Remarks on observations of lattice results

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Non-perturbative renormalization

- The quasi-LF correlation operator is multiplicatively renormalized Ji, JHZ, Zhao, PRL 18', Ishikawa et al, PRD 17', Green et al, PRL 18'

$$[\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_B = e^{\delta m|z|}Z\,[\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_R$$

- UV divergences independent of external state, can be removed as a whole by dividing different quantities
  - RI/MOM: off-shell quark state Alexandrou et al, NPB 17', Stewart, Zhao, PRD 18'
  - Ratio: hadron state at zero momentum Radyushkin, PRD 17'
  - VEV: vacuum expectation value Braun, Vladimirov, JHZ, PRD 19', Li, Ma, Qiu, 20'

- Linear/logarithmic divergences can be removed separately, with δm being fitted from
  - Wilson loop Musch et al, PRD 11'
  - Gauge-fixed Wilson line… Green et al, PRL 18'

Brief summary of observations

- Caution: all observations remain to be tested by independent calculations of other groups

- Hadronic matrix elements of $O_{\gamma_t} = \bar{\psi}(z)\gamma_t W(z,0)\psi(0)$ have universal UV divergences
  - Tested on clover and overlap fermion, for pion and nucleon

- RI/MOM, VEV of O_{γ_t}, mass counterterm fitted from Wilson loop or gauge-fixed Wilson line do not seem to work well for cancellation of linear divergence
  - Potentially plagued by computational systematics, bad signals, chiral symmetry breaking effects…

Viable renormalization strategies

- Hybrid renormalization with ratio scheme at short distance
  Ji, JHZ et al, NPB 21'

$$\tilde{h}^R(z,a,P_z) = \frac{\tilde{h}(z,a,P_z)}{Z_X(z,a)}\theta(z_S - |z|) + \tilde{h}(z,a,P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S,a)\theta(|z| - z_S)$$

- Self-renormalization LPC (Huo, Su et al), 2103.02965
  - Renormalization factor can be extracted from the bare hadron matrix element alone

$$\ln \mathcal{M}(z,a) = \frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + g(z) + f_{1,2}(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\rm QCD}]}\right] + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\rm QCD})}\right],$$

$$g(z) = m_0 z + g'(z)$$

  - Eliminate all divergences and discretization errors if data at several lattice spacings are available
  - Independent of valence fermion formulation and smearing

Discussions

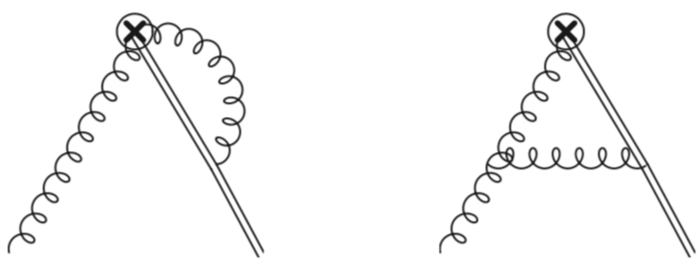
- Other renormalization options may also be used once we understand what is happening with lattice simulations

- Taking the RI/MOM matrix element as an example
  - It exhibits different divergent behavior from the hadronic ones, in particular at small lattice spacings, no matter whether chiral fermions are used or not

- Natural to expect that the problems are related to the off-shellness of external states
  - Gauge variance
  - Mixing with contributions that vanish in physical external states

Discussions

- Power divergences are sensitive to breaking of gauge symmetry
- Example: Gluon quasi-PDF
  - Additional linear divergence apart from those originating from Wilson lines observed in early calculations, which turn out to be artifacts JHZ et al, PRL 19', Li et al, PRL 19', Wang, JHZ et al, PRD 19'



$$I_1 = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{4-d}(A_a^\nu n^\mu - A_a^\mu n^\nu) n\cdot\partial \mathcal{Q}_a/n^2 - \frac{\pi\mu}{3-d}(n^\mu A_a^\nu - n^\nu A_a^\mu)\mathcal{Q}_a + reg.\right\},$$

$$I_2 = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{4-d}\left[\frac{1}{4}F_a^{\mu\nu}\mathcal{Q}_a + \frac{1}{2}(F_a^{\mu\rho}n_\nu n_\rho - F_a^{\nu\rho}n_\mu n_\rho)/n^2 + \frac{1}{2}(A_a^\mu n^\nu - A_a^\nu n^\mu) n\cdot\partial\mathcal{Q}_a/n^2\right]\right.$$
$$\left. + \frac{\pi\mu}{3-d}(n^\mu A_a^\nu - n^\nu A_a^\mu)\mathcal{Q}_a + reg.\right\},$$

Discussions

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- It is worth investigating the quark RI/MOM matrix element following a similar strategy
  - Beyond one-loop in continuum
  - Beyond one-loop in lattice perturbation theory

Lattice action

- Clover action ($O(a)$ improvement)

$$S_q^{\mathrm{clv}} = S_q^{\mathrm{w}} + a c_{\mathrm{sw}} \sum_x \bar{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) \delta_{x,y} \psi(y)$$

- m^{cri} can be suppressed from $O(\alpha_s/a)$ to $O(\alpha_s^2/a)$ with a suitable clover coefficient
- But there are other dim.-5 operators

$$\begin{aligned}
L_1^{(1)}(x) &= \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \; , \\
L_2^{(1)}(x) &= \overline{\psi}(x)\overrightarrow{D}_\mu(x)\overrightarrow{D}_\mu(x)\psi(x) + \overline{\psi}(x)\overleftarrow{D}_\mu(x)\overleftarrow{D}_\mu(x)\psi(x) \; , \\
L_3^{(1)}(x) &= m \; \mathrm{tr}\left[F_{\mu\nu}(x)F_{\mu\nu}(x)\right] \; , \\
L_4^{(1)}(x) &= m \left(\overline{\psi}(x)\gamma_\mu\overrightarrow{D}_\mu(x)\psi(x) - \overline{\psi}(x)\gamma_\mu\overleftarrow{D}_\mu(x)\psi(x) \right) \; , \\
L_5^{(1)}(x) &= m^2 \, \overline{\psi}(x)\psi(x) \; .
\end{aligned}$$

- Elimination of which requires using equations of motion or rescaling of bare coupling and mass

Auxiliary heavy quark propagator

- The auxiliary heavy quark satisfies

$$(n \cdot D - \delta m) S_Q(x,y) = \delta^4(x-y)$$

- with the solution

$$S^0_Q(x,y) \sim e^{\delta m (x^z - y^z)} \theta(x^z - y^z) \delta(x^0 - y^0) \delta^2(\vec{x}_\perp - \vec{y}_\perp) L(x^z, y^z)$$

- There could be additional terms Maiani et al, NPB 92'

$$S_Q(x,y) \sim e^{\delta m (x^z - y^z)} \theta(x^z - y^z) \delta(x^0 - y^0) \delta^2(\vec{x}_\perp - \vec{y}_\perp) \{ L(x^z, y^z) + \frac{\mathcal{S}^1(x^z, y^z)}{2\delta m} + \ldots \}$$

$$\mathcal{S}^1(x^z, y^z) \sim \int_{y^z}^{x^z} dw \, L(x^z, w) D_i^2(x, w) L(w, y^z)$$

- Perturbative corrections lead to results like

$$e^{\delta m (x^z - y^z)} \theta(x^z - y^z) \delta(x^0 - y^0) \delta^2(\vec{x}_\perp - \vec{y}_\perp) [1 + g^2 (\frac{X}{a} + \frac{W}{2\delta m a^2})(x^z - y^z) + g^2 (Y + \frac{X}{\delta m a})]$$

- $1/a^2$ terms in the 2nd piece absorbed into mass, $1/a$ in the 3rd piece into heavy quark WF renormalization
- But for $\delta m \sim 1/a$, $1/(\delta m a)$ becomes finite

Operator mixing

- In auxiliary field language, local operators $\bar{q}(z_2)Q(z_2)$, $\bar{Q}(z_1)q(z_1)$ are of the lowest mass dim., mixing with higher-dim. operators suppressed as $O(a^n)$ effect
- In coordinate space, the one-loop correction to quark bilinear operator contains the following structure

$$-ig^2 C_F \frac{-i\pi^{d/2}}{8\pi^d} \int_0^1 dt \int_0^1 d\alpha \bar{q}(v\lambda) \not{v} \Big[v^2 t^{2\epsilon-1} \lambda^2 q(\alpha t v \lambda) \frac{\Gamma(1-\epsilon)}{(-v^2\lambda^2)^{1-\epsilon}}$$
$$-\lambda \frac{\bar{\alpha} t^{2\epsilon}}{2} \gamma^\nu (v\cdot\gamma) \partial_\nu q(\alpha t v \lambda) \frac{\Gamma(-\epsilon)}{(-v^2\lambda^2)^{-\epsilon}} \Big].$$

- The 1st term is an operator of the same mass dimension
- The 2nd term involves an operator of higher mass dimension multiplied by the distance $z^\mu = \lambda v^\mu$