

# Lattice data

- Wilson-clover fermion on 2+1 flavor HISQ configurations.

$$a = 0.06 \text{ fm}$$

$$a = 0.04 \text{ fm}$$

$$P_z = 0 \text{ GeV}$$

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$$48^3 \times 64$$

$$64^3 \times 64$$

$$m_\pi = 300 \text{ MeV}$$

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- X. Gao, et al., PRD102 (2020).

- RI'/MOM renormalization factors.

$$n_z = (p^x, p^y, p^z, p^t)$$

$$= \{ \{2, 2, 4, 2\}, \{2, 2, 5, 2\}, \{2, 2, 6, 2\}, \{3, 3, 4, 0\}, \{3, 3, 5, 0\}, \{3, 3, 6, 0\}, \\ \{3, 3, 4, 3\}, \{3, 3, 5, 3\}, \{3, 3, 6, 3\}, \{4, 4, 4, 0\}, \{4, 4, 5, 0\}, \{4, 4, 6, 0\}, \\ \{4, 4, 4, 4\}, \{4, 4, 5, 4\}, \{4, 4, 6, 4\} \}$$

$$p_R(a = 0.04) = (4, 4, 4, 4) \propto (9.8, 9.8, 9.8, 9.8)$$

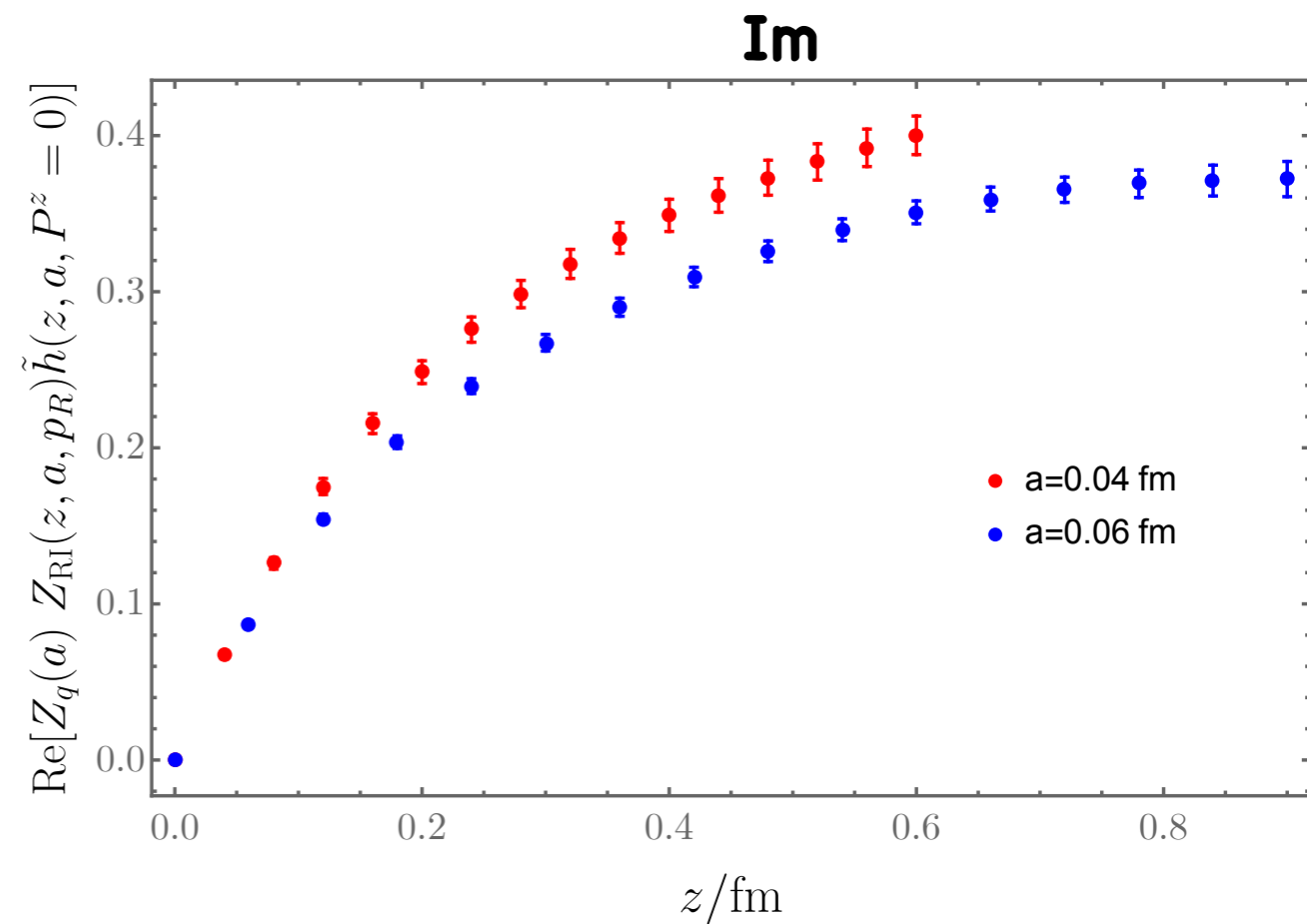
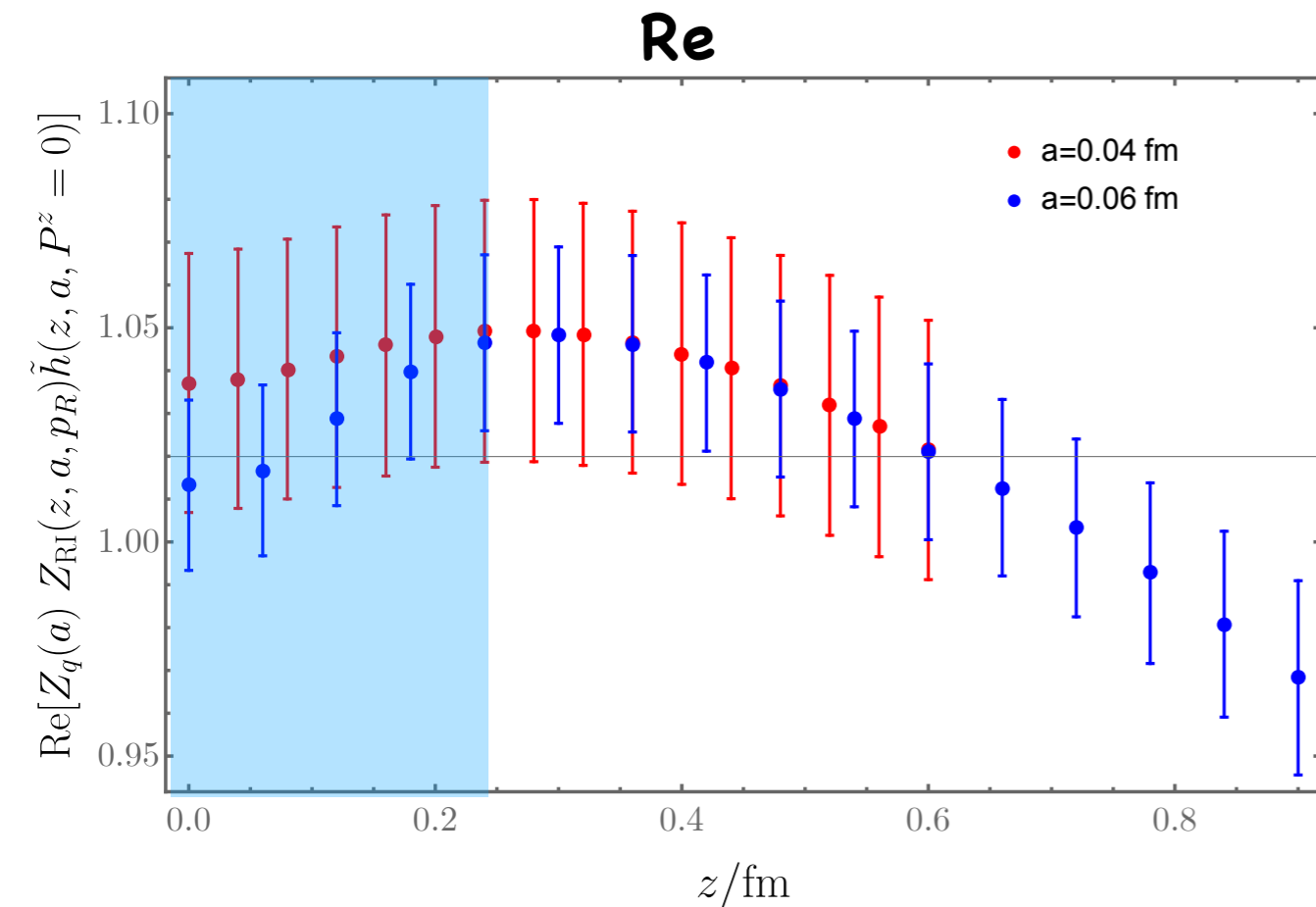
$$p_R^2(a = 0.04) \propto 19.635$$

$$p_R(a = 0.06) = (4, 4, 6, 4) = (8.7, 8.7, 13.1, 6.5)$$

$$p_R^2(a = 0.04) \propto 19.144$$

# RI'/MOM renormalized matrix element

$$Z_q(a) Z_{\text{RI}}(z, p_R, a) \tilde{h}(z, P^z = 0, a)$$



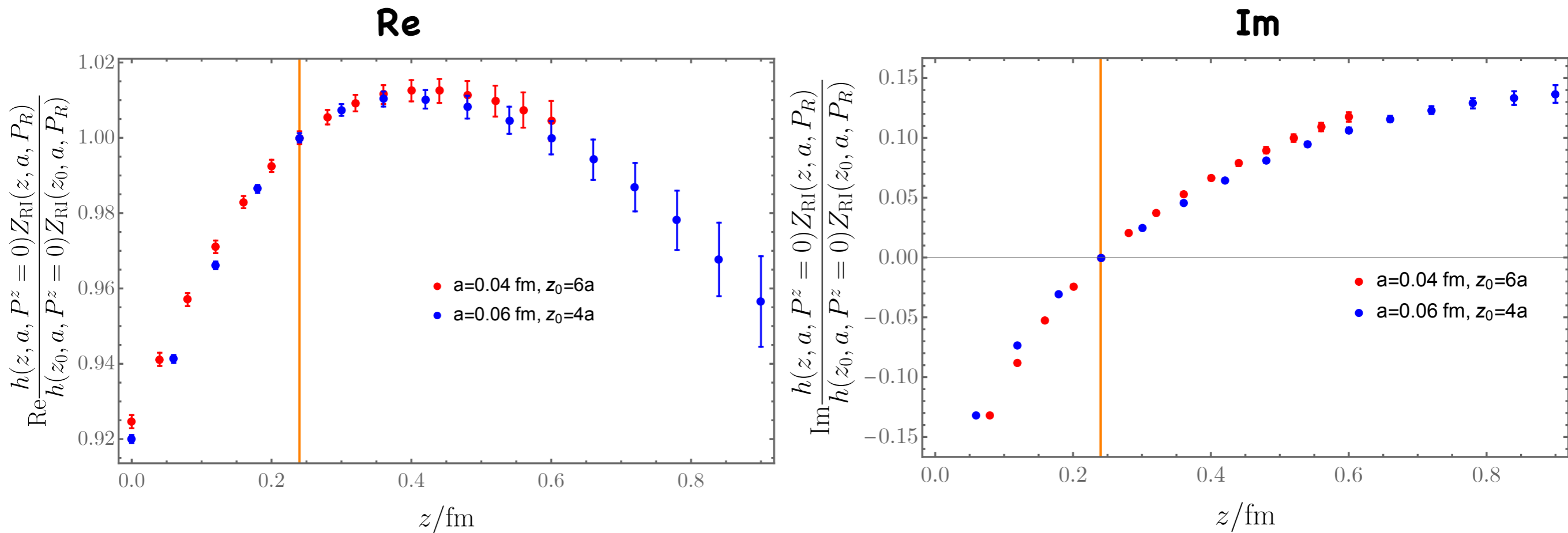
- Error dominated by that of  $Z_q(a)$ ;
- Discretization effect at small  $z$  considerable;
- Can we normalize to 1.0 at  $z=0$  by using the  $z=0$  matrix element?

$$Z_q = Z_q^0 + B/(ap_R)^2 + C(ap_R)^2(1 + c_4\Delta^{(4)} + c_6\Delta^{(6)})$$

$$Z_q \Big|_{a=0.04 \text{ fm}} = 1.03(3) \quad Z_q \Big|_{a=0.06 \text{ fm}} = 1.02(2)$$

# Ratio of RI'/MOM renormalized matrix elements

$$\frac{Z_q(a) Z_{\text{RI}}(z, p_R, a) \tilde{h}(z, P^z = 0, a)}{Z_q(a) Z_{\text{RI}}(z_0, p_R, a) \tilde{h}(z_0, P^z = 0, a)} = \frac{Z_{\text{RI}}(z, p_R, a) \tilde{h}(z, P^z = 0, a)}{Z_{\text{RI}}(z_0, p_R, a) \tilde{h}(z_0, P^z = 0, a)}$$



- Subpercent level difference mainly because the RI'/MOM renormalized matrix element differs by less than 1% at  $a=0.06\text{fm}$  and  $a=0.04\text{fm}$ .

# Wilson-line mass renormalization

- Check of continuum limit:

$$O_B^\Gamma(z, a) = e^{\delta m |z|} Z_{j_1}(a) Z_{j_2}(a) O_R^\Gamma(z)$$

$\delta m$  determined from static quark-antiquark potential.

A. Bazavov et al.,  
TUMQCD, PRD98 (2018).

$$\lim_{a \rightarrow 0} e^{-\delta m(z-z_0)} \frac{\tilde{h}(z, a, P^z = 0)}{\tilde{h}(z_0, a, P^z = 0)} = \text{finite} \quad z, z_0 \gg a$$

