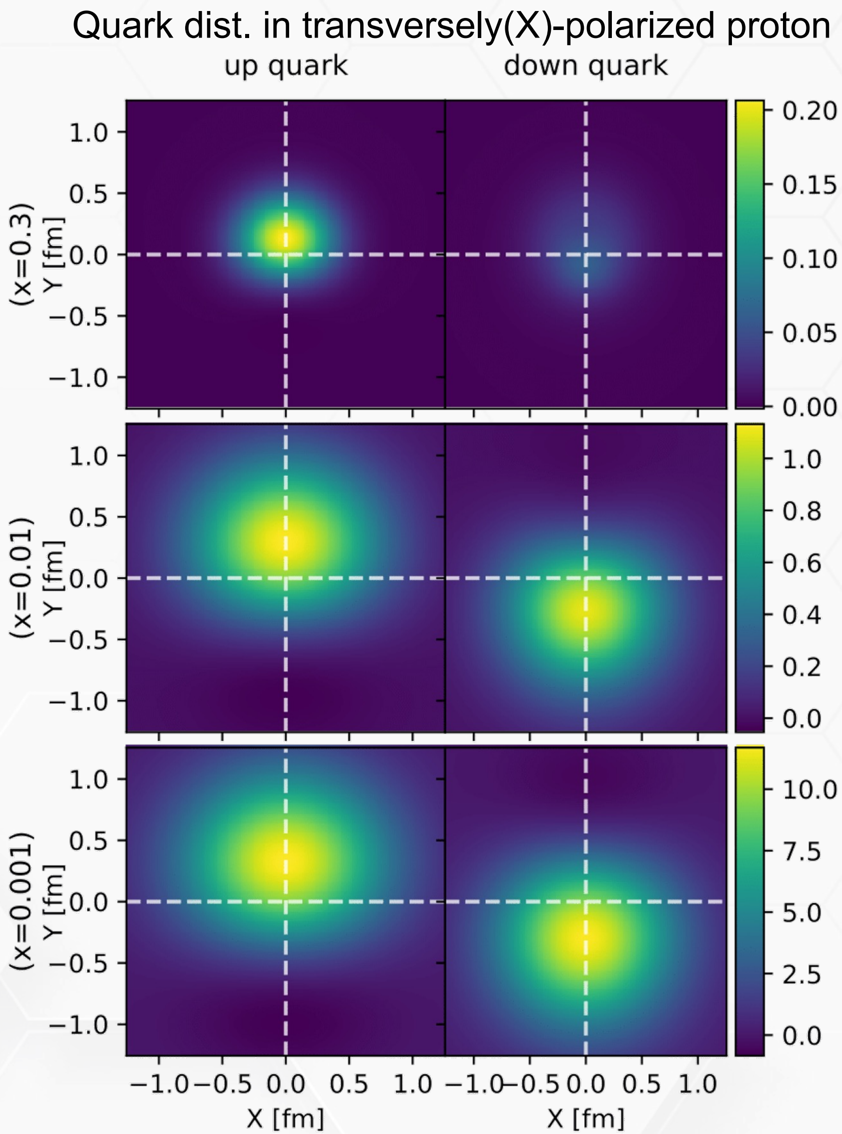
Of course we can also study the nucleon tomography via:

$$\rho_{q/g}(x,\boldsymbol{b}) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} H_{q/g}(x,-\boldsymbol{\Delta}^2)$$

And the ones for transversely polarized proton

$$\rho^X_{q,\mathrm{In}}(x,\boldsymbol{b}) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \left[H_q(x,-\boldsymbol{\Delta}^2) + \frac{i\Delta_y}{2M}\left(H_q+E_q\right)(x,-\boldsymbol{\Delta}^2)\right]$$





Outlook and summary

To summary

Summary

- ✓ Implemented all four leading-twist GPDs for up, down quarks and gluons
- ✓ Full NLO accuracy for GPD evolution, DVCS, DVMP, DVJ/ψP (?)
- ✓ Include the state-of-the-art lattice calculations of GPDs and moments
- ✓ Flexibility of parameterizations examined with all input constraints.
- ✓ GUMP1.0 GPDs are released, and more analyses are on-going!

What's more?

Extend to more observables

- More meson productions data: vector meson and others, J/ψ photoproduction
- Implementing the strange flavor (ϕ meson productions)

Improving the accuracy

- Bayesian inference method for fitting
- Input/estimate some Next-to-Next-to-Leading-Order (NNLO) corrections
- Possible NNLO GPD evolutions?
- Kinematic corrections in DVCS; Mass corrections for J/ψ production 4

Precision and benchmarking

- Open GPD evolution code in moment space
- Benchmark of LO/NLO GPD evolution precision

Thank you!

GUMP parameterization

Moments of GPDs are polynomials of ξ, so they can be written as

$$\mathcal{F}_j(\xi,t)=\mathcal{F}_{j,0}(t)+\xi^2\mathcal{F}_{j,2}(t)+\xi^4\mathcal{F}_{j,4}(t)+\cdots$$

The first term describes GPDs at $\xi=0$, and is parameterized as:

$$\mathcal{F}_{j,0}(t)=NB(j+1-\alpha,1+\beta)\frac{j+1-\alpha}{j+1-\alpha(t)}\beta(t)$$

Euler Beta Function ↑   Regge trajectory ↑ $\alpha(t)=\alpha+\alpha' t$

- Beta function $B(j+1-\alpha,1+\beta)$: corresponds to the PDF ansatz $x^{-\alpha}(1-x)^{\beta}$
- Regge trajectory: modify the small-x behavior at different t in the form of $x^{-\alpha(t)}$
- The residual term $\beta(t)$: motivated by the measured t-dependence from experiments.

GUMP Param.: ξ-dependence

Generally, we model the ξ-dependent terms to be proportional to the forward ones:

$$\mathcal{F}_j(\xi,t)=\mathcal{F}_{j,0}(t)+\xi^2\mathcal{F}_{j,2}(t)+\xi^4\mathcal{F}_{j,4}(t)+\cdots$$

$$\mathcal{F}_{\underline{j,2}}(t)=R_2\mathcal{F}_{\underline{j-2,0}}(t)$$

$$\mathcal{F}_{\underline{j,4}}(t)=R_4\mathcal{F}_{\underline{j-4,0}}(t)$$

The shift in j will enhance ξ-dependent terms, which might or might not be a good choice.

$$\frac{\xi^2\mathcal{F}_{j,2}(t)}{\mathcal{F}_{j,0}(t)}=R_2\xi^2\boxed{\frac{\mathcal{F}_{j-2,0}(t)}{\mathcal{F}_{j,0}(t)}}\quad\text{for}\quad j\geq 2$$

Even when R_2 is of order $\mathcal{O}(1)$, the extra factor due to the shift can still be large.

While the shifted moments method can describe the data (DVCS,DVJ/ψP ,DVρP) well, hints from models like holographic QCD can be help. We can confront it with the data.

Angular momentum contributions