Down with the Breit Frame- Space-Time Interpretation of Nucleon Form Factors

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QGT folk know the 3 dimensional variables are light front: $(x, \mathbf{p}_\perp)$

$(x, \mathbf{b}_\perp)$

That seems to be a minority opinion!

Four recent talks at LBL and many papers report results depending on $\mathbf{r}$

So I posted "On the Impossibility of Obtaining Time-Independent, Three-Dimensional, Spherically-Symmetric Densities of Confined Systems of Relativistically Moving Constituents"
2507.14388

Form Factors and 3 problems with the Breit Frame

$F(q^2) = \langle p' | \widehat{\mathcal{O}}(x^\mu = 0) | p \rangle, \quad q = p' - p$ $\qquad \hat{\mathcal{O}} : J^\mu, A^\mu, T^{\mu\nu}$

Form factors ⟶ cross sections

Many seek spatial interpretation: $\rho(\vec{r}) = \int d^3q \langle p' | \widehat{\mathcal{O}}(0) | p \rangle e^{i\vec{q}\cdot\vec{r}}$

- 1. 4 dimensional scalar $q^2 \to \vec{q}\cdot\vec{q}$ time ? Breit frame: $\vec{p} = -\vec{q}/2, \vec{p}' = \vec{q}/2, q^0 = 0$
  Lorentz contraction prevents spherical symmetry
- 2. density is $\psi^*\psi$, SAME initial and final state wave function
- 3. What, where when is $x^\mu = 0$? $|p\rangle, |p'\rangle$ plane wave states everywhere in space

Then- how to do it right on light front

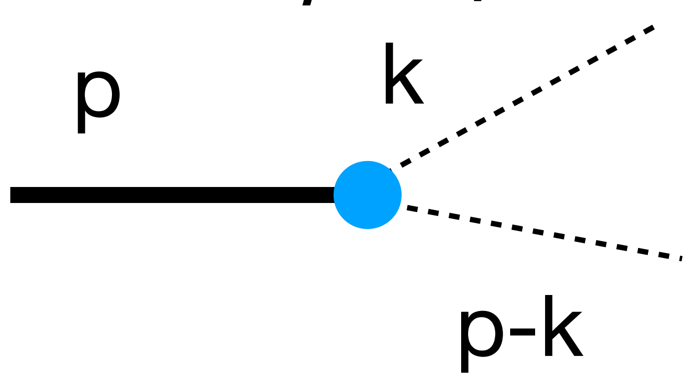
Problem 2 $\psi^* \psi$

Non-relativistic systems: separate relative and center of mass coordinates. The initial and final relative wave functions are same.

NO such separation for relativistic constituents

Schroedinger eq. ⟶ Bethe-Salpeter eq.

ψ depends on the total momentum



$$\psi_p \propto \frac{1}{k^2 - m^2 + i\epsilon} \frac{1}{(p-k)^2 - m^2 + i\epsilon}$$

Not a product f(p) g(k)

Initial | Final

$$p^\mu = (\sqrt{\vec{q}^2/4 + M^2}, -\vec{q}/2), \quad p'^\mu = (\sqrt{\vec{q}^2/4 + M^2}, \vec{q}/2); \quad (p-k)^2 - (p'-k)^2 = 2q \cdot k \neq 0$$

Initial final wave functions are different, in general no density

Problem 3- where? -Lorce, Pasquini …. Wigner distribution

Plane waves, momentum exact, in Breit position claimed exact

Uncertainty principle violated!

Wigner distributions try to avoid problem

Three-dimensional Breit-frame spatial distributions

system on average, is at rest and at the origin

Relativistic phase space density operator

$$\rho_{R,P} = \int \frac{d^3\Delta}{(2\pi)^3\, 2P^0}\, e^{-i\Delta\cdot R}\, |P-\tfrac{\Delta}{2}\rangle\langle P+\tfrac{\Delta}{2}| \qquad \langle \hat{\mathcal{O}}(X)\rangle_{R,P} = \mathrm{Tr}[\hat{\mathcal{O}}(X)\rho_{R,P}]$$

$$\langle \mathbf{K}\rangle = \int d^3K\,\mathbf{K}\int \frac{d^3\mathbf{\Delta}}{(2\pi)^3\, 2P^0}\, e^{-i\Delta\cdot R}\,\langle K|P-\Delta/2\rangle\langle P+\Delta/2|K\rangle = \int d^3K\,\frac{\mathbf{K}}{E(K)} = 0,$$

Parity gives - 0, but

$$\langle \mathbf{K}^2\rangle = \int d^3K\,\frac{K^2}{E(K)} = \infty\,.$$

Similarly $\langle \mathbf{r}\rangle = 0, \quad \langle \mathbf{r}^2\rangle = \infty$

Fluctuations are infinite, position and momentum not defined

Problem 3 when? Meissner, Epelbaum

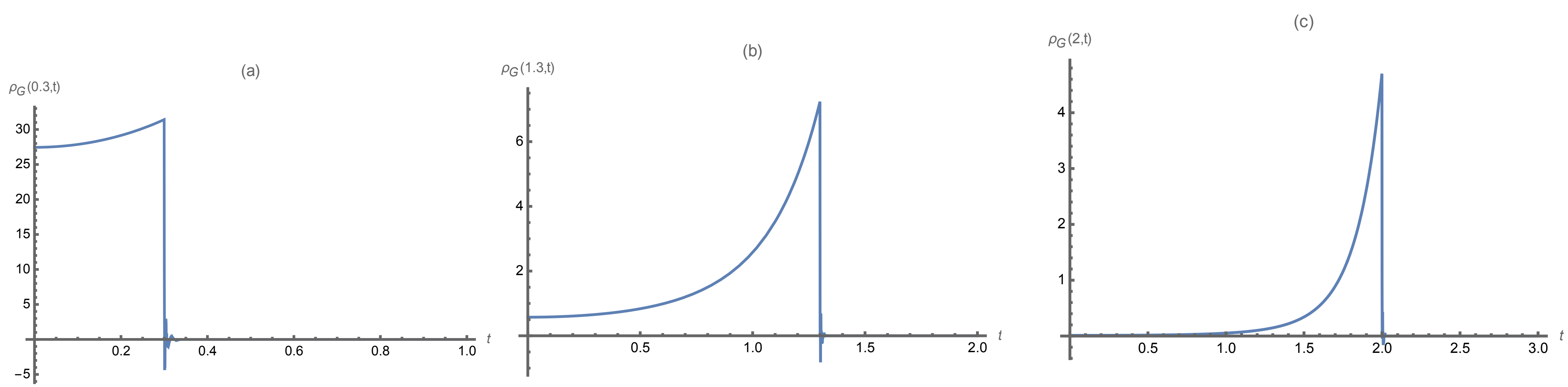
Use a clever wave packet to get 3 D density

$$\langle p' | \hat{\rho}(\mathbf{r}, t=0) | p \rangle = e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r}} 2\sqrt{\mathbf{q}^2/4 + M^2}\, F(q^2)$$

$$F(q^2) = \frac{1}{2\sqrt{\mathbf{q}^2/4 + M^2}(2\pi)^3} \int d^3r e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r}} \langle p' | \hat{\rho}(\mathbf{r}) | p \rangle$$

Left side is relativistic scalar- right side is not

$$\rho(t, r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{1} d\alpha \frac{1}{2} F((\alpha^2 - 1)\mathbf{q}^2) e^{i\alpha|\mathbf{q}|\, t}$$



Light front, infinite momentum frame: Obtaining Time-Independent, Two-Dimensional, Cylindrically-Symmetric Densities of Confined Systems of Relativistically Moving Constituents

Freese & Miller PRD 103,094023

$$\text{LF energy } P^- = \frac{\mathbf{p}_\perp^2 + M^2}{2P^+}, P^+ \to \infty, \text{no energy transferred}$$

Use wave packets to define transverse position

$$\int dx^- \langle \Psi | \hat{\mathscr{O}}(x^+, x^-, \mathbf{x}_\perp) | \Psi \rangle = \sum_{\lambda\lambda'} \int \frac{dP^+ d^2\mathbf{P}_\perp}{2P^+(2\pi)^3} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\mathbf{P}_\perp \cdot \Delta_\perp x^+/P^+} \langle \Psi | P^+, \mathbf{p}'_\perp, \lambda' \rangle \frac{\langle P^+, \mathbf{p}'_\perp, \lambda' | \hat{\mathscr{O}}(0) | P^+, \mathbf{p}_\perp, \lambda \rangle}{2P^+} \langle P^+, \mathbf{p}_\perp, \lambda | \Psi \rangle e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{x}_\perp}$$

Time dependence vanishes if $P^+ \to \infty$

Gaussian wave packet spatial

$$\int dx^- \langle \Psi | \hat{\mathscr{O}}(x^+, x^-, \mathbf{x}_\perp) | \Psi \rangle = (2\sigma)^2 \int \frac{d^2\mathbf{P}_\perp}{(2\pi)} e^{-2\sigma^2 \mathbf{P}_\perp^2} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \frac{\langle P^+, \mathbf{p}'_\perp, \Lambda | \hat{\mathscr{O}}(0) | P^+, \mathbf{p}_\perp, \Lambda \rangle}{2P^+} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{x}_\perp} e^{-\frac{\sigma^2}{2}\mathbf{\Delta}_\perp^2}$$

σ is width of transverse position, take to 0 *after* integration

If matrix element has no $p_\perp$ factors

$$\rho_{\mathscr{O}}(\mathbf{x}_\perp) = \int \frac{d^2\Delta}{(2\pi)^2} \frac{\langle P^+, \mathbf{p}'_\perp, \Lambda | \hat{\mathscr{O}}(0) | P^+, \mathbf{p}_\perp, \Lambda \rangle}{2P^+} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{x}_\perp}$$

2 D Fourier T of Form Factor

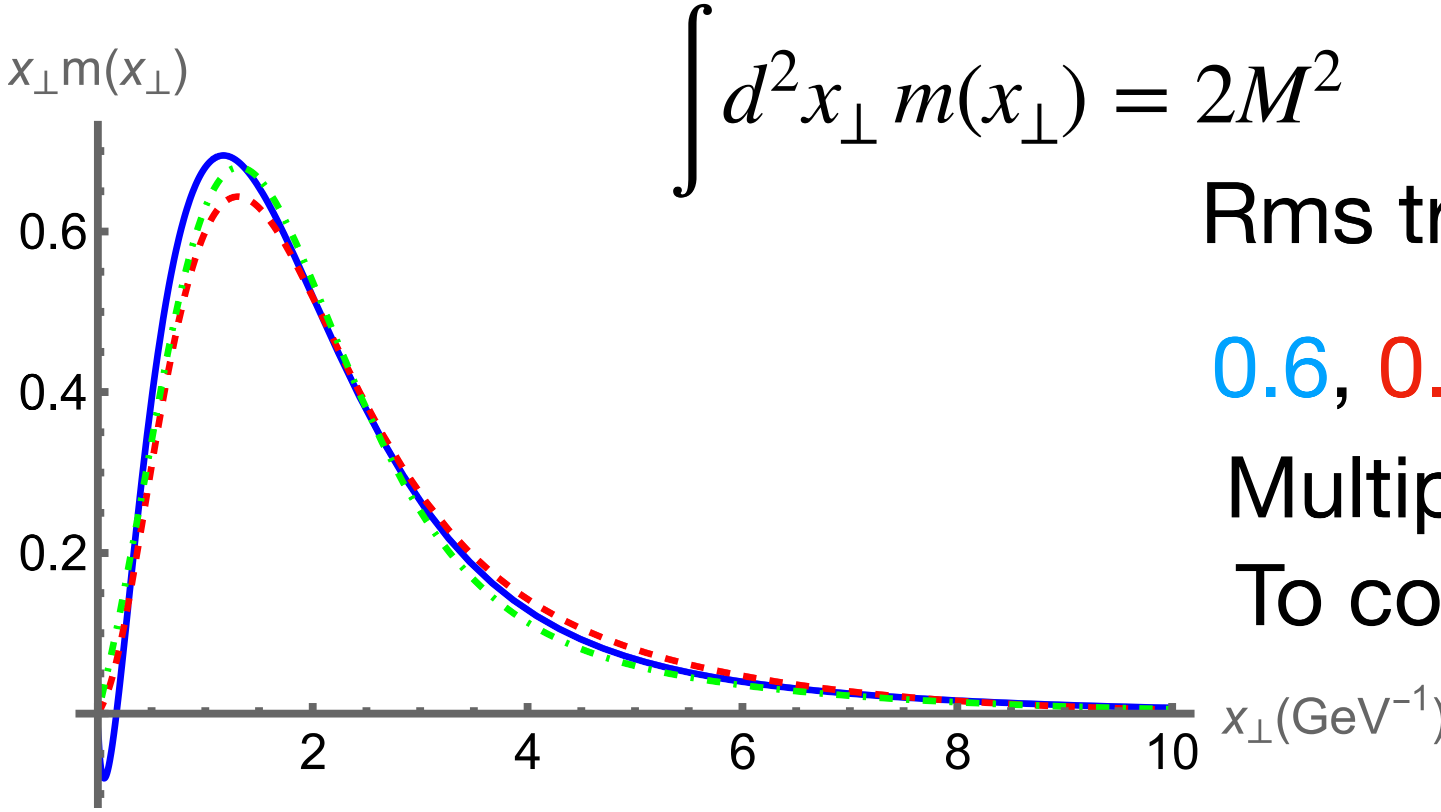
Gravitational mass density from trace of EMT

$$\bar{u}(p',\lambda)T^{\mu}_{\mu}(0)u(p,\lambda) = 2MG_s(\Delta^2) = 2(M^2-\Delta^2/4)A(\Delta^2) + J(\Delta^2)\Delta^2 - \frac{3\Delta^2}{4}D(\Delta^2)$$

Mass density

$$m(x_\perp) \equiv \frac{P^+}{M}\int dx^-\langle\Psi|T^{\mu}_{\mu}(x^-,\mathbf{x}_\perp)|\Psi\rangle = \int\frac{d^2\Delta_\perp}{(2\pi)^2}2MG_s(\Delta^2)\,e^{-i\Delta_\perp\cdot\mathbf{x}_\perp}$$

$$\int d^2x_\perp\, m(x_\perp) = 2M^2$$



Rms transverse radii

0.6, 0.54,0.52 fm

Multiply by $2/\sqrt{3} = 1.15$

To compare with 3 D still small

Mass radius < electromagnetic radius

Form factors from lattice

Hackett, Pefkou, Shanahan PRL,132,251904

Summary

Impossibility of obtaining time-independent, three-dimensional, spherically-symmetric densities of confined systems of relativistically moving constituents

Can get time-independent, two-dimensional, cylindrically-symmetric densities of confined systems of relativistically moving constituents

Mass density can be defined and lattice gives radius smaller than electromagnetic radius